PROCEEDINGS
Jorge Nuno Silva (Ed.)

BOARD GAME STUDIES

COLLOQUIUM XI

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Jorge Nuno Silva (Ed.)

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Contents

Introduction

Jorge Nuno Silva

The *Board Game Studies Colloquium XI*[1] took place in the Museum of Science of the University of Lisbon, organized by the Ludus Association, in April of 2008.

The sequence of yearly colloquia, started in London in 1990, had one more term. It is hard to put in words the meaning that this event had for the Portuguese organizers. The subject of Bord Games is an emergent one in Academia and the University of Lisbon is proud to have played a small role in its development.

This colloquium was one of the activities linked with games that the Ludus Association has been recently engaged in. The exhibition *Mathematical Games Throughout the Ages*[2], that was first seen by the participants, the organization of the *Portuguese Championship of Mathematical Games*[3] deserve mention.

Besides the cultural and technical studies that Board Games suggest, we promote the practice of the best ones among the Portuguese school community.

The next pages give an overview of the talks we attended. The subjects are scattered, we invite the reader to jump immediately into any appealing paper.

[1] http://ludicum.org/bgs08/
[2] http://wwmat.mat.fc.ul.pt/~jnsilva/Exhibition/index.html
[3] http://ludicum.org/cnjm

The Sociable *Game of the Goose*

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This is an expanded version of the paper presented to the Board Game Studies Colloquium in Lisbon in April 2008.

References in the form [Ciompi n] are to the Goose-*game site set up and maintained by Dr Luigi Ciompi at* `http://www.giochidelloca.it` *where legible images and further information on each particular game may be viewed by typing the number n into the* **codice** *field on the* **archivio** *page.*

Overview

The Game of Goose *(Jeu de l'Oie, Giocodelloca, etc.)* is a simple race game played with dice on a spiral track, with the usual tokens, the aim being to arrive exactly at the winning space, numbered 63 in traditional versions. The track is provided with favourable spaces, each traditionally marked with a goose, and with hazards, involving payment into the winner's pool, and in most cases delaying the player's progress. In this traditional form, the game is one of pure chance, the movement of the tokens being entirely determined by the throw of the dice. Indeed, the course of the game can readily be simulated by computer, with no human interaction [Seville, 2001].

However, there do exist variant versions, in which the human aspect of the players becomes significant and the players are required by the rules to interact in ways that are not confined to movement of a token according to the dice throw or the payment of determined stakes. Into this category fall many, but not all, educational variants. There are also a few games in which different rules apply according to the gender of the player. Then, there are games in which the player is called upon to undertake the playing of a particular role, with or without influence on the actual play. And finally there are games — such as those involving forfeits — where the player is called upon to perform a particular action. These last exhibit some crossover into the category of party games (jeux de societé).

This paper discusses these variant categories, placing them in their historical and social context against the background of the development of race games over four centuries, and briefly indicates the relevance of this analysis for the modern designer of games.

The traditional Game of *Goose*

Before discussing variant forms, it is helpful to review the social context of the game in its 'traditional' form [Seville, 1999]. The *Game of Goose* is historically the most important spiral race game ever devised. It has its roots in the Italy of Francesco de Medici (1574–87), who, as [Carrera, 1617] reports, sent it as a present to King Philip II of Spain. The game took hold there and elsewhere in continental Europe, where it is still played. When John Wolfe introduced it into England on the 10th June 1597 [Stationers' Hall register, London] it was called *The Newe and most Pleasant Game of the Goose*, though its princely roots led it often to be labelled as 'Royal', as in the oldest surviving English board: *The Royall & Most Pleasant Game of Ye Goose* printed by John Overton at the Black Lion in Exeter Exchange [in the Hannas collection sold at Sotheby's, London in 1984 and dated in the catalogue as c. 1660]. Although the game was regarded as a suitable diversion for a Dauphin of France, the game had also a popular following and was regularly played for gambling stakes by men in taverns.

Figure 1 Goose as a gambling game in a tavern — detail from *Het nieuw en vermaekelyke GANSEN-SPEL*, Charles de Goesin-Disbecq, Ghent, end 17th C (author's collection [Ciompi 992])

Over the years, this rough gambling element diminished and the game became one that could be played in respectable mixed company [Figure 2] or indeed in the family with children [Figure 3], until by the 20th century it became regarded as a children's game [Figure 4].

Figure 2 Goose in respectable mixed company — detail from Bowles's *ROYAL and Entertaining GAME of the GOOSE*, Carington Bowles, London, mid 18th C (author's collection [Ciompi 927])

Figure 3 Goose as family entertainment — detail from *Nieuw Vermakelijk Ganzenspel*, Vlieger, Amsterdam, late 19th C (author's collection)

Figure 4 Goose as a children's game — detail from *Het aloude Ganzenspel*, Daan Hoeksema, Netherlands, early 20th C (collection of Christine Sinninghe Damsté)

Variants of *Goose*

The question arises as to what games can properly be regarded as variants of traditional *Goose* [Seville, 1999]. The present paper is essentially concerned with unicursal race games played with dice or an equivalent random number generator such as a teetotum (a spinner in the form of a small top). Very many such games are derived from *Goose*, though not all: for example, *Snakes and Ladders,* which may be regarded as unicursal if the ladder/snake excursions from the track are implemented by forward or backward movement, has a separate historical pedigree of comparable age [Parlett, 1999].

Perhaps the most compelling evidence of the influence of *Goose* is if the game has favourable spaces embodying the *Goose* rule of moving the token past the **goose** space to the extent of the throw. Another is the reverse overthrow rule, seen in most versions of the traditional game (though not in Spain), whereby a player overshooting the winning space must count backwards from it until the throw is fully used. Again, the hazard spaces in their particular rules and/or iconography may reflect those in the traditional game, most notably **death,** on space 58, which requires the player to begin the game again, or the **prison** or the *well,* both of which require the player to remain unless and until rescued by another, who must then suffer the same fate. In the examples set out below, the extent to which each game resonates with traditional *Goose* will be indicated.

Educational Race Games

Educational race games based on *Goose* are a French invention of the 17th century. The earliest known game of this type is Mariette's *Jeu Chronologique,* dated 1638 [D'Allemagne, 1950, p. 44] designed to teach History. It was followed by games designed to teach Geography, the Arts of War, Heraldry — indeed, all the accomplishments required of the noble cadet class studying in the colleges of France. These were expensive games produced from finely engraved copper plates, predominantly in and near the Rue St. Jacques in Paris, as opposed to the more down-market provincial productions of games for amusement, from woodcut blocks.

It is evident from these games that some participation was expected of the players. For example, in Duval's *Le Jeu des Princes de L'Europe,* published by Nicolas Berey in 1662 [Ciompi 541], each of the 63 circular spaces of the spiral track is a small map of a region or country of Europe, with France as the winning space. A map of Europe is in the centre. A note to the rules advises as follows: "He who would take some profit in Geography concerning the knowledge of Europe should take care to say the names of the countries where he arrives and to read those of the towns that are found there" [present author's translation]. Duval's game is clearly based on *Goose*, as is evident from the careful grouping of countries and provinces to produce the canonical 63 spaces. And many of the particular rules for the individual spaces derive from this source, for example **Candie** at No. 57: Must be arrested to serve against the infidels and must stay until another takes his place (cf. the *Goose* prison rule). Duval's games also take from *Goose* the fact that their rules are concise enough to be included on the playing surface, so that the game could be played quite quickly, as compared with more erudite educational games where there was continual reference to a separate and often lengthy book of rules.

In England, the development of educational race games began about 100 years later than in France [Shefrin, 1999]. Whitehouse [1951] gives the first dated game of this kind as that invented by John Jefferys in 1759: *A Journey through Europe, or the Play of Geography,* published by Carington Bowles in London [Figure 5]

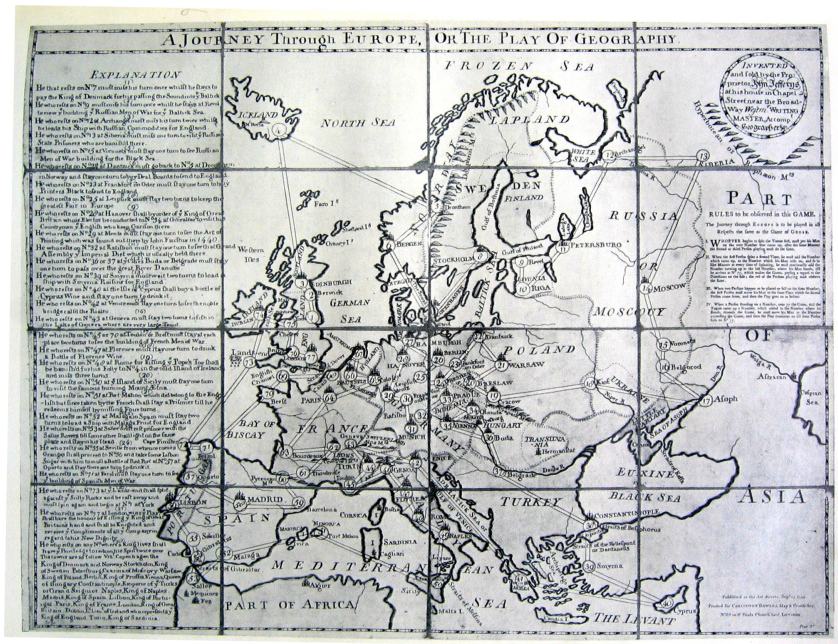

Figure 5 The earliest dated English educational game, John Jefferys' *A Journey through Europe, or the Play of Geography*, Carington Bowles, London, 1759 (Whitehouse, 1951, plate 1)

The rules to be observed in the game begin: "The Journey through Europe is to be played in all respects the same as the *Game of Goose*. Whoever begins to spin the Totum first must place his man on the very number that turns up...". The 'totum' (later known as the teetotum) was used because in this period the use of dice was frowned upon, as being associated with gambling and cheating. The rules then explain how the next move is to be made by adding the next number spun, that the directions given for the various places encountered are to be followed, and that if the number spun carries a man past the winning space (London, at 77), the excess is to be counted negative — i.e., reverse overthrows are played, just as in the *Game of Goose*. Indeed, the game has considerable structural similarity to traditional *Goose*. The doubling forward of the throw when a goose space is encountered is found in Jefferys' game, where the equivalent spaces are '...any number where a King lives', and the player who lands on such a number has the privilege of reckoning his spin twice over. The rules for the individual hazard spaces are instructive, for example: "he who lands on No. 42 Venice must stay one turn, to see the noble bridge called the Rialto". But no player-participation is required to reinforce the educational message. There is, though, an element of role-playing for the winner: "He who rests on No. 77 at London wins the play, shall have the honour of kissing the King of Great Britain's hand and shall be knighted and shall receive the compliments of all the company in regard to his new dignity".

Before the end of the century, English educational games were beginning to follow the lead of the French in requiring active participation from the players. For example, in Wallis and Newbery's *Royal Genealogical Pastime of the Sovereigns of England* [Ciompi 980], published in 1791, the rule is expounded as follows: "As an encouragement to the player for the attention he may pay to the useful Science of Genealogy, he will be entitled to move one number forward when he can tell without looking into the description of the game what King immediately preceded and followed that number on which chance may have thrown his pyramid; and if he can tell the date in which such King was born began his reign and how long he reigned he shall be allowed to move one number more forward." This game, though it is deliberately made to look like a genealogical chart, is in fact a unicursal race game, played with 'pyramids' as moveable tokens and using an eight-sided 'totum' in place of dice. The educational 'utility' of the game is clearly set out: "This being a scientific game in which the amusement and the instruction of the parties are equally considered, we hope that the young player will not think much of exercising his memory to acquire a perfect knowledge of it. Most games are calculated only to promote little arts and cunning but this, while it will undoubtedly amuse, will not a little contribute to make the players acquainted with the genealogy of their own King" (George III — the winning space at No. 52). Reverse overthrows are played, as in *Goose*. There is no equivalent of the *Goose* doubling rule but landing on Henry VIII results in the instruction: "as his treatment of his queens was so unjustifiable, the player must go back to No.1" — recalling the **death** rule and being an early example of "go back to square one".

The use of a separate rule booklet, following French precedents of the 17th century, is common in the many English games intended to teach history. Much more detail could be given in this form than could be shown on the playing surface but at the cost of slowing down the game considerably. Typical of such games is Wallis' *New Game of Universal History and Chronology* [Ciompi 854]. Published in 1814 with George Prince Regent as the winning space at No. 138, the game was accompanied by a 24-page booklet, of which pages 3 to 17 set out the rules to be observed on the individual spaces, which constitute a chronological track of historical events. The pages that follow give an 'Outline of History' associated with certain of the historical events that are judged to be particularly important, with the rule: "Where a player is directed to read the history of an event, in another page, he shall have the privilege, after so doing, of spinning again, and be rewarded with a counter from each player". The choice of important events is to our modern minds a little obscure, e.g. No. 5 Babylonish and Assyrian monarchies founded A M 1787. However, none can quarrel with the selection of No. 10 Birth of Moses AM 2433 or No. 16 Birth of Homer ("if you can say who he was and what he wrote receive 2 from each player; otherwise place 6 [counters] on 13 [Trojan War] and learn there"). The selection of the Birth of Mahomet (No. 58) as an important historical event is an encouraging indication of some breadth of religious understanding in Regency times; but this positive view was evidently too challenging for the owner of the present author's copy of the booklet where the relevant rule has a chilling manuscript addition — begin again. This is an example of another kind of social interaction with the game — the imposition of local or family rules.

Our final example is a game published in Germany in about 1933: the *Reise der Deutschland (Tour of Germany)* [Ciompi 526]. At first sight, this looks like so many of the games published from the end of the 19th century to promote tourism, especially in Switzerland. But there are differences! First, this is the divided Germany that resulted from the Treaty of Versailles, after World War 1, as sown by the swathe of white that represents the Polish (Danzig) Corridor. It is significant that the 'Tour' swings confidently through this region (by motor bus and with the appropriate documentation). Next, though there is a general atmosphere of rural life, supported by the images of country workers in traditional costume, a warship lurks in the Baltic Sea. And the detailed account of the tour begins in Berlin, with reference to the opening by Hitler of the first Reichstag of the Third Reich. This game evidently had a political message — but, as the rules printed on the envelope make clear, the message was not aimed at the tourist. At various points in the tour, the player is required to repeat accurately a short verse. For example, at No. 19, on the River Oder, two piles of crops are shown, one labelled 'with potash' and the other, a smaller one, 'without potash'. The player landing there is required to repeat after the leader (the oldest player):

> *Kartoffeln, Futter und Zuderrueben*
> *Kali und Stallmist besonders lieben.*

(potatoes, mangold and sugar beet like potash and stable-manure very much). If this is done without error, the player advances to No. 24. The game is in fact a promotional device for fertilisers produced by the German firm of Kainits. The social interaction in this game is evidently complex, with messages at various levels being imparted, and reinforced through authority of seniority in the social circle.

Over the centuries, therefore, educational race games have developed from those in which learning is incidental to the enjoyment of play, to those in which success in learning is tested with rigour and rewarded by favourable moves in the game itself. And, like other effective learning tools, they can be used to promote a variety of ends, not all of them necessarily overt or altruistic.

Male/Female rules

A few race games have rules that differ according to the gender of the player. One might expect the various games of courtship and matrimony to fall into this category but not all do. Thus Crepy's *Nouveau Jeu de l'Himen* (Paris, 1725) [Ciompi 790] has no such differences of rule. However, the same firm's *Les Etrennes de la Jeunesse* (1713) [Ciompi 921] has different tracks for the two sexes and markedly different rules: this game is treated below as a game of forfeits (section 7). In the 18th century English game of *Courtship and Matrimony* [Ciompi 978] (publisher unknown) there is an interesting and highly thematic rule difference. Though the track is 64 spaces in length rather than 63, this game is clearly derived from *Goose*, the favourable spaces where the throw is counted again being denoted by the titles of popular ballads of the period, several of which are familiar as the tunes used in John Gay's *Beggars' Opera* of 1728. Most spaces are governed by rules that are not gender specific but the **Prison** at No. 55 has the following rule: "Pay 1 into ye pool stand there & lose 3 turns of throwing unless released by another coming in. N.B. If one of ye other sex comes in it is a Fleet marriage and you win the Game and divide the pool" (A Fleet marriage was a marriage which took place in the Fleet Prison in London, which claimed to be outside the jurisdiction of the Church. Disgraced or pretending clergymen often conducted them, for a fee. Such marriages were in fact legal until the Marriage Act of 1753).

The Dutch game of *Sint Nicolaas* (Saint Nicholas) first published by G Theod. Bom about 1858 [Ciompi 832] affords an example of another kind. This is a 63-space game with favourable spaces of the *Goose* type, marked by boots and shoes containing the presents traditional in Holland for the season of the Saint's day, in December. As is usual with *Goose*-

games, where the geese are spaces at intervals of 9 there are special rules to deal with an initial throw of 9, which would otherwise give an immediate win by jumping all the way to 63. The standard *Goose* rule is that if the throw, with double dice, is by 6 and 3, the move is to space 26, whereas if it by 5 and 4, the move is to space 53; these two spaces are traditionally marked with images of dice, to remind the players of the rule. However, in the *Sint Nicolaas* game, the special rule is that an initial throw of 6 and 3 leads to space 25 if the thrower is a man and to space 26 if a woman: these two spaces respectively are marked with a young woman and a young man, both eminently marriageable. But an initial throw of 5 and 4 leads to space 51 if the thrower is a man and to space 53 if a woman: these two spaces respectively are marked with an old woman and an old man, both well beyond the age of marriage. These throws would no doubt have led to much general amusement among the company.

An example of a different kind is provided by *The New Royal Game of Goose* a 19th century English game [Ciompi 586] where the track is in the form of a goose. Although the track has 63 spaces and does have **Goose** spaces (though not at traditional numbers), the rules are very idiosyncratic. One such rule is at space 57, which shows a man with a pipe: here, the rule is that the player 'must, unless a lady' go back to 47 — plainly, no lady ever smoked a pipe!

These male/female rule variations, though rare, are interesting in confirming that the games concerned were intended to be played in mixed company, even at periods of history when there may have been disapproval of dice games and gambling generally.

Role-playing

An extension of the recognition of specific male/female roles is role-playing of a more diverse kind. One might have thought that race games such as *Goose* would lend themselves to such variations, with corresponding variant rules that would perhaps wittily reflect the role assumed by, or assigned to, the individual player. However, such is the strength of tradition — fortified by the merit of the traditional rules in furnishing an exciting but fair game — that examples are hard to find in close variants of *Goose*, notwithstanding the huge range of thematic treatments over several thousands of published games.

Thus, in the football game, *Guioco del Calcio* (Marca Stella, 1920)[Ciompi 113], suitable roles are assigned to players dependent on their initial throw e.g. double six leads to space 34 (of 56) and the role of *Arbitro* (referee) is assigned; roles of Captain etc are similarly assigned. Yet it is specified that the players assigned these roles thereafter follow exactly the same rules as everyone else.

For role-playing with appropriate and specific rules, we may look to a race game published by the firm of Spear in several countries and languages: *Cat and Mouse* (c. 1920). In this simple race game, playing figures of two cats and four mice are provided as tokens. The specific rules differ for the two species: for example, when a mousetrap is encountered, the mouse is out of the game but the cat proceeds without hindrance.

A variation on this theme by the Swiss firm of Karlit, published in the 1950s, is interesting because of the ingenious mechanism by which the role-playing is enforced. Here, the cat is represented by a large marble whereas the mice are small marbles. The track is composed of holes of two sizes punched in the horizontal playing surface, which is raised above the base of the box. When a mouse encounters a trap (large hole) it falls through, whereas the cat remains safe; both mouse and cat are safe on the small holes.

It is interesting to reflect on the paucity of examples of role-playing in *Goose* variants. Perhaps one reason may be that to alter the rules non-uniformly will give one or another player an advantage that the remainder will perceive as 'unfair'. True, it is difficult to construct diverse rules within a single game that would all be fair with respect to each other, especially in a game as complex as *Goose*, with its traps, delays and re-starts. However, if the discrepancy were not gross, then some unfairness might add spice to the game e.g. by winning when in a role that was (or was thought to be) unfavourable.

Forfeits and Actions

As mentioned above, there have been attempts to combine race games with forfeits and other human actions, though it has to be admitted that some of the resulting games are far from *Goose* both in concept and in playing terms

In Crepy's *Les Etrennes de la Jeunesse* (Paris, 1713) [Ciompi 921], the females ('Dames', further referred to as shepherdesses) play on the left-hand of two circular tracks, while the males ('Cavaliers' or shepherds), who must be in equal number, play on the right. The two circles touch (one is tempted to say 'kiss'!) in the middle of the sheet, and it is here that the winning spaces are, each being marked with a crowned heart. Initially, each 'shepherdess' chooses her 'shepherd' to sit on her left, the choice being first made by the highest thrower of the dice, and so on. The rules say that the game 'se gouverne à peu près de la jeu de l'oie'(the rules are quite like those of *Goose*) though in truth they are peculiar to the game. Typical of the rules is that for **inconstance** (inconstancy) on the left track. The unfortunate shepherd who lands on the butterfly that marks this space must submit to being tied to his chair by his shepherdess, using her scarf. But there are penalties for the ladies, too: at **La Jalousie** (jealousy), the jealous one must go and hide behind a curtain or half-open door, missing two turns and paying to the pool. The game ended for the males (for example) when one of them reached the crowned heart, where their circle touched the other. If no female had reached the corresponding heart in the other circle, the males waited until this had occurred. The two winners, on these two hearts, would then share the pool and 'seront unis ensemble' (will be joined together). Clearly, this was a game that depended for its success on having much leisure and the right company! However, the rules do contemplate the possibility of there being only two players, one of each sex, who then compete to see who reaches their crowned heart first and do not share the pool — the state of matrimony rather than courtship?

Forfeits are certainly an integral part of the game of *Din-Don ovvero Tutte le Strade conducono a Roma* (Ding-Dong or really All Roads lead to Rome — Caroccio, Milan, 1933)[Ciompi 316]. Here, the object is to get from a starting place to the centre space, Rome. The starting places are all Italian cities — but cities like Mogadiscio and Pola are included from the Italian colonies, reflecting the sense of empire that developed under the Fascist regime. It is not a unicursal game and is far from Goose in concept. It is played with a single die, two faces showing 'Din' (being favourable), two showing 'Don' (unfavourable) and the remaining two being blanks so that the player does not move. The game is evidently meant to be played in a bar, so that drinks can be bought when directed, and indeed several of the playing spaces are hostelries. A forfeit to be dreaded in such circumstances is shown by a pair of scissors, the meaning being: 'cut off the tongue', i.e. remain silent for the rest of the game — a truly terrible punishment!

A danger of combining human actions with the playing of a race game is that the game is apt to be slowed down unacceptably. An ingenious way of avoiding this trap was found by the inventor (P. Louwerse) of the Dutch game *Schoolmeester en Collectant* (Schoolmaster and Collector for Charity) dating from about 1875. Here, in addition to usual penalties such as paying to the pool, landing on certain spaces requires the player to 'sit with puffed cheeks' or 'with an agonised face', cry "ooh, ouch!" according to the scene depicted in the unfolding story. (The help of Christine Sinninghe Damsté in suggesting the inclusion of this game and translating the instructions is gratefully acknowledged).

Finally, in this short catalogue of games requiring human rather than mechanical actions, mention should be made of the Dutch games advertising biscuits. These were to be played with biscuits as stakes and indeed special biscuits were also made to serve as playing tokens. An example is the *Nutrix Kabouter Spel* (Leiden) [Ciompi 968], a unicursal game of 63 spaces depicting a fairyland journey, with various imaginative delights and perils, and quite evidently a *Goose* variant. Each player begins with 10 Nutrix biscuits and puts two of these in the pot. At various points on the journey, the player is directed to eat a biscuit from the pot; but a worse punishment is to have to eat two of one's own store of biscuits — and begin the game again.

Discussion

In surveying the field of goose-related race games, it is striking that — with the exception of educational games — so few of them call for any involvement of thought or judgement. Yet these games have been produced and marketed successfully in thousands of versions and millions of copies over four centuries. There is obviously a human need for 'non-mind' games that, like *Goose*, are cleverly constructed to provide uncertainty and excitement within a reasonably short — but not instant — time frame. This need is more complex than the gratification associated with gambling. It involves social interaction — competition, seeing who will win, learning how to win without offending others, how to lose with equanimity — and, of course, in a non-mind game, everyone is equal. *Goose* rules are therefore ideal for the family environment, where young and old can compete on equal terms. The design of the game provides variety of experience on the board: there are many but not too many favourable spaces and hazard spaces, nicely contrasted with the 'plain vanilla' spaces. The fear of the principal hazards — **well**, **prison**, **death** — may be acute but the penalties are not final and the unfortunate player is not banished from the game — indeed, he or she may even win from an apparently hopeless position. Being 'rescued' by another of the social group — perhaps someone who does not even like you — adds to the frisson. And the 'reverse overthrows' rule is brilliantly conceived to maintain tension to the end — there is none of the boring waiting for an exact throw on a space near the finish, as happens in many other race games. Indeed, because the death space is within reach by a reverse overthrow, a slowly approaching 'tortoise' can humble the proud 'hare'. And the game can end in a couple of throws or, just occasionally, can go on for many rounds: variety and unexpectedness is built in.

Designers of race games depart from these principles at their peril.

[Mascheroni & Tinti, 1981, p. 78, present author's translation] comment on the tedium of some of the geographical games dating from the 18th and early 19th centuries:

> The information was always of the same kind: the world was divided into four sections. The principal cities were specified. There were notes on economic resources but these were concerned only with gold, diamonds, commerce in porcelain, and silk...
>
> For movement along the track, it was necessary to refer to an extremely long series of rules that in practice consisted of a reward of another throw or the payment of a penalty.

The need for repeated reference to a detailed rulebook, unlike in *Goose* where the simple rules were apparent from the face of the game, was a worthy but stultifying device, aimed at imparting detailed knowledge but sacrificing playing values. By contrast, designers like Duval kept to the spirit of the original game and used wit to make their educational points. For example, in his *Jeu des Princes de l'Europe* [Ciompi 541], the rule for **Muscovy**, though a familiar *Goose* rule, adds a comment that is entirely memorable, making clear just why the player may not stay on the space concerned:

> Must advance once according to the points on the dice. The Muscovites do not permit entry to their country, yet you pay.

The design of successful race games with variants of *Goose* rules is therefore not easy, by comparison with the relatively simple task of providing thematic variations around the invariant skeleton of the traditional rules. Testing such variants in practical play would have been daunting in the past and even now computer simulation is not without programming effort.

It may be for these reasons that games with complex variant rules have never substantially displaced the traditional game, something that may explain the virtual absence of goose-derived games that allow through their rules for individual role-playing.

As far as forfeits and actions are concerned, their introduction tends to slow the game down to the point where the character of a *race* game is lost:

the game then takes on a wholly different character: that of a spectacle where waiting for one's turn is made bearable by watching the antics of one's fellow players.

It is hoped that these comments may be of some use to present-day game designers who, though the resources at their disposal are now incomparably greater than the simple engraved or lithographed playing sheet, might do well to remember that the psychology of social interaction, as mediated by board games, has not changed over the centuries to a like extent.

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Jogos Matemáticos, A Portuguese Project

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This is an expanded version of the paper presented to the Board Game Studies Colloquium in Lisbon in April 2008.

Abstract

Ludus Association and other mathematical associations organize yearly the Portuguese Championship of Mathematical Games. Since 2004, this tournament has been growing: from 500 students in 2004 in Lisbon to 1200 students in 2009 in Covilhã. Students aged 7 to 17, from all the country, join, each year, at a Portuguese city to play one of six different abstract games.

Figure 1 Posters of the first four editions of the PCMG.

The Benefits of The Project

We don't intend to prove that board games practice improve the cognitive performance of the young students. We *believe* that board games can offer very good mental exercises that, sometimes, are close to mathematical thinking. There is some evidence about the positive correlation between board games practice and mathematica achievement [1], however this kind of study is not our task. There are some easily spotted skills associated with our subject:

- Focusing - Children learn the benefits of observing carefully and concentrating.
- Visualizing - Children are prompted to imagine a sequence of actions before it happens.
- Thinking Ahead - Children start thinking before acting.
- Weighing Options - Children understand clearly the advantage of good choices.
- Thinking Abstractly - Children develop the ability of grouping situations, games, positions etc so they apply the same tactics in similar situations.

Board games do not replace the traditional mathematical curriculum. We just state that some board games seem closer to mathematics than, for instance, swimming, and the implementation of games seems a very good thing to do.

How it Works

The Choice of Games

We pick games with no chance devices and no hidden information. There is an huge number of board games that satisfy these criteria. To make the choice we assess the games accordingly to their quality. They must have easy rules. Using the terms of [2], simple rules help game clarity and help the student to understand the basic dynamics and to concentrate in the tactics and strategy. The elected games must have some depth, ie, allow several levels of sophistication in their playing level. The chosen games must be a dramatic, ie, they must allow volt-faces and traps, sequences of errors, missing wins and defeats, etc. The elected games must also have good interaction between opposite pieces. A game without interaction is merely a double race. Another important factor is decisiveness in the sense that if a player got a substantial advantage, winning should be an easy task. It is important to choose games with simple material. If possible, we choose games with recyclable material, i.e., the gaming parts can be used to play a big number of good games. This idea allow us to diversify the tournament throughout the years.

The games should also converge to an end, no matter how the players move, for the organization to be sure that the matches evolve smoothly.

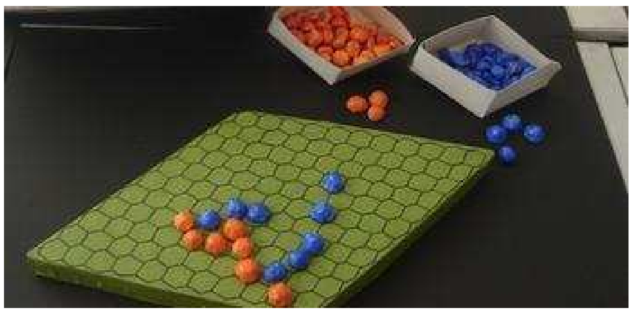

Figure 2 The game of HEX, constructed by Portuguese students.

With these guidelines we show, in the next table, the choices for the first four editions of PCMG:

Level	PCMG1	PCMG2	PCMG3	PCMG4
1st (6-10 years)	DOTS'N BOXES POLYHEDRON PUZZLES WARI	DOTS'N BOXES TRAFFIC LIGHTS WARI	DOTS'N BOXES TRAFFIC LIGHTS WARI	DOTS'N BOXES TRAFFIC LIGHTS WARI
2nd (10-12 years)	PAWNS POLYHEDRON PUZZLES WARI	HEX TRAFFIC LIGHTS WARI	HEX TRAFFIC LIGHTS WARI	HEX TRAFFIC LIGHTS WARI
3rd (12-15 years)	AMAZONS PAWNS WARI	AMAZONS HEX WARI	AMAZONS HEX WARI	AMAZONS HEX WARI
Secondary (15-18 years)	AMAZONS HEX PAWNS	AMAZONS GO HEX	AMAZONS GO HEX	AMAZONS HEX SLIMETRAIL

Some of the chosen games have a relevant cultural component. Games like GO and WARI are very old, being a part of human history. Some African students enjoy seeing their traditional game in a national championship of other country.

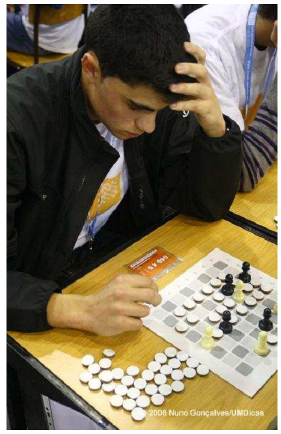

Figure 3 A student playing AMAZONS.

We implemented a very interesting parallel competition, called *Invent Your Game*. Some students designed new games with the materials existing in the championship games.

In the fifth edition, in Covilhã, we introduced special boards to allow the participation of blind students.

Figure 4 TRAFFIC LIGHTS board for blind students.

Mathematical Institutions Support and Interaction with Schools

The average budget of the project, in Portugal, is approximately 50000 euros. It is possible to get some support from scientific, educational, cultural and technological institutions.

It is important to guarantee that the information about the tournament and games reaches many schools. With the help of established institutions it is possible to manage and use sufficiently large mailing lists to reach the schools. In Portugal, we have the support of the two main mathematical institutions of the country (Association of Teachers of Mathematics and Portuguese Mathematical Society). In parallel, Ludus Association focus on the promotion of recreational mathematics and abstract games.

The interaction with schools and students is fundamental. The organization should pay attention to the following factors:

1. Along the year, before the final tournament day, the organization must supply or lend gaming material, since many schools do not have games. The schools should be able to get the boards or the information about how to get them. Another way is to build them, so, the information about this process must be easily known too.

2. Many teachers and students do not know the game rules. This information must be known and easily accessed. The organization should have a written document with the rules and must guarantee that the document is distributed by enough schools throughout the country.

3. It is important to organize teacher formation and motivate open days in schools for students and teachers.

4. The organization must help the schools with the preliminary phases. In the final tournament day the participants are the school champions. So, the schools must previously qualify their champions. Sometimes the teachers need some help to understand how to organize tournaments, etc.

5. The information about parallel events like *Invent Your Game* or participation of blind students must also be well known by teachers and schools.

6. The organization should try to have good prizes for the best classified players. In Portugal it is usual to have prizes like computers, calculators, digital cameras, books, etc.

7. The organization must have a proper website with all the required information. The website must have *more* than the needed information: it should include stuff about abstract games, links to online clubs, etc. In Portugal, the url address is http://ludicum.org.

The "Tournament Day"

In our opinion it is very important the existence of an yearly "special day". Our project is centered in a day when the students play a final tournament deciding who are the ultimate champions. There are several reasons for the importance of this day:

1. The day remains in students memory. Every year the older students remember who were the previous champions, how interesting was the previous tournament, how fun it was, etc. Those factors help students to participate. In a few years, almost every students and every teachers will know about the existence of the project and remember the past "special days".

Figure 5 PCMG–fourth edition.

2. The existence of a "special day" helps the organizers to invite politicians, institutional presidents, other VIPs and, so, attract some media to the event.

3. With the existence of this special event, the organization can bring gaming practice to many regions. Every year the final tournament is played in a different city.

4. The existence of a "special day" helps the organization of exhibitions, mathematical events, recreational activities, etc. It is important to be sure that the "special day" remains in teachers memory too.

Final Remarks

In the five first editions of the PCMG we got the amazing numbers ≈ 500, ≈ 650, ≈ 665, ≈ 1100 and ≈ 1200 participants in the final. These statistics show that, yearly, tens of thousand students play mathematical games in Portuguese schools (since each student in the final is the winner of a school local tournament).

There are about 1760 high schools (public and private). Yearly, the average number of inscriptions is about 180 schools. So, more or less 10% of schools participate in our project.

In *Invent Your Game* we had 30 invented games. This is a very good number for this kind of contest. We have strong evidence that computer games do not replace mathematical board games. Students still like to play mathematical board games: they are immortal.

We believe that the practice of mathematical games helps the development of mathematical thinking. We did not make any scientific study to support this. However, we have thousands of students playing the PCMG. It provides a very good opportunity to explore scientifically the question. Some steps are being given in this direction.

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An English riddle: Chess and Draughts in medieval England

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Language: a disregarded tool at describing ancient board games

In many ways, the language can contribute to our knowledge of board games in times past. In this article I show how we can use the language to prove that in medieval England chess was a minor board game and draughts a major game. But first I give the information you need to understand my claim.

Necessary information in advance

We modern readers rather easily understand medieval texts. The reason is that our sentence structure and our vocabulary are largely the same as centuries ago. Every modern language uses many medieval words, for the greater part with an unchanged meaning.

Important for this article is the percentage of medieval words used today in an unchanged sense. The computer can provide some figures. For Dutch it is more than 97%, the result of an investigation of the Dutch linguist Nicoline van der Sijs, published in Stoep 2007:135. This figure is not itemised; we may assume that a word with a concrete sense as a name for a board game or for other phenomena in our social world, like sun, river, house, horse, is not susceptible to a change. I do not have figures for the English language, but I do not have a reason to expect a significantly different result.

Medieval England: the names for chess and draughts changed

An exception is more interesting than the rule. *Chess* is considered to have been a well-known medieval board game — read chess historians — but in England in the 14th c. this board game received a new name. This is surprising; even more surprising however is that we encounter the former name for *Chess* used as the name for '*Draughts*'. This is demonstrated in the following two tables:

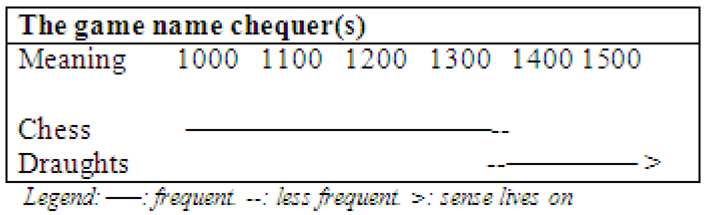

Table 1

Names for chess	**Names for draughts**
c. 1000 – c. 1400 *chequer*	.... – c. 1400 *morris*
c. 1400 – ... *chess*	c. 1400 – ... *chequer(s)*

Table 2

Such a change of meaning, the reflection of a change in reality, is rare, but nevertheless so remarkable that linguists gave it a name, see below [Bree 1990:153–155].

How to explain these changes?

To explain tables 1 and 2, we must combine language and board game. The linguist wants to establish if it regards a drag or a push chain — see below —, to fill up his scanty material. But to decide he needs knowledge of board games in England in the 14th c.

In the case of a drag chain, *Chess* players would reject the name chequer and chose a new word to give their game a name: chess. Thus:

Drag chain.	Chess players exchanged the name *chequer* for the name *chess*
	Draughts players pinched the name *chequer* for their game of draughts

Table 3

This option is plausible on one condition: that English chess players changed the rules of their game to such an extent that they invented a new board game, a game that needed a new name. But as far as we know this did not happen.

Consequently, we conclude we have to do with a push chain: English draughts players started to call their game *chequer*, and then chess players felt obliged to look for another name for their game. Thus:

Push chain.	Draughts players gave to draughts the name *chequer* 1)
	Chess players were compelled to choose another name: *chess*

Table 4

Linguistics: a rich source for the study of board games

Table 4 gives information on the positions of chess and draughts in medieval England: draughts was a major board game, chess a minor game, insignificant. Numerous other linguistic examples, embodied in Stoep 2007, confirm this finding, not only for England but also for the European continent. The two games kept their position until the second half of the 19th c., when chess acquired more popularity and rose in estimation, whereas draughts socially slid back into a humble state.

Note

1) Draughts was transferred from the lined board to the chequered board, see Stoep 2007:157–165.

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At the crossroads of trade in the Indian Ocean - a ludic exploration

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Research on the dispersal of board games, with the possible exception of chess, is frustrated by a lack of historical evidence pertaining to rules and playing practices.This is compensated by numerous systematic descriptions of board games that detail board and rules showing patterns of similar games across the continents as they are played today.

The rules of *mancala* are notoriously diverse and variants are less remarkable than instances in which largely the same rules are played in more than one geographical location. These places with near identical playing practices form a pattern that in many cases follow known trade routes of the region. Similar routes can be found involving *draughts* or *alquerque* games, which seem to have a similar diversity of boards and rules but which have received much less attention in the literature.

The possibilities of rules are diverse so that the introduction of a game through a single traveler cannot always explain strong similarities over large distances. The presence of a series of near identical games across wide geographical areas is best explained through intensive contact (de Voogt 1999).Such contact is possibly established through trade, assuming that the traders also play, which is frequently attested for shipmen. A second, equally convincing, mode of dispersal is migration in which a group of players settlesin another area. For instance, it is generally assumed that slave trade has brought games from West Africa to the Caribbean (Herskovits 1932).

Unlike rules, the board and playing context do not necessarily accompany the dispersal. *Mancala* boards are not known to travel except as prestige gifts (Walker 1990) in which case they are not accompanied by playing rules. Players are known to have traveled to seek out other players, but this is only attested for competitive games (de Voogt 2005). As such, board games are not an *object* of trade. Their dispersal is better compared to music (for the Swahili coast, cf. Askew 2003), which is played by people but which does not necessarily include the dispersal of the instruments.

This study concentrates on one four-row *mancala* game and one so-called *alquerque* variation with a brief mention of *draughts*.The examples are taken from the Seychelles and the Maldive Islands and compared to games found on the African and Asian mainland.

Makonn and *hawalis*

Mancala games have rows of holes on a board or in the sand as well as a set of identical counters, commonly seeds, shells or stones. The object of the game is to capture the majority of the counters or to empty one or more rows of the opponent. During a move the counters are spread in (counter) clockwise direction one-by-one in consecutive holes. The last counter of such a sowing determines the continuation of the move; the move may end, the move continues with the counters from a specified hole or the player continues by making a capture.

The rules of *mancala* are diverse and apart from the description above there is little that groups these games together. Men, women and children are known to play in contexts that can be competitive, ceremonial or entertaining. The boards are often made of wood but holes in clay or sand are equally common with occasional examples of other material such as terra cotta or stone (see de Voogt 1997, Walker 1990).

On the coast, playing counters are most often cowry shells or caesalpinia bonduc seeds that are found in abundance on the tropical coasts. Other areas use stones, particularly in the sand since they are somewhat larger and less likely to fade among the grains of sand.

The boards have been divided into two-, three- and four-row boards. In terms of rules, they are one circle and two circle games, meaning that a player shares the rows when counters are distributed (two- and three-row games) or that a player has two rows for him or herself. In the latter case, captures can be made where the two rows of each player meet.

Capturing in *mancala* takes on many forms and books and websites have been filled with their variations (e.g. www.manqala.org, Murray 1952). The rules for spreading and capturing counters most clearly distinguishes one game from another since the size of the board and the number of playing counters sometimes varies according to the wishes of the players, mostly to influence the duration of the game. There are few games in which the number of holes per row structurally affects the playing rules.

In four-row *mancala* games Murray (1952) distinguished captures that are taken from the board and those that are re-entered into the game. The consequences of capture rules for the remainder of the rules make them a promising distinguishing feature.

Oman boasts a four-row *mancala* game, one of few found outside Africa, by the name of *hawalis*. The game is played by men in clubs. Despite its recent decline it can still be found in Muscat near taxi stands where card and other games are also practiced. The Omani fixed the board to four rows of seven holes and usually play in the sand using stones. Direction changes for sowing, special houses and re-entry of captured counters are unknown for this game.

The game strongly resembles descriptions that have been made of Mozambiquan *mancala* games. In the area between Oman and Mozambique this game appeared remarkably absent until a field study in 2007 revealed a similar game in the Seychelles.

The Seychellois still play *makonn*, as it is called. Players on Mahé recall playing the game as far away as Diego Garcia, prior to its British occupation, and outlying atolls of the archipelago. The game is most likely played in Mauritius as well based on information from the Seychelles. *Makonn* is fixed on four rows of ten holes but despite the larger board, the rules and thereby the playing strategies are remarkably similar to those of *hawalis*. The following elements of the rules are identical in both games (for the complete rules of both games, see de Voogt 2003 and de Voogt, in press):

- counters are always played counter-clockwise
- counters are captured from the front and back row together
- captured counters are not re-entered in the game
- a hole with a single counter cannot be used for play
- if all holes of a player contain only a single counter, the rules change:
  – the player can move singles, but not into an occupied hole, and the player can capture counters of the opponent in the same way as before

The Seychelles were uninhabited until the end of the seventeenth century when French settlers took possession of the islands (Scarr 2000). African slaves were introduced inthe course of the eighteenth century and slavery has continued well into the nineteenth century. The supply of slaves is said to originate from Mozambique and sometimes occurred via Zanzibar. There is no link between the Seychelles and Oman in their history or in their present contacts. Rather, one has to assume that the slaves that entered the Seychelles played the same game as the Africans that settled and introduced the game in Oman. The opposite route in which Omani traders introduced the game to the East African mainland and that the game spread from here to the Seychelles is ruled out. No other four-row games appear in Oman or near Oman while two-row games in the Middle East are well attested.

Draughts, *danm-la-tete* and *wagu thinhamma*

Draughts is an European invention and was brought to the Indian Ocean in different forms. The Seychelles play draughts on a 10×10 board with Anglo-French rules, meaning that the king travels and captures across an entire diagonal and that individual checkers capture forward and backward, but move only forward (see Kruiswijk 1966, van der Stoep 1994). The British protectorate of the Maldives did not successfully introduce the British form of *chechkers*, instead the Maldivians prefer their own form of *chess*, known as *raazuvaa* that was introduced via the Indian subcontinent.

The game *danm-la-tete*, "draughts with a head", is a variation of an earlier *draughts* form, also referred to as *alquerque*. Instead of squares on a board it consists of intersecting lines. The pieces are only allowed to move from one intersection to the next via a connecting line. As in *draughts* the pieces jump over the opponent's piece to make a capture. The game is played in many variations and has been described for large parts of Asia but is also known in Africa. *Danm-la-tete* is the variation played in the Seychelles and similar boards can be found in the Maldive Islands where the game is called *wagu thinhamma*. Both games are well-known for the Indian sub-continent where they may have originated (Balambal 2007).

Danm-la-tete is found in the same so-called "baka" bars where *makonn* and *draugths* are also played. Its board is usually drawn on wood or chalked on stone. In the Seychelles, players have applied *draughts* rules to the *danm-la-tete* board. They create kings when they reach the far end of the board and these pieces may often move and capture backwards across more than one point, something they are not allowed to do in the beginning phase. The National Archive (1991) in the Seychelles published a short description, otherwise this game is not described in the literature. The following data were collected on Mahé, Seychelles, in 2007:

> The National Historic Museum in Mahé, Seychelles, shows one *makonn* and one *"danm-latet"*. The latter has four triangles attached to a squaregrid of 5×5 intersections created by five horizontal and five vertical lines in addition to four diagonal lines that connect the triangles. The rules were not attached or known and the wooden board was shown with checker pieces.
>
> Louis Lazeau in Mahé made a *danm-la-tete* board identical to the one on photo 1. He played the game mostly in high school and had not played in over ten years. It has 16 pieces. Captures are mandatory and made as in draughts both in forward and

backward directions. Multiple captures are optional. A blocked opponent or a wiped-out opponent has lost the game. Pieces reaching the back row of the triangle make a king that crosses and captures across more than one point in one direction. As in draughts the king cannot jump across two or more pieces that are adjacent to each other. Captures with a king do not come before other captures.

Lusette Azémia plays the game with children and added the rule that failure to capture leads to forfeiting that piece, as is common in some draughts games.

Richard Louis of 19 years old, played the game when he was 16 and when he tried to make a board he added four triangles. He was not able to complete the board but confirmed that they would play in fours.

Robin Pierre Mare, born on La Digue island in the 1930s, demonstrated *makonn* in his bar where a 10 × 10 *draugths* board was present and a *danm-la-tete* board with two triangles (photo 1).

Photo 1 *Danm-la-tete* as found in a bar on Mahé, Seychelles

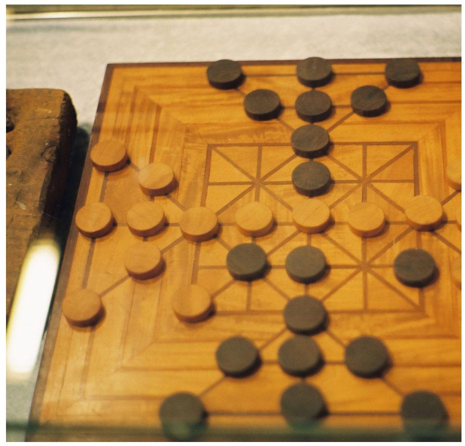

Photo 2 *Danm-la-tete* as found in the National Historic Museum on Mahé, Seychelles

Wagu or sometimes *baburu thinhamma* means large or "negro" *thinhamma*. It shares the name *thinhamma* with a *morris*-game that is also played on the Maldives. Those Maldivians who play *thinhamma* are not always familiar with the game of *wagu thinhamma* even after being shown a photograph of a local *wagu thinhamma* board. The two games only share a drawn configuration of lines and similar pieces. Wagu *thinhamma* is better compared with *draughts* since it has little relation to strategies and moves required in *thinhamma*. The simplest configuration is identical to *damn-la-tete* as found on the Seychelles.

Wagu thinhamma can combine a complex configuration with a simplicity of production. The lines are drawn on a board and additional lines are easily applied. The following date were collected in 2003:

> Moosa Ismail (70) was one of few to remember how to draw a *wagu thinhamma* board on South Feydhoo, Addhoo Atoll. His board is relatively small and much smaller than the board that was briefly shown to him on a photograph.
> Zaynab Ibrahim (53) of Feridhoo, Ari Atoll, drew a *wagu thinhamma* board of 5 × 5 with two triangular additions containing another six positions. Captures are made as in draughts by

jumping over the opponent's piece(s). One may traverse only across the drawn lines. Capturing is not obligatory and a multiple capture is not mandatory when possible. One may move forward or backward and moves are allowed in all sections of the board. The winner has captured all the opponent's pieces. No promotion is made when reaching the far end of the board.
Saud, the island chief, and Mohammed Husayn (86) pose for *wagu thinhamma* in Feridhoo, played according to the rules above.
In Nilandhoo, many players of *wagu thinhamma* appear:
Hauna Ibrahim (43) draws a design of 6×6 with six positions in each triangle. An unusual design since the intersections created by the diagonal lines are not used as playing fields.
Aishath Naasira (32) and Abdulganee Mohammed (34) demonstrated a game of *wagu thinhamma* on the board belonging to the Endherimaage house. It is a 9×9 board with six positions in each triangle. Again, capturing is not mandatory, neither are multiple captures over single captures. The triangle does not know special rules. It is noted that a draw is possible in this game.
Boards without players were found in other households on the island. Such as a 7×7 board with *thinhamma* on the back and a 5×5 board in the Irudheyamaage house.
Sirumeena Abdulrahman (20) and Ismail Waheedh (30) demonstrated the 7×7 board. They maintain that captures can be made backwards but moves are always forwards. Again, capturing is optional. In Noorange house, grandmother Habeebaa Moosa and the father of thehouse played on a 7×7 board with six pieces in each triangle. They named the central position on the board "raskan" (or "rasge" in neighboring islands), which is the king's house. On each of the far sides of this position, i.e. the farleft and the far right, is "mudhinkan" which relates to persons outside the lines of prayer. These three positions are considered strategically importantand were marked on the board.

Photo 3 *wagu thinhamma* as found in the Maldive Islands

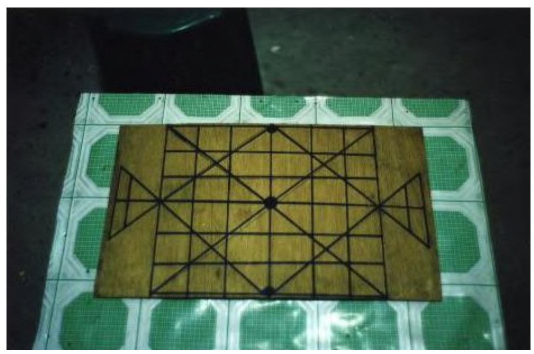

Photo 4

Photo 5

The above description shows that *wagu thinhamma* has a limited distribution in the Maldives. Only two islands had players while only one person was found outside these two islands, an older man in Addhoo. It is likely that there are players else where in the Maldives, but all the islands that were visited in both northern and southern parts of the Maldives did not indicate that the game was popular or even known. Feridhoo is a rather small island and was quickly searched. In Nilandhoo many families are familiar with *wagu thinhamma*. Boards vary from 5×5, $6times6$, 7×7 and 9×9 as is also illustrated in Maloney's work on Maldivian culture (1980). The triangle always contains six positions, with the exception of one board made on Addhoo. Variations in rules are minor. The only clear variation is the rule that pieces can only move forward. All other rules are consistent.
With the absence of *wagu thinhamma* in Fuamulaku, large parts of Addhoo, Huvarafushi and even Kandholodhu, it is possible to conclude that *wagu thinhamma* has a limited distribution that is concentrated in the central part of the Maldives. In this limited distribution, variations of size are common, and sizes differ from house hold to household. In contrast, the sizes of *raazuvaa* and *thinhamma*boards do not differ in any of the islands. This is not surprising, since the difference in *wagu thinhamma* boards does not affect any of the playing or capturing rules while they would affect such rules in *raazuvaa* and *thinhamma*. *Wagu thinhamma* and *thinhamma* have little in common but a name. Indeed, the name *wagu thinhamma* is probably derived from *wagu thinhamma*, a sign that it was a later introduction. They are both drawn on boards or in the sand but their contrasting distribution pattern does not indicate that they obtained the same distribution or popularity.

Trade and play

Both *damn-la-tete* and *wagu thinhamma* are games coming from India where the game was knowneven before the settlement of the Seychelles. Its distribution can be linked to the arrival of Indian indentured laborers in the Seychelles and long-term trade relations with the Maldives. It is unlikely and there are no historic reasons to assume that the Maldives and the Seychelles had long-term trade contact.
Makonn and *hawalis* are games traveling from Mozambique during the slave trade, possibly via Zanzibar. There is no trade or other connection between the Seychelles and Oman that point to anyother mode of distribution.

Draughts was introduced by the colonial powers. Its rules have been partly applied to *danm-la-tete* in the Seychelles but seem to have made no such influence in the Maldives where British *checkers* is hardly known.
Both the Maldives and the Seychelles show that games travel without boards and pieces but with players. Once introduced, they coexist and are not necessarily limited to environments in which they were introduced. For instance, games with an African origin are now played by Arabs in Oman and games with an Indian background are played by a diverse group of Seychellois. Trade, particularly trade in people, helps to understand the distribution of board games in the Indian Ocean. The understanding of their popularity and the social environments to which they may be confined require further research.

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The development and dispersal of l'Attaque games

Fred Horn and Alex Voogt

Modern versus traditional games

The study of board games concentrates on board game materials, such as boards and pieces, and game concepts, which include rules for moves and positions. The dispersal and development of materials and playing rules are central in the historical studies that exist on board games from the beginning of the twentieth century dominated by Culin (1895), Murray (1913,1952) and Bell (1960, 1969) up to the present with a recent overview from Parlett (1999). Even though Murray was an anthropologist by training (Wendling2002), his histories of board games resulted in a strong historical focus on games from academic disciplines such as archaeology, philology and art history with themes relating to development and dispersal.
Modern and traditional games are understood differently within this context of mostly historical studies. Modern games maybe distinguished from traditional games because their history is not only limited in years but also considered to be of a different kind. Murray, for instance, did not discuss games that had been invented in the United Kingdom in the twentieth or even the nineteenth century. Although Murray does not explainthis selection, it is generally thought that games invented by a known individual and distributed by a games company are to be treated differently. Historical studies on such modern games concentrate on tracing patents and discussing the history of games companies (cf. Whitehill 1999) or occasionally the artwork of the printed paper (Goodfellow 1998). On the other hand, the traditional games are seen as part of a long historical development and require research in the field and an inventory of variations such as those made for mancala games (de Voogt 1999) in order to answer questions on development and distribution which seem trivial in the case of modern games.
Both the development and the dispersal of modern games may be in the hand of a games company that determines the written rules and introduces

the game to a chosen market. Only the company changes the written rules and the distribution is regulated through locally acquired patents. As a consequence studies on the distribution and development of modern games are unlikely to exist outside the study of games companies and inventors.
The following history has the game *Stratego* at its center. It is traced back to the game of *l'Attaque* and related to Jun Qi as it is found in China. It has a history prior to its patent and distribution in China that took place outside the control of games company.

The game of *l'Attaque*

A study of modern games may have legal implications.The patents refer to the uniqueness of a board game that may include the design of the board, the pieces and the rules. When their origins are questions so are their intellectual property rights. Fortunately, most battles have been completed for the game that is here referred to as the game of *l*'Attaque.
The game of *l*'Attaque has at least four elements that together create a game that can be distinguished from all other board games: the ranking of pieces, the hidden position of the pieces, the presence of both static and movable pieces, the capture mode and the general design of the board. In addition, the military ranks and the conquest of a flag make the game easily recognizable.
l'Attaque was most probably developed at the beginning of the twentieth century or perhaps as early as 1880 in France. Boutin (1999) states a patent deposited by Hermance Edant in 1909 based on a game she developed in the 1880s. A patent on a game, called *Jeu de la Guerre*, was filed in 1907 by Julie Moller for quite possibly a similar game.
After the First World War, the game was also published under the translated name *Attack* by the London-based firm H.P. Gibson & Sons, Ltd. So far, this is the only publication of this game at the time.
The board consists of 9×10 squares. In the center of the board there are three obstructions in the shape of three lakes with a size of two squares each. There are movable and immobile pieces. The first are ordered by rank. Each higher rank is stronger than all lower-ranked pieces. Two, or in rare cases four, players fight each other with an army of pieces while the rank of the pieces of one player is hidden from the opponent. The players may position the pieces on their side according to their own views. The purpose of the game is to conquer the flag of the opponent.
Each piece moves one square orthogonally and in some cases more than one

square. Pieces are captured after they are engaged in aduel with a neighboring piece. The strongest piece then remains on the board.
This general description of the rules is valid for all *l'Attaque* games, varitions mostly relate to the size of the board and army. *l'Attaque* is resembling *Stratego*, a game that became internationally successful when it was published by Jumbo in the Netherlands. Before establishing a possible link between *l'Attaque* and *Stratego*, the latter's recently uncovered patent history is recounted.

Patent history of *Stratego*

On April 20, 1942, the name *Stratego* was registered by Van Perlstein & Roeper Bosch N.V. in the Netherlands. Four years later the game was published by Smeets & Schippers in Amsterdam. Between 1948 and 1949, the game was also produced using the brand name Clipper. In 1951 the license to produce the game seems to have been returned to Mogendorff by Smeets & Schippers.
This series of events is the prelude to the negotiations between Mogendorff and Hausemann & Hötte N.V. in Amsterdam about the publication of the game under the latter's brand name Jumbo. These talks between Mogendorff and the Jumbo representative de Graaff are said to have taken place between 1952 and 1957. On February 28, 1958, Van Perlstein & Roeper Bosch N.V. sign over the property rights of *Stratego* to Jacques Johan Mogendorff. Correspondence between Mogendorff and de Graaff confirm this and also note that an international registration is still to completed and that *Attack* seems to be a copy of the game *Stratego*. Further research shows that the Hausemann & Hötte company has a 1920 *l'Attaque* game in their archive suggesting that the similarities between the two games were known to them.
On March 7, 1958, Smeets & Schippers N.V. also declared that Mogendorff can use the name and the appearance of the *Stratego* game. An official mention in Merkenblad confirms Mogendorff's registered name under number 130494. From May 17, 1958 until April 8, 1960, the *Stratego* name becomes registered in a series of countries, including England, South Africa, Australia, USA and Canada.
Meanwhile, on June 10, 1958, Mogendorff and Hausemann & Hötte agreed to publish and distribute the game *Stratego* for Europe. A second agreement for the rest of the world was added with a royalty agreement on April 21, 1961. In August of that same year, Mr. Mogendorff dies and his heirs make a new agreement with the company that transfers all the rights to Hausemann & Hötte, a situation that has persisted up to this day.

1 The Canadian connection

The 10×10 board of *Stratego* requires a higher number of pieces than the board of *l'Attaque*. The principles of play appear near identical to each other. The route of dispersal is partly revealed by the presence of an intermediate game, owned by the late Mr . Voorn and discovered in Leiderdorp, the Netherlands.

Voorn's father had made one issue of a game he called *Tek*, presumably in 1942. The game had been made at the instigation of a shot-down Canadian pilot who was in hiding.The leaflet with the rules and the instructions on how to manufacture the game is still in existence. The maker had been unfamiliar with *Stratego*.

Both Voorn and Mogendorff had been residing in The Hague during the Second World War, and it is suggested that such a contact may have transmitted the idea of a *l'Attaque* game to Mogendorff.

Although the firm Gibson had published *Attack* before, both the name and the rules of the game had not been registered by Gibson in England. But in 2003, the heirs of Gunger Sigmund Elkan from Vancouver, Canada, commenced a lawsuit against Hasbro, the license holder in the USA of *Stratego* since their grandfather had registered a game named *Strategy* on June 1, 1948 and a booklet with the same name registered on May 25, 1948 in the USA. A vague copy of the rules was entered as well that mentioned a 10×10 board but did not mention any blocking fields. The case was not strong but

shows a possible American connection. They mention Mr. Elkan who was to have invented the game in the Second World War after which he emigrated from Europe to Canada. Where in Europe is unknown and all involved, Mogendorff, Elkan and Voorn have since died so that their possible connection cannot be established.
This short history of l'Attaque and Stratego shows that multiple inventors and different routes of dispersal are possible and likely.

That transmissions of an idea took place with and without the help of patent offices that only assist in settling an ownership debate but not a chronology of events and inventions. This transmission of an idea is futher illustrated with a chinese game for which no patents are known.

The Chinese connection

A number of different companies produce *Jun Qi*, or flag game, under various names, including *Liuzhangi, Siguodanzhanqi* (played with four people) and even with an English translation of their name *Superduty Amry Chess* (sic) or *ChaojiLuzhan Qi*. Rules and descriptions can be found in Lhôte (1994) and on the internet (www.chessvariant.com/oriental.dir/tezhi.html). They come in hard plastic or carton boxes, which depict tanks, army planes and helicopters. The pieces are small, i.e. 1.5 cm, rectangular blocks made out of plastic and with printed or relief Chinese characters. The board invariably consists of a plastic white sheet with red print. These sheets are also common for other board games popular in China, including Chinese chess. Some luxury editions do not have printed carton but fabric-covered boxes, which contain massive plastic stone-like pieces with engraved characters but of a similar size. Few if any of the games are accompanied by game rules, even though the rules appear quite complicated and are of at least two different kinds.

All pieces are blank on one side and have Chinese characters on the other side indicating the (military) rank of a piece. Sometimes the characters for the Army Flag also feature a flag symbol. Pieces move one square at a time and enter combat as in *Stratego*. Except that the board features a railroad on which pieces move unlimited empty squares in one direction along the track. The Engineer may also take corners on the railroad within one move. The board consists ofa grid of 13 rows of five intersections each. Each player places his pieces on their six rows of five intersections. The middle or seventh row is left empty and has only three positions thereby limiting the connection between the two camps.
Each side has five army camps, usually depicted as round, which are safe havens in which pieces cannot be captured but which are also left empty in the initial set-up. Therefore, only 25 pieces for each player are used in the line up. Diagonal lines connect the five camps with all surrounding positions, i.e. eight connecting lines. The third and the fifth row count two of those camps and the fourth row has one in the middle which is connected to the remaining four. There are two encampments at the bottom of the board of a different shape either of which should contain the flag in the initial set-up in certain variations of the game.
A railroad is depicted on the lines connecting the intersections of the first and fifth file of each player with the exception of the back rows as well as the second and sixth row on either side also including the lines that connect the two opposing camps on the seventh row.
The game is won when one side moves the flag of the opponent to one of their own encampments. As in *Stratego*, in case of a stalemate the party with the greatest number of moving pieces wins.

There appear three variations available in Beijing. One requires each player to design a positionas in *Stratego* and continue as described above. A second allows for the game to be played with four persons on an enlarged board that also shows extra bends of the railroad. The third variation requires all pieces to be the same blank color on the outside and different color characters on the inside. These pieces are mixed and put at random face down on the squares used for positioning. With each turn a player may turn a piece or move a piece that is already turned and belongs to their color. Other rules stay the same.
This brief description of *Jun Qi* identifies a number of similarities to *Stratego* that are not likely to be incidental. *Stratego* was never directly introduced to China but when *Stratego* was introduced to the United States, it may have transmitted to Asia and developed into a local flag game as early as the 1950s. The long-time study of Chinese games (Culin 1895, Schlegel 1869, Röllicke 1999) has concentrated on their introduction to the West rather than the other way round. The particularities of a railroad and the marked places on the board warrant further research of Chinese appropriation processes in post-war China.

Development and dispersal

The development and dispersal of *l'Attaque* is complex and crosses many borders. The combination of r ules that have made the game unique and suitable for a patent and has also made the connection with China more probable. The transmission and transformation of board games in China is largely unexplored for modern games and opens up an area of board games research as complex and revealing as that of traditional games.
The history of a modern board game is not always in the interest of board games manufacturers and the hesitation of proclaimed inventors and patent

owners is understandable in the light of the above. However, the context in which these games develop and the transformations they experience when they are transmitted from one to the other reveal the processes of development and dispersal that has been at the center of board games research since the beginning of the twentieth century.

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Table 1: Name, rank and number of playing pieces

Strength	*l'Attaque*	#	*Tek*	#	*Stratego* Clipper/Jumbo	#	Computer *Stratego*	#	*Strategy*	#	*Jun Qi*	#
1	Spy	1	Spion	1	Spion	1	Spion	1	Spy	1	Bomb*	2
2**	Scout	8	Soldaat	8	Verkenner	8	Verkenner	8	Scout	5	Engineer	3
3	Sapper	4	Ingenieur	4	Monteur	5	Mineur	5	Engineer	4	Platoon commander	3
4	Sergeant	4	Sergeant	4	Sergeant	4	Sergeant	4	Corporal	4	Company commander	3
5	Lieutenant	4	Luitenant	4	Luitenant	4	Luitenant	4	Sergeant	3	Battalion c.	2
6	Captain	4	Kaptein	4	Kapitein	4	Kaptein	4	Lieutenant	3	Regiment c.	2
7	Major	2	Majoor	2	Majoor	3	Majoor	3	Captain	2	Brigade c.	2
8	Colonel	2	Kolonel	2	Kolonel	2	Kolonel	2	Major	2	Division c.	2
9	Brigadier-General	1	Generaal	1	Luitenant-Generaal	1	Generaal	1	General	1	Army c.	1
10	Commander-in-chief	1	Veld-maarschalk	1	Generaal	1	Maarschalk	1	--		Commander in Chief	1
	Mine	4	Mijn	4	Mijn	6	Bom	6	Mine	4	Mine	3
	Flag	1	Vlag	1	Vlag	1	Vaandel	1	Flag	1	Flag	1

* Bombs may destroy any other rank but is automatically destroyed itself.
** Scouts are allowed to move across any number of empty squares and the engineer in Jun Qi is allowed to move anywhere from one station to another along the entire railroad.

Table 2: Board designs

	l'Attaque	#	*Tek*	#	*Stratego* Clipper/Jumbo	#	Computer *Stratego*	#	*Strategy*	#	*Jun Qi*	#
Board Center (size)	9 x 10 rivers (2 squares)	 3	9 x 11 woods (3 squares)	 3	10 x 10 no-man's land lakes (Jumbo) (4 squares)	 2	8 x 10 emblems (2 squares)	 2	10 x 10 ?		5 x 13 grid mountains (1 square) free zones flag positions	 2 10 4

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Toward a Classification of Non-Electronic Table Games

Bruce Whitehill

This is an expanded version of the paper presented to the Board Game Studies Colloquium in Lisbon in April 2008.

Abstract

Game theorists have long attempted to devise a method of classifying or cataloguing the myriad types of games that exist and have existed. Historians examine specifics within board games, cataloguing games by their method of play. Modern game companies separate their game product line into children's games, family games, and adult games. Players use divisions according to the level of strategy and the type of game, such as board game, card game, skill & action game or party game. Classification by characteristics, such as game mechanisms, is differentiated from ways of sorting, such as by complexity or theme.
This author examines whether all the factors used to differentiate games can be employed in one system of classification, and attempts to determine if they can be applied to all games, past and present. Further, an effort is made to classify "indoor" games and, in greater detail, board games.).

Introduction

The Need for Classification

Why classify games? The need to classify games comes with the yearning to understand the similarities and differences between games old and new across cultural boundaries, and to develop a system of description and terminology that assists in simplifying communication about games. The desire to compare and contrast is held by both historians and modern players.

History

Around the turn of the 20th century, games researcher Stewart Culin examined games of the world and, especially, of American Indians. He categorized games as games of chance, games of dexterity, and "games of pure skill and calculation". He called these "classes of games". He broke down games of chance into two categories, dice games and guessing games, which he described as games of concealment. Culin's main writings in 1896 and 1907 were produced when the commercial game industry in the United States was little more than 60 years old.

Half a century later, H.J.R. Murray, in his 1952 book, *The History of Board Games Other Than Chess*, proposed a classification of games that included race games, war games, hunt games, alignment or configuration games, and mancala games. At that time, manufactured games in America were still primarily for children, and the industry itself was still in its infancy. Murray's primary interest was in examining ancient and classic games.

It wasn't until 1962 that the 3M Game Company began to produce games of strategy and calculation aimed at adults, an impetus which repeated itself in Germany four years later when 3M used its world-wide marketing presence to introduce these games to the continent; the style of strategic, tactical games synonymous with that country today has added the words "German game" or "Eurogame" to the vernacular.

Since the late 20th century, games have evolved considerably, and more and more types of games have been introduced to a public that devotes more time to play than its ancestors did.

In the 1999 *The Oxford History of Board Games*, author David Parlett compares the system of classification of board games used by Murray with the similar system used by R.C. Bell and one advanced by himself. Since the 1980s and, indeed, even since Parlett's book, there has been a great increase in the number and types of games that have made it to market. This makes it more urgent—and more difficult—to develop a system of classification that works for *all* non-electronic games.

What Is a Game?

Before one can classify or categorize games, one must define what a game is. A game is a pastime, a form of play, in which players compete, each trying to emerge the winner *according to a specific set of rules and a predetermined end*; the winning player is the first to reach a particular goal, be it to attain a certain position or accumulate the pre-requisite requirements. In a solitaire game, considered by some purists to be more a puzzle than a game, a player is either competing against himself—trying to better a previous score or accomplish a goal in a shorter amount of time—or is competing against the game itself, in a sense playing against the game's inventor, trying to fulfill the author's objectives.

Many activities are referred to—sometimes mistakenly-as games. One of the problems is the overlap and lack of distinction between certain games and some sports. We "play" at sports just as we "play" a game. In most sports, all players play at the same time, as opposed to the turn—order used in games. But there are many exceptions. In the sport of golf-also called the "game" of golf-players play one at a time (in an order that usually changes throughout the game). Sports usually require a field or a hall or arena, whereas games can be played within a small confine. Those pastimes, then, that fit the requirements of rules and a prescribed end that are played in a large outdoor area are sports, whereas those played inside the home or similar structure are games. However, this leaves us with an in-between area, the "backyard", where such pastimes as *Croquet*, *Horseshoes*, *Quoits*, and *Boce* are played and referred to alternately as both games and sports. Similarly, some "street games" such as *Hopscotch*, played on the "sidewalk" or asphalt, blur the line between play and a game. It is interesting to note that all these examples and others that come to mind are pastimes that require some physical action and activity.

Initial Classification

The difficulty in developing a system of classification for games-even after we have agreed on a terminology-is that there are so many overlaps (where a game may be classified in two or even more categories), so many exceptions, and an inordinate combination of different variables.

The first classification that is needed, then, is the division of games into

Indoor Games and Outdoor Games, with the understanding that **Indoor Games** may be played outdoors, such as *Wari* played on a front porch or chess played in the local park; but **Outdoor Games**, in their normal fashion, cannot be played indoors. Lawn games such as *Quoits*, *Croquet* and *Lawn Bowling* have been miniaturized to allow *Ring Toss*, *Table Croquet* and *Carpet Bowls* to be played indoors; even *Hopscotch* has been transformed into an indoor game played on a vinyl mat.

Once indoors—if we stay with a linear classification of games—we need to differentiate between games played with no implements (save for, perhaps, those needed for record-keeping) and those games that use some materials. Games with no implements are usually called "parlor games" or, more recently, "party games", and in earlier times included an entire category called "forfeits", in which someone would have to answer a question or riddle or perform some dexterous activity, and, if unsuccessful, would then have to take the mild consequences, mostly deemed humorous in those times.

The latter division, "Games with implements", can be broken down into three sub-categories: pencil & paper games, games using common household objects, and classic or proprietary games that have a set of materials, be they unique or standard game pieces. These are the games referred to by R.C. Bell and others as "Table Games". Pencil-&-paper games, often designed for two players, include such classics as *Battleships*, *Boxes*, and *Noughts & Crosses* or *Tic-Tac-Toe*. It is interesting to note that *all* of these pencil-&-paper games have been made commercially, employing anything from pencils and a pad to a slate to a more elaborate molded plastic or carved wooden base.

Games using household objects include hidden object memory games and games such as a *Shove Ha'Penny* variant in which a coin is finger-flicked across a table top, with the aim being to get the coin as close to the far edge of the table without falling off.

Classic games and proprietary games constitute a large category today, and here is where devising a system of game classification becomes complex. There are three primary systems that are possibilities for classification: the historic, put forth primarily by Murray; the "company model", used by today's game companies; and the players' system of classification, used informally by many game players today. Certainly these three groups—the historians, the manufacturers, and the players—are interested in much of the same information, but the emphasis of each is quite different. The historians are looking primarily at the origins of games, the materials employed, and the games' social and cultural significance; the manufacturers are categorizing their games according to the intended audience—that is to say, the consumer; and the players are looking at (and for) special mechanisms that make games unique and/or interesting to play.

If we can successfully set up a linear system—and this could prove an impossibility—then that would mean establishing a hierarchy. But each group making the assessment would want, at the very least, its own system to be at the top of the pyramid. However, as we have entered an age of computers and databases, and have access to instant "Find" and "Sort" possibilities for our data, it may be possible to combine the three classifications, allowing the hierarchy to change depending on the end user.

There are many attributes to a game that are important in analyzing and describing a game, but they may not necessarily be part of an overall classification. These include:

- Theme, as it relates to historical, geographical or socio-cultural subjects presented in or promoted by the game;
- Strategy vs. Chance (Luck), two factors which should total 100%;
- Simplicity vs. Complexity, which cannot be assigned a percent value but can be shown as a range.

Whereas such things as game implements, style of board, illustrator, condition, source, and notes or background information, among other things, can be useful in sorting games listed in a database, only certain basic elements should be employed in designing and clearly defining an uncomplicated system of games classification.

Historical Interests

Many 19th century and early to mid 20th century games as well as games of antiquity are still played today, such as *Chess*, *Checkers*, *Chinese Checkers*, *Halma*, *The Game Of Goose*, *Mill*, *Mah Jongg*, *Reversi*, *Snakes & Ladders*, *Mancala* games and the more luck-based games such as *Parcheesi* and

Backgammon. Games of antiquity such as *Senet* (*Senat*), *The Royal Game Of Ur*, *The Game Of Twenty*, and Fifty-Eight Holes include games that originated 5000 years ago. Historians try to decipher not only where and how a game originated, but also how it might have been played. There is also considerable interest in whether the game traveled to other parts of the world, and, if so, how and where, and in what way it may have changed over time and distance. A game may suggest something about the social order of a particular society in those times, just as information about the culture could suggest what purpose the game served.

Sub-categories of prime interest for historical research, then, would include land of origin; the range of areas where the game is/was known to be played; existence of rules of play; variations of the gameboard, materials, or rules in different areas; and the eras during which the game was played.

Commercial Manufacturing

Since the mid to late 1800s, game companies in the United States and Europe have been manufacturing games as products which must then be sold for public consumption, whether it be through direct (retail) or indirect (wholesale) means. Game companies put their games into categories to make them easier to market. The categories are based primarily on the audience to whom the game is targeted—children, family, or adults—or to the theme or style of game.

The three main categories used by many mass-market game companies are: Children's Games (including educational games); Family Games (games suitable for the entire family, or, at the very least, which adults won't be too bored playing with children); and Adult Games (games which have either "adult" themes or a relatively complex method of play). Other categories, which have changed and will continue to change over time, include:

- <u>Party Games</u> - games for groups usually of five or more; the games are often word games or games that require physical actions or stunts.
- <u>Trivia Games</u> - a separate category that normally falls under Party Games, requiring individuals or teams of players to answer trivial questions.

- Skill & Action Games - dexterity games that require players to throw, roll, balance, build, or otherwise manipulate materials or objects. Some Skill & Action games actually require no skill and are all action, such as in some top-spinning games.

An exploration of game company catalogs also reveals that companies classify games according to price and theme; Milton Bradley catalogs from the early 1900s, for example, had a section of low-priced "5 Cents" games, while in the 1980s there was a special group of one-dollar games. Some catalogs in different years had pages devoted to games representing a particular popular theme of the time, such as cartoon games or mystery games.

Players' Games

Game players have a few select primary interests: they want the game they are playing to be fun and/or challenging. They are interested in the game mechanism (how it plays), or the level of interaction with other players that the game creates. In some cases, they even may be interested in what they may learn from the game. Whereas historians are interested in the oldest games, players pay attention to the newest of games, making a game's "date" of indirect importance. Many players are also interested in the theme. One popular website lists over 70 categories it uses as themes in its classification of non-electronic games. Serious players, who are more attuned to the game market, are also often interested in knowing the manufacturer, the game's author, level of strategy, and duration (games can last from ten minutes to several hours).

The 7 Categories of Table Games

To begin, it is necessary to have an overall classification of indoor games, or "table games" as we refer to them here. The seven main categories are:

- Board Games
- Card Games
- Dice Games
- Word Games
- Dexterity Games

- Domino Games & Linear Tile-Laying Games
- Memory Games

One cannot expect a classification of games to be without exceptions and overlaps. Board games, for instance, may have cards or use dice, and a dice game or card game may employ a small gameboard, but the distinctions are made according to the primary material or mode of play. A game in which cards are laid out on a table and then played upon could be considered a board game rather than a card game.

A board game consists of, predominantly, a game board that directs the movement or style of play. Boards might be carved in wood, etched in stone, printed on linen, or commercially manufactured with lithographed paper pasted to cardboard.

A card game can be one of two types of games:

1. games played with a standard pack of playing cards (example: *Rummy*, *Pinochle*, *Skat*, *Canasta*);
2. games which use a special set of cards (examples: *Old Maid*; *Authors*; *Snap*; *Pit*; *Quartet*; *Schwarzer Peter*, *Bohnanza*).

Games played with playing cards are considered generic, while games with special cards are usually proprietary, though some may have fallen into the public domain. Dice and markers or playing pieces might be included with a card game.

Dice games are those that use dice exclusively or as the primary mode of play. *Yahtzee* and *Liar'S Dice* are prime examples.

Word Games are games that use words or phrases as the principle method of play, regardless of the materials. *Scrabble* is a word game played on a board (and would be considered by many to be a board game); *Perquackey*, a popular game for many years, used letters on dice-like cubes to form words; other popular word games were pencil-&-paper games. Many modern word games also fit under the classification of Parlor Games.

Dexterity games have been referred to by other sources as "Skill & Action" games. These are games in which objects may be rolled, thrown, pushed, flipped, finger-flicked, balanced built, or otherwise moved, the outcome depending on the skill or manual dexterity of the player. However, some Skill & Action games, such as many of those involving tops, depend primarily on the "action", as the player has little control (or skill) over the object's movement.

Domino games share a category with linear tile-laying games. A modern linear tile-laying game, *Tutankhamen*, forms a pathway, the object being to move your pieces strategically and be the first player to reach the end. Though the name "dominoes" is usually used to refer to both the playing pieces and the basic game that is played with it, there are many different games that can be played with dominos, allowing "domino games" to stand as its own category.

Memory games are worthy of their own classification since they represent a particular style of game with the same game mechanism, whether the game is played with a deck of cards, a molded plastic three-dimensional unit, or as a TV-based licensed product.

A Classification of Board Games

The predominant category of games is Board Games. One of the main purposes of this paper has been to devise a system that would serve not only as an overall classification of games but specifically as a classification of board games, designed to pull together and update the proposals of Culin, Murray, Bell and Parlett.

The category of Board Games can be subdivided into games with fixed boards and those with variable boards-that is, game boards which are laid out in sections at the start of a game and therefore will most likely be different each time the game is played; the most popular example: *The Settlers of Catan* and the many variants in that series.

Here, then, are the seven categories that encompass fixed-board and variable-board board games.

- **Games of alignment**: the object is to line up your pieces in a straight line or particular order; examples are *Pente* and four-in-a-row games such as Hasbro's *Connect Four* and its many knock-offs.

- **Race games**: the object is to move one or more playing pieces across or around a gameboard in order to be the first player to arrive at a prescribed destination; the game might have one piece per player or a number of pieces (often four) that must reach the end position. There are three types of Race Games:

  1. Path game: Players begin at Point "A" and race along what may be a circuitous path to Point "B", though there may be detours and shortcuts along the way; examples are S*Snakes & Ladders*, which has one playing piece per player, and the game of India, in which each player has four pieces.
  2. Track game: Players begin at Point "A" and race along a usually oval track near the perimeter of the gameboard back to Point "A"; one circuit of the track is the norm, but more may be required; players normally are controlling one playing piece. Examples are most horse race or car race games.
  3. Goal game: the object is to be the first player to get all his pieces from a starting "home" area to a target zone, usually an opponent's home area. Examples are *Halma* and *Chinese Checkers*.

- **Games of Capture**: the object is to capture an opponent's pieces or to capture the most (or best) territory; examples are *Mancala* games, in which the playing pieces are neutral, and *Chess*, in which all playing pieces have a clearly defined power and movement, and *Reversi* or *Othello* and combinatorial games in which territory is captured either through expansion or by changing an opponent's pieces to your own. Games of capture can be **symmetrical** (players have identical pieces) or **asymmetrical**, in which each player has either a different number of pieces or pieces that have different strengths from the opposing player's pieces; piece strength may vary according to the type of space on which the piece stands. Games of capture can be classified into five subcategories:

  1. Placement Only: pieces are placed on a gameboard during turns but are not moved; example: *Reversi/Othello*.
  2. Movement Only: pieces are pre-positioned on a gameboard as per the game's criterion and are moved during turns; examples: *Checkers/Draughts*, *Chess*.
  3. Place & Move: pieces are placed as chosen by the players, then moved; example: *Stratego*.

4. War & Simulation: themed games which represent real or fictional battles; armies need to capture or kill (remove from play) opposing troops; examples: Risk and Axis & Allies.
5. *Mancala* Games: abstract games with a defined system of cyclical movement during which neutral pieces are captured and brought to your home cells.

- **Building Games**: the object is to build, develop or acquire ownership in objects or geographic areas; examples: *Acquire*, *Carcassonne*, and *Catan* games (see also immediately below).
- **Trading & Negotiation Games**: these are games in which players are required to interact in order to negotiate goals or barter for materials they need; examples are *Diplomacy* (which is also a War & Simulation game), *Clue/Cluedo*, and *Catan* games.
- **Games of Survival**: the object is, simply, not to lose your piece or pieces as parts of the gameboard disappear; examples include *Survive/Atlantis*, in which pieces of a volcanic island in the middle of the gameboard and removed, forcing islanders to vie for boats to get to the mainland, and *Isolation* (Lakeside, 1978), Niek Neuwahl's *Arctic*, both of which require a figure to be able to continue moving on the gameboard as parts of the board are removed from play, and *Vineta*.
- **Other**: Historians and researchers are loath to have a "miscellaneous" category because the implication is that their system has flaws or needs a "catchall" for exclusions or exceptions. But in a system of classification of games, not only is there likely to be a great deal of overlap of categories and subcategories, but also the development of ideas and technology in the future will introduce games that do not conform to the current system. The system is not broken because of this, but it must contain a category that allows for growth. At some point, a number of games that have found their way into the "Other" category will be seen to be similar enough to one another that a new, specific category can be formed and added to the system. In this way, this proposed system of classification of games and, more pointedly, board games, will serve to include not only ancient games but also games not yet invented.

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Alekhine's Death: Murder or natural causes?

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This is an expanded version of the paper presented to the Board Game Studies Colloquium in Lisbon in April 2008.

Abstract

On the 24th of March 1946, the World Champion Alexander Alekhine was found dead in his bedroom at the Hotel do Parque in Estoril. Over the causes of his dead there are some theories. This paper supports the murder hypothesis.

On March 24th 1946 the World champion Alexander Alekhine was found dead in his hotel room at the Park Hotel in Estoril. He was sitting on an armchair; in front of him was a table where his dinner laid; next to him was a chessboard, chess pieces left untouched. Alekhine had his overcoat on and looked as if he was asleep. A fact worth mentioning is the existence of two photographs taken by Luís Lupi, Francisco Lupi's father, a chess mastermind and regular partner to Alekhine. Luís Lupi was known for having connections with both the Portuguese political police (PVDE) and the North-American secret services.

Figure 1 Alekhine's body at the Park Hotel in Estoril.

There are several versions regarding the World champion's death. The Portuguese police tried to convey the idea of a suicide, caused by a laceration in the throat by means of a razorblade. Another version defends the theory of heart failure. Nonetheless, the official version, and the most commonly accepted, is that he died by asphyxiation due to a piece of meat caught down his throat. This was the cause of death determined after the autopsy by a pathologist named António Ferreira, who passed away recently in the United States where he resided. Let's have a look at a document written by Dr. António Ferreira on the 8th of September 1967:

"I was present at Alexander Alekhine's autopsy which took place in the Department of Legal Medicine, of the Medical Faculty, University of Lisbon. Alekhine had been found dead in his room in an Estoril hotel under conditions that were regarded as suspicious and indicated the need of an autopsy to ascertain the cause of death. The autopsy revealed that Alekhine's cause of death of (SIC) asphyxia due to a piece of meat, obviously part of a meal, which lodged itself in the larynx. There was no evidence whatsoever that foul play had taken place, neither suicide nor homicide. There were no other diseases to which his sudden an unexpected death could be attributed."

Nevertheless, this doctor could have only been an assistant during the autopsy for he was still a student at the Medicine Faculty at that time, as stated by himself to a Yugoslavian reporter, Bjelica, a statement that was later published [Bjelica, 1993]:

“Yes, I was present during Alekhine’s autopsy. Back then I was a medicine student in Lisbon. It was mere chance that I was on duty; thus I had the opportunity to attend the autopsy. I didn’t have the slightest idea of whose body that was. A colleague of mine asked me, quite carried away by the solemnity of the moment: ’Do you know who that is? It’s Alekhine.’ I was speechless. ”

Oddly enough and according to GM Anthony Saidy’s recent investigations, GM Kevin Spragget found that, without giving any detail whatsoever as to how and when, Luís Lupi had told him in a private conversation that Alekhine’s death wasn’t due to asphyxiation but due to a bullet injury. We also know that Dr. António Ferreira told some friends that Alekhine’s body had been found on the street in front of his hotel and that he had been shot. He also confessed to have been pressured by the Portuguese governmental authorities to state in the autopsy report that he had died of asphyxiation. As we already wrote in our book [Markl, 2001]:

“What we find peculiar is the peaceful position of Alekhine’s body. It is hard to believe that a person victim of asphyxiation would die with the serenity shown by the photographs and recorded in the files. Dr. António Ferreira himself states that the asphyxiated man fell on the floor. Supposedly it is easy to understand that in such a situation the victim would have kicked and choked in convulsion. But none if this is shown in the pictures.”

The photos are suspicious. It is taken for granted that when a person dies in such a place, the Police are present at the scene and no one is allowed to touch the body or contaminate the crime scene. However, the photographs taken from different angles by Luís Lupi prove that something has been changed. In the up-right corner we see a piece of furniture over which we see a glass and a book and a folded newspaper that mysteriously disappears in another photograph. This strengthens the theory that a stage has been set up to cover up the incident.

Figure 2 Alekhine's body at the Park Hotel in Estoril (compare with Figure 1).

The suspicions that my book stimulated were confirmed by a news article sent to me by GM Boris Spassky (ex-world champion), who had also been working on the events of Alekhine's death, writing a text later published in a Moscow weekly chess magazine.Spassky told me that, in a lunch he had in Paris in 1985 with Aristides Sain, a Romanian industrial who at that time lived in Estoril and was acquainted with Alekhine, he told him that in the night of the 23rd to the 24th March he received a telephone call from the police notifying him that a man had been found dead in the street. On his pocket he had a telephone book with his telephone number and so they asked Sain to identify the body. The Romanian rushed to the place and identified Alekhine. This comes to confirm what Dr. António Ferreira said: Alekhine had been murdered.

Who would be interested in such a deed and who carried out the crime? According to GM Kevin Spraggett, Alekhine was killed by a super-secret Death Squad created by the French Resistance. This Squad's main objective was to estimate and alphabetically register each and every French collaborator with the Nazi Regime. It is good to bear in mind that Alekhine had been previously naturalized as French. The convicted list had 200.000 names upon it. But this is not our opinion though.

On February the 4th 1946 Alekhine received the following letter from Mikhail Botvinnik (ex-world champion):

World's Championship.

Mr. A. Alekhine!
I regret that the war prevented the organization of our match in 1939. But I herewith again challenge you to a match for the world's chess championship.If you agree, a person authorized by myself and the Moscow Chess Club will conduct negotiations with you or your representative on the question of conditions, date and the place where the match should be held, preferably through the British Chess Federation.
I await your answer, in which I also ask you to state your ideas about the date and the place of the match. I beg you to state your ideas about the date and the place of the match. I beg you to send a telegraphic reply, with subsequent postal confirmation, to the Moscow Chess Club.

February 4th, 1946
Mikhail Botvinnik

A controversial arousal settles amidst the Soviet Chess Federation after Botvinnik's proposal. In an interview [Vainshtein, 1993], Boris Vainshtein, the former president of the Soviet Chess Federation and ex-member of the secret services NKVD, describes the session of 1945 in which a voting took place to decide whether the match was to be held or not:

"When the polemic issue of Alekhine's participation was proposed for voting, I declared that it simultaneously arouse a question concerning the continuity of my position as president. If the committee voted in favour of Alekhine's participation, then I would automatically cease my presidency. The vote was 5 to 4 against the match. Obviously I gave up my vote, since I was the president and member of the committee and since I had advanced the motion of my departure. Voting was done by raised hands and I recall that both Kotov and Ragozin (Botvinnik's trainer to be) were against Botvinnik's idea. In that moment, one of the assistants (I think it was Abramov) told Kotov: but Sasha, we decided in the party's committee meeting that the match was to take place. Kotov mumbled: 'I didn't know that... Then we shall have to repeat the voting'. We voted once again and this time around every party member raised their hands accordingly to the party's guidelines. Nevertheless, Viacheslav Ragozin, and I cannot emphasize this enough, voted against the match."

I can only assume that a movement was created in order to prevent the actual realization of this event due to this division. The London Tournament occurred between the 14th and the 26th January 1946. Alekhine wished to enter the tournament and asked Francisco Lupi to put in a good word for him among the best chess players attending the competition. Lupi was there when the *great court* or the *great judgement* took place. The assembly was gathered by Max Euwe and Arnold Denker; George Thomas, Ossip Bernstein, Tartakower, List, Friedmann, Medina, Abrahams and Herman Steiner attending. It was deliberated that Alekhine was to present himself to the French Chess Federation authorities and make his own defense case towards the accusations made against him.

This meeting occurred the day before the tournament. It was held forehand because the organization decided they needed proof that the invitation made previously to Alekhine had been redraw.

In [Parr, 2005] dedicated to the memoir of the late Arnold Denker, Larry Parr wrote as follows:

"When we began writing The Bobby Fisher I Knew and Other Stories in the early 1990s, he said that the time had come to admit the wrong he had done to Alexander Alekhine. Although a half century had passed since a player's meeting was held during the 1946 "Victory" International in London to discuss Alekhine's alleged collaboration with the Nazis, Arnold averred that it seemed as if that meeting had happened yesterday. "I found myself in anguish", he wrote of that meeting. "Back in the Depression years of the early 1930s, Alekhine lavished me with kindnesses-free dinners, superb analysis sessions, instructive practice games... He even chose me as his partner in consultation games. This king of chess treated a young, unknown player like a prince. He became my hero and chessic guiding light. And now, I found myself going along with the condemnatory herd, repaying the currency of kindness with the coin of unproved accusation... To this day, nearly a half century past, I regret that more of us did not act like... Dr. Savielly Tartakower, who publicly pleaded Alekhine's case and then, facing down the entire group, proceeded to take up a collection for the stricken champion, who was penniless in Portugal." Arnold said that he could feel Alekhine's sense of abandonment at the hands of chess players."

Faced with the Soviet Chess Federation's decision, the political power decided that killing Alekhine was the only way to prevent the match from happening. In my opinion, the secret police of both countries (the Soviet Union and Portugal - PVDE) managed to arrange a plan of action along with a hotel employee, so it seems, who allegedly gave Alekhine some poison. This might have been the cause of the symptoms that led Alekhine to go out to the chemist. The watchful criminals were waiting outside and fired the shot that killed him.

The date for the attack was meticulously calculated, for in the afternoon of the 24th of March a telegram came in the Park Hotel. It came from England and it confirmed the match. Meanwhile, it also stated that a French visa was being issued and sent. Someone was able to inform the conspirators of this telegram.

An improbable but also current idea is that Botvinnik was a NKVD Colonel and was intimately linked to the murder. The intention was not to play against Alekhine. This makes little sense because if indeed the match took place and considering Alekhine's physical and emotional state, Botvinnik's would very likely be the winner. The fact that the match didn't occur forced a tournament in which Botvinnik had to face some excellent players of the time and that surely didn't guarantee an easy victory. We mustn't forget that Botvinnik had already defeated Alekhine in the Avro International Tournament (Amsterdam, 1938) and that they have come to a tie in the Nottingham Tournament in 1936.

Unfortunately we have no documents to insure the good course of our investigation. The Portuguese secret police (PIDE-DGS) files were either destroyed or sent back to Moscow after the April Revolution in 1974. We can only to hope that one day the files will eventually show up and thus allow us to shed some light over the details involving Alexander Alekhine's death.

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Elements of chance and skill in games

David Parlett

> *In the course of a simultaneous display... I [once] said to one of my opponents, "Tell me, Mr McMahon, how long did it take you to learn to play Chess so badly?" He replied, "Sir, it's been nights of study and self-denial".* Geral Abrahams[1]

Like Gaul, games are anciently and popularly divided into three parts: games of skill such as *Chess* and *Go*, games of chance such as *Snakes & Ladders* and *Roulette*, and games of mixed chance and skill such as *Backgammon* and *Bridge*. Such categorization is patently inadequate. It is slightly more adequate to demolish the divisions and regard chance and skill as polar opposites of a single continuum, so that any given game — or any given instance of one — may be regarded as involving x per cent skill and $(100 - x)$ per cent chance.

But then skill and chance are themselves inadequate terms. Games involve many different forms of chance, some of which are perceived rather than real. A more appropriate term for this end of the spectrum is uncertainty, or unpredictability as to the outcome of a game. All games by definition involve a degree of uncertainty, for if the outcome of a game were ever entirely certain or predictable there would be no point in playing it.

At the opposite end of the spectrum lies the antidote or counter to uncertainty, which is the degree, if any, to which you can control or at least influence the outcome of a game. The opposite of uncertainty is better characterized as controllability rather than skill, as skill itself is not an atomic property: there is no such thing as a single, universal "skill at games" but rather many different types of skill. People tend to play those games for which their particular talents suit them, or, if their talent is not one of controllability, to which they are most attuned by temperament.

I am interested in exploring the elements of uncertainty or types of chance that may be encountered in games, and the corresponding elements of skill or types of controllability that may be employed to counter them.

[1] Abrahams, Gerald, *Brains in Bridge* (1962), Preface.

This exploration takes me into specialized disciplines, such as mathematics, psychology, and pedagogy, in which I have absolutely no qualifications or expertise. I write purely as a games enthusiast and inventor, and can only hope that my comments might be found to have some bearing on (a) the classification of games, (b) inventing games, and (c) games appreciation.

1 Elements of uncertainty (chance)

Randomisation

The primary, most fundamental and oldest embodiment of uncertainty is the occurrence of randomizing events such as the cast of lots or dice, for which reason many suppose games to have originated in the practice of divination. Equally fundamental, but historically more recent, is the randomization of an initial position, which is classically embodied in the deal of playing-cards from a shuffled pack.

Compulsion (lack of choice)

An element of uncertainty occurs in randomizing games that give you no choice of play. A classic example is the Indian ancestor of our *Snakes & Ladders (Chutes & Ladders)*. Why such a game should continue to exist is well explored by Salen & Zimmerman in *Rules of Play*[2]. I would paraphrase their argument by suggesting that *Snakes & Ladders* may be regarded as an overlap between, on one hand, the playing of games, and, on the other, the performance of plays, a point which I think would have appealed to Johan Huizinga. Compulsion also overlaps with divination, in that it is an essential property of Fate. The opposite of compulsion is choice, or free will, which provides an essential opportunity for the exercise of skill.

Hidden information

Careless commentators tend to lump randomization together with imperfect information, but in fact there is a significant difference between the types of informational imperfection. Taking *Backgammon* and *Bridge* as exemplars:

- *Backgammon* starts from a predetermined opening array and all subsequent moves are visible to both players. To this extent (only) the game

[2]Salen, Katie, and Zimmerman, Eric, *Rules of Play* (MIT Press, Cambridge, Mass., 2004), p. 179.

is one of perfect information at least as to the present and the past. The element of randomness is introduced by the unpredictable roll of dice, so the type of uncertainty involved here may be characterized as "future imperfect information".

- *Bridge*, on the other hand, starts from a randomized opening array. Thereafter, however, it is entirely free from uncontrollable randomizing eventualities. It is, therefore, a game of "past imperfect information".

The differences are significant in that they call for distinctive skills in order to exert some degree of control over their outcome.

- In *Backgammon*, skill consists in gradually setting up positions in which you can benefit from a greater proportion of possible future casts than your opponent, whom you try to manoeuvre into such a position that very few possible casts are favourable.
- In *Bridge*, skill consists in deducing or inferring the lie of cards in other players' hands, initially by means of the auction, and subsequently by playing your cards in such a way as to uncover existing information in time to take advantage of it. Bridge and other intelligent card games are therefore not so much games of imperfect information as games of perfecting information.

Besides perfect and imperfect information, further distinctions may be drawn between *certain* and *uncertain* information, and complete and incomplete information; but let this suffice for the moment[3].

Inequality and indeterminacy

Inequality is associated with a random and therefore indeterminate opening array such as the initial distribution of cards. Its effect is reduced by sufficiently increasing the number of deals that constitute a whole game. It is also inherent in asymmetric board games like *Hnefatafl* and *Fox & Geese*, where the two players have different forces and different objectives; but here, too, it is easily overcome by playing an even number of games and alternating the positions. You might say that it is a feature of most combinatorial games, at least in so far as it is usually advantageous to move first.

Here it might be relevant to note that one of the ways in which games evolve is by deliberate alteration of the rules in order to reduce potential

[3]See Wikipedia, s.v. *Complete information.*

inequalities of players' experience. If you subscribe to the online newsgroup `rec.games.abstract` you will be aware of a current interest in devising forms of *Chess* with indeterminate openings, such as *Fischerchess* (aka *Chess960*). In these variants the opening array is unknown in advance, but is fully open before play begins so there is no lack of information. What is lacking here is an experience of playing with a particular configuration of pieces. On these grounds one might propose novelty as a carrier of uncertainty, and, for its counter, experience as a component of skill.

Opacity

An element of uncertainty, or at least uncontrollability, is induced by the opacity of a game. By *opacity* I mean the opposite of *clarity*, a property I think Robert Abbott was the first to point in an article entitled "Under the Strategy Tree".[4] Abbott writes:

> Clarity is essentially the ease with which a player can see what is going on in a game... [It] has nothing to do with simplicity, or even with elegance. Edward de Bono's L-Game is elegantly minimal — it uses only four pieces and is played on a board of only 4×4 squares. It is not, however, clear. I find it very hard to picture what the board will look like when I turn my 'L' over, I find it harder still to visualize my opponent's responses, and it's impossible for me to look ahead to my next move. A game can be simple yet lack clarity, and conversely a game can be complicated but still clear. Playing a game soon reveals its degree of clarity. The greater the clarity of a game, the farther you can see into it, and therefore the greater its depth for you.

Perplexity (= perception + complexity)

Chance and skill may differ according to whether they are perceived from the inside, subjectively by a player, or from the outside, objectively by an observer. As Salen and Zimmerman put it, "the perception of randomness is more important than randomness itself". They quote the example of Chinese Checkers with four or more players:

> As the game unfolds..., the centre... becomes crowded with a seemingly random arrangement of pieces... even though every

[4] Abbott, Robert, "Under the Strategy Tree — the concept of Clarity in games", *Games & Puzzles* (Series 1), No 36, May 1975, pp. 4–6.

> single move... is the result of a player making a strategic choice about where to play next. If you closed your eyes and opened them only when it is your turn to move, it might seem like the board is merely reshuffling itself, particularly in the middle... game, when the centre area is most crowded. This feeling of randomness is only an illusion, however, as there is no formal chance mechanism in the game. [Hypothetically] logical players... wouldn't feel any randomness: they could look at the board and immediately trace every move back to a series of strategic decisions. However, for human players, this feeling of randomness is an important part of what makes the game fun to play... [T]he feeling of randomness creates a sense of open-ended possibility and players are rewarded for taking advantage of chance configurations on the board... Seeing a pattern emerge out the chaos that allows you to jump a piece back and forth all the way across the entire length of the game board is a moment of wonderfully meaningful play.[5]

Another subjective perception of chance comes into play when a Chess master plays a novice. Both have perfect information as to the moves and changing positions, but whatever the weaker player does, the stronger sees through it to what led him to make that move, what he has in mind, what situation it might ultimately lead to, and how to circumvent or take advantage of it. The weaker, however, will often be baffled by the master's unforeseen move with what are to himself unforeseeable implications. As far as the novice is concerned, he might as well be confronting a completely random move determined by the roll of a die, and exhibiting no perceptible past cause or future effect. As Philip Ross puts it:[6]

> The feats of chess masters have long been ascribed to nearly magical mental powers. This magic shines brightest in the so-called blindfold games in which the players are not allowed to see the board...

Ross's reference to magic reminds me of the converse of perceived but unreal chance, namely that of perceived but unreal skill. An example is provided by Dennis Tedlock:[7]

[5] Salen, Katie, and Zimmerman, Eric, *Pop. cit.*, p. 176.

[6] Ross, Philip, "The Expert Mind" *Scientific American*, August 2006.

[7] Tedlock, Dennis, Introduction to Culin, S, *Games of the North American Indians — Volume 1, Games of Chance*, Nebraska University Press, 1992, p. 23.

> We might call it a "game of chance," which is what Culin calls similar games in this book, but that expresses the point of view of an observer. Meanwhile, the participants constantly think in terms of strategy, pitting their wishes against chance in momentary acts of magic, which is what we all find ourselves doing when we throw dice. / [A] paradox of the Zuñi game of wooden dice is that, technically, it is not what Culin calls a "game of dexterity," and yet the players do try slightly different ways of handling the sticks, as if they could influence the outcome of a throw. [...] So if there is any dexterity here, it must remain on the side of magic [...]

Extrinsics

We might mention, for completeness, the existence of chance factors that are extrinsic to the game. I first became aware of these in my schooldays, when my friend's mother raised some objection to our playing *Chess* on a Sunday. "It's a game of skill", we protested, not a game of luck". "Well", she replied, "You're lucky if you win, aren't you?". It was at this point that I realised that (a) luck and chance are not the same thing, and (b) you are indeed lucky if in a Chess tournament you happen to be drawn against a weaker player, or one whose opening you have just been mugging up, or one who happens to be feeling unwell at the time. Of such chance factors, no more need be said.

2 Controllability (skill)

Cheating

The best way to exercise control over the outcome of a game is to cheat. This pits maximum controllability on your behalf against baffling uncertainty on the part of your victim, who, if sufficiently gullible, may look upon your constant success as a form of magic. And why not — when you consider that cheating and magic are little more than differing interpretations of the same conjuring trick?

Memory — past and future

The importance of memory is obvious, but its ramifications are subtle and it would be impossible to outline them here without going into disproportionate detail.

Abrahams, in The Chess Mind,[8] holds that memory is easily overestimated, especially if it is taken to imply remembering a number of standard openings. Very long retentiveness, indeed, is "often a concomitant of minds lacking in originality". More important than the consciously recollected "is that set of mental habits which smooths the action of the mind", a capability best described as "technique", and most relevant in the endgame. Of greatest significance, however, is what he refers to as "holding in [one's] mind a clear conception that is in part constituted by the memory of what will have happened, i.e. what has already happened as a mental event".

Here we find ourselves talking about the forward visualisation involved in combinatorial games like Chess. What do we mean by forward visualisation? At first sight we mean looking ahead to our next move and to the sequence of moves likely to result from it. This has been described as examining the branches of the strategy tree, and is something that computers are very good at. In human terms it seems like a form of memory, only in reverse, in that we are following a sequence forward into the future rather than backward into the past. I have always described this ability as mental "projection", in that we are projecting ourselves into the future. Abrahams refers to it simply as "vision". But in fact "future recall" or "reverse memory" is a pretty good term for it, as experiments have shown that exactly the same parts of the brain light up as when it is engaged in tracing backward memories.[9]

Apperception and experience

Against this, however, must be set the discovery that Chess masters do not normally go down this analytical route on a step-by-step basis. Capablanca, on being asked "How many moves do you see ahead?", is said to have replied "Only one — but it's always the best one". As Philip Ross observes:[10]

> He thus put in a nutshell what a century of psychological research has subsequently established: much of the chess master's advantage over the novice derives from the first few seconds of thought. This rapid, knowledge-guided perception, sometimes called apperception, can be seen in experts in other fields as well. Just as a master can recall all the moves in a game he has

[8]Abrahams, Gerald, *The Chess Mind*, London 1951.

[9]Marshall, Jessica, "Future recall: your mind can slip through time", *The New Scientist*, 24 March 2007, pp. 36–40.

[10]Ross, *op. cit.*

> played, so can an accomplished musician often reconstruct the score to a sonata heard just once. And just as the chess master often finds the best move in a flash, an expert physician can sometimes make an accurate diagnosis within moments of laying eyes on a patient.

How do they do this? They see total situations as in a photographic memory, to such an extent that it seems to an outside observer — or to their hapless opponents — more like a stroke of intuition than a process of cerebral analysis and future recall. Ross notes that it was in 1894 that Alfred Binet, co-inventor of the first intelligence test, hypothesised that Chess masters achieved an almost photographic image of the board, but he soon concluded that the visualization was much more abstract, resembling, rather, the same kind of implicit knowledge that the commuter has of the stops on a subway line. The expert relies not so much on an intrinsically stronger power of analysis as on a store of structured knowledge, enabling him to reconstruct any particular detail at will by tapping a well-organized system of connections. A weaker player, confronted with a difficult position, may calculate for half an hour, often looking many moves ahead, yet miss the right continuation, whereas a grandmaster sees the move immediately, without consciously analyzing anything at all. In brief, experts rely more on structured knowledge than on analysis.

Ross goes on to invoke the theory of information "chunking" developed by [Herbert A.] Simon and [William] Chase, of Carnegie Mellon University. Simon explained the masters' reconstruction of Chess positions with the aid of a model based on meaningful patterns called chunks, enabling them to manipulate vast amounts of stored information that would be expected to strain the working memory beyond its normal capacity to contemplate more than seven items at a time.

It occurs to me that a metaphor other than chunking, but achieving the same effect, might be termed "networking". The novice, examining a complicated *Chess* position, sees little more than a collection of pieces occupying specific squares, unconnected with each other, like a cloud of dots on a page. The master, however, upon glancing at the same position, immediately "sees", in his mind's eye, not just a collection of discrete dots but also a network of interrelationships between them.

Intuition

An alternative explanation is offered by Lois D. Isenman in her paper on Intuition.[11]

> Intuition as a bridging function brings the power of the unconscious into conscious thought. Through intuition, the unconscious with its vast memory banks, its associative accessing system, its speed, and its ability to process multiple items in parallel, greatly enriches the ability of conscious mental activity to manipulate logic and construct empiric tests... As dreams demonstrate, the unconscious frequently communicates in the language of symbols. In symbol formation, each object can be represented by multiple associative categories. In symbolic expression, any one of these aspects can stand for the whole item; however, symbols often simultaneously encode a number of different levels, presenting a richly textured and very often surprising understanding of the object under consideration. In the unconscious, in effect, each item is categorized by all its different component parts, as well as its descriptive, situational, and affective associations. Intuitions very frequently come through to awareness in symbolic form and tend to share in the rich and unexpected quality that characterizes unconscious mental processes... Associative processing, an important component of symbol formation, plays a central role in intuition whether or not intuition is expressed in consciousness in symbolic form.

Intuition may be closely allied to the skills of deduction and inference required of intelligent card games. Does it also shade into elements of extrasensory perception as might be the case in the game of Pelmanism, also known as Memory? If so, are we also encroaching on the borders of what some might categorise as "magic"?

Inference and deduction

When I was a teacher in my early twenties I played a lot of *Chess.* At one school most of the staff played *Kriegspiel*, even those members who did not normally play Chess. I soon discovered, to my surprise, that I was better at *Kriegspiel* than at *Orthochess.* Then I noticed that the annual Kriegspiel

[11] Isenman, Lois D., *Toward an understanding of intuition and its importance in scientific endeavour*, 1997.

tournament was invariably won not by the strongest Chess players but by the strongest Bridge players. It was obvious that one of the skills demanded by *Kriegspiel* was that of deduction or inference as to the positions of the playing pieces; that this was a skill particularly demanded of card-players; and that it might therefore be appropriate for me specialize in card games rather board games. In reading the literature on games I also became aware that *Chess* players tend to look down on card-playing on the grounds that cards are not games of perfect information, with the further implication that games of perfect information require more skill than games of imperfect information, which are therefore to be equated with games of chance. Mortimer Collins, in Attic Salt (1887), writes:

> There are two classes of men, those who are content to yield to circumstances, and who play Whist; and those who aim to control circumstances, and play Chess.

But of course this is nonsense. Information is not absent from strategic card games: rather, it is released gradually as cards are played or announcements made, and much of the information that has not yet been revealed is to be deduced or inferred — or even 'intuited' — from that which has. The acquisition of information is as much the goal of strategy in strategic card games as the positional moves made as a result of the knowledge acquired. Indeed, in the higher trick-taking games positional moves may be made specifically for the purpose of acquiring information, even at the expense of loss of material — a device equivalent to the gambit at Chess.

Specific and peculiar skills

Another question from personal experience. My favourite abstract board game is the game of *Pentominoes* originally proposed by Solomon Golomb and sometimes referred to as Golomb's Game. Why do I enjoy this game so much more than games of the *Chess / Draughts* variety? I would say that it involves what I call the packing skill. It's interesting that I happen to be very good at efficiently packing suitcases, loading excessive amounts of luggage into the car when going on holiday, and finding new ways of rearranging my expanding collection of books and videos without taking up much more space than they did when I started.

In what way does Pentominoes differ from (say) *Chess* or *Draughts*? My first observation is that it is a game of placement rather than movement. It starts with an empty board and play proceeds with each in turning placing a piece but not thereafter moving it. The same applies to another game I enjoy, namely *Reversi (Othello)*. On these grounds, I often wonder whether I might have had some aptitude for *Go*, also a game of placement rather than movement, had I only discovered it earlier in life.

This is one example of how the classification of games may relate to a classification of different types of skill involved in playing them, and perhaps also taste and temperament. It is not surprising that some people will play only word games, and some only war games or fantasy games.

Types of intelligence

I have long been seeking a classification of mental skills that might be applicable to games classification. In other words, can we classify games by reference to the skills required in playing them rather than directly by reference to the contents of the games themselves? Eric Solomon tells me that this was the basis of a booklet he was planning some years ago, but found too difficult to follow through. The most promising line of enquiry that I have discovered derives from Howard Gardner's concept of "multiple intelligences".

Gardner originally distinguished seven types of intelligence as follows: Linguistic, Logical-mathematical, Musical, Bodily-kinaesthetic, Spatial, Interpersonal, and Intrapersonal. He later added an eight, designated Naturalistic. I will comment on these in a different order, which I think more relevant to their application to games.

Spatial intelligence: the potential to recognise and manipulate the patterns of wide space (those used, for instance, by navigators and pilots) as well as the patterns of more confined areas (such as those of importance to sculptors, surgeons, chess players, graphic artists, or architects).

> Obvious in most board games, with possible exception of *Mancala*; irrelevant to most card games (even *Patience*) except those designed to imitate board-game activity.

Logical-mathematical intelligence: the capacity to analyze problems logically, carry out mathematical operations, and investigate issues scientifically.

> Is this involved in the type of forward thinking relevant to most abstract board games? Does it relate to the deduction/inference skill of most card games? If not, how does deduction/inference

> fit into Gardner's schema? Should we not also posit some kind of creative intelligence?

Musical intelligence: skill in the performance, composition and appreciation of musical patterns.

> I'm not aware of any musical games, but it is worth noting the frequent association of skill at Chess with advanced musical skills. Many good musicians are good Chess-players, and vice versa. In medieval universities, music and mathematics were closely linked.

Linguistic intelligence: sensitivity to spoken and written language, the ability to learn languages, and the capacity to use language to accomplish certain goals.

> At first sight this would appear to be relevant only to word games. But consider what the essence of language is: it is the ability to encode our multi-dimensional experience of the world and our interaction with it into a linear stream of a limited number of discrete sounds, from 20 to 50 according to the language we use. This seems to me clearly related to both the logical-mathematical and the musical intelligence.

Interpersonal intelligence: a capacity to understand the intentions, motivations and desires of other people and, consequently, to work effectively with others.

> In other words, a theory of mind. If we convert the phrase "work effectively with others" into "work effectively against others", we find this obviously fundamental to the skills required to play any strategic game against a live opponent. An interesting sidetrack in this day and age, of course, is how relevant this ability is to playing against computer software. Interpersonal intelligence is especially relevant to all intelligent card games.

Intrapersonal intelligence: a capacity to understand oneself, to have an effective working model of oneself — including one's own desires, fears and capacities — and to use such information effectively in regulating one's own life.

> *Poker.* Need I say more?

Bodily-kinaesthetic intelligence: the capacity to use one's whole body or parts of the body (like the hand or the mouth) to solve problems or to fashion products.

> Obviously relevant to outdoor sports and to games of dexterity, manipulation, and hand-eye coordination. It would appear to be related to spatial intelligence.

Naturalist intelligence: relates to observing, understanding and organizing patterns in the natural environment. A naturalist is someone who shows expertise in the recognition and classification of plants and animals.

> (No suggestions.)

Re "creative intelligence"

There must be such a thing as creative intelligence, but I'm not quite sure where it fits into Gardner's scheme of things. I would take it to be at least related to, if not a form of, the skill of inference already mentioned. Abrahams observes:[12]

> What distinguishes the player of any of the best-known card games from the player of one of the main board games is that the former frequently analyses, whereas the latter always synthesises...

Synthesizing is a form of creativity, and inference a form of inductive reasoning, classically tested in Abbott's celebrated game of Eleusis[13] — which, *pace* Abrahams, is not a board game but a card game.

Intelligence is not the same as skill

Interesting as the concept of multiple intelligences may be, I feel that it doesn't entirely answer my enquiry into the skills involved in game-playing; for a skill is not the same as an intelligence: rather, it is the application of an intelligence, which is not only a skill in itself but also introduces another potential element of differentiation into the subject. In brief, I have still not been able to track down a classification of mental skills as distinct from the mere intelligences on which they may be based.

[12] Abrahams, Gerald, *Brains in Bridge*, 1962, p. 132.

[13] Abbott, Robert, *Abbott's New Card Games*, New York, 1963, pp. 73–92. See also `<http://www.logicmazes.com/games/eleusis/index.html>`

Endquote

Arthur C Clarke's Third Law of Prediction[14] states 'Any sufficiently advanced technology is indistinguishable from magic'. Perhaps we may say the same of any sufficiently advanced intelligence.

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[14]Clarke, Arthur C., *Profiles of the Future*, 1962.

Draughts and Academies des Jeux

Jürgen Stigter

Introduction

How do you know when and where a game was played and how popular it was?

The evidences come from written accounts and illustrations, game artifacts found and linguistic analyses. But the reason for mentioning a game — or not — are erratic, so it is very difficult to draw firm conclusions from this evidence.

E.g., it may be the case, that *Chess* was a "sexy" game, about which much was written, though it may not have been played much, while on the other hand draughts was often played, but not a game you would like to write about. It would not be interesting to explain its rules, because these were well-known!

Following a query from Arie van der Stoep,[1] I went through the many editions from *Academies des Jeux* (which all contain *Chess*). A frontis, first published in the 1721 edition shows that *Chequers* was played and must have had some popularity, but there is no reference in the text itself and no description of the game in any edition until the end of the 18th century! Can one conclude that *Chess* was less well-known?

Draughts and *Academies des Jeux* in the 17th century

The standard work for books on (board)games upto 1700 is Manfred Zollinger's *Bibliographie der Spielbücher des 15. bis 18. Jahrhunderts. Erster Band: 1473–1700.* Stuttgart: Anton Hiersemann, 1996. For the bibliographical description of such books, I'll refer to description with the number in this bibliography. For académies des jeux in French, I will also refer to another standard work, Thierry Depaulis's *Les loix du jeu. Bibliographie de*

[1] Chequers historian.

la littérature technique des jeux de cartes en francais avant 1800 / Suivie d'un supplément couvrant les années 1800–1850. Paris: Cymbalum Mundi, 1994.[2]

The chronological search in my library starts with *La maison des jeux academiques*. In the edition of 1668,[3] the following request is placed at the end of the table of contents: "Le Lecteur est prié, s'il a quelques Ieux parciculiers qui ne soient point dans ce Receuil, de les vouloir donner, pour adjouter a ce Liure, & le Libraire fera vne honneste reconnaissance."[4]

But without result: still *no Chequers* in *any* later edition, though this game was well-known already in 1654. Kruijswijk[5] cites the first edition (1654) which places *Tricque-trac* above *Chess* and *Chequers*:

"Car les dez, les eschecs, les dames, ... ce sont tous les jeux communs et populairs, où il y peut avoir beaucoup de fraude, et peu d'esprit. Mais au grand tricque-trac, il n'y a que les gens d'honneur qui le pratiquent, et encore les plus spirituels, actifs et vigilants, qui le peuvent comprendre."[6]

Another quote given by Kruijswijk shows that *Chequers* was very popular in 1668; it is taken from Francois Mallet's *Le iev des dames*:

"on en sera facilement persuadé, si on considère, qu'il n'y a point d'honnêtes maisons, òu il n'y ait un damier au moins, et souvent deux et plus, et qui même il n'y a personne qui ne joue aux dames, ou au moins qui n'en connaisse le jeu, qui est aussi commun chez les rois, princes, seigneurs, gentils-hommes, et bourgeois, que chez les soldats, matelos, artisans et autres personnes populaires, et qu'il n'y a pont de personne de qualité, ni de cavalier d'honneur, qui allant en campagne, ou ã la guerre, ne fasse porter un damier dans son équipage."[7]

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[2]Both [Zollinger 1996] and [Depaulis 1994] are still in print.

[3]La — maison — des ievx — academiqves, — contenant — vn recveil general de tous les Ieux diuertissans — pour se réjoüir & passer le — temps agreablement. — Et augmentée de la Lotterie plaisante. — A Paris, — Chez Estienne Loyson, au Palais, — a l'entrée de la Galerie des Prisonniers, — au Nom de Iesvs. — M. DC. LXVIII. — Avec privilege dv roy. [= Depaulis #16, Zollinger #121 (Anonymous), refers to first ed. Zollinger #103, ed. 1654: La Mariniere [Depaulis #11: Jean Pinson de La Mar(t)iniere ?]].

[4]"The reader is asked, in the case that certain games are not in this collection, to give these, to add to this book, & the publisher will show an honest gratitude." [Zollinger Nr. 121, p. [viii]; my translation].

[5][Kruijswijk 1966], p. 94, from p. 41 of *La maison academique* 1654 (Zollinger no. 103). Repeated on p. 109 of the 1668 edition.

[6]Because dicing, chess, draughts, ... are all common and populair games, where there can be lots of cheating and little thinking. But the grand *Tricque-trac*, is only played by gentlemen, and then only the most clever, active and vigilant ones who can understand it. [My translation].

[7][Kruijswijk 1966], p. 95, from p. 267 of Mallet 1668 (Zollinger No. 122): "you will be

From the index of Zollinger, it is clear that very little was written on *Chequers*: till 1700, only six books with *Chequers* (of which five were published in Spain) are listed, while *Chess* has over hundred entries and the "noble" game of *Tricque-trac* twenty six!

Interestingly, the use of *j'adoube* (I adjust) in *Chess* appears to have been derived from *Tricque-trac*. It is already used in 1668:

"En ce jeu c'est vne maxime tres-inuiolable, & qui deuoit estre aussi des premieres, que Dame touchée, Dame joüée, si l'on ne dit ce mot, i'addoube, horsmis dessous le bois; car si l'on en peut tenir dans ses mains, à plus forte raison y peut-on toucher innocemment, & sans en estre repris, & encourir ny perte ny contrainte de joüer: C'est mon auis que ie soumets à tout autre meilleur en ses raisons, ou en ses regles & pratiques."

The next book I looked at was *The complete gamester* after Charles Cotton (first ed., 1674[8]). In all editions, only Gentile Games are given — hence no *Draughts*! *Chess* is often called "the royal game", apparently with the double meaning of game of and game for kings. The fifth edition of the *The complete gamester* (by Richard Seymour, 1734) is "Written for the Use of the Young Princesses". In the description of *Chess*, the draught-board (*Chequers* board) is used to explain the *Chess* board and to be used instead, showing that *Chequers* was better known and more often played than *Chess*:

"III. The Theatre upon which this Game is acted, is a chequered Board, half black, and half White, painted like a Draught-board, which may serve for this Use upon Occasion."[9]

easily convinced, if you consider, that there are no honest houses, where there is not at least one *Chequers* board, but often two or more, and that even no one doesn't play *Chequers*, or at least knows the game, which is as common to kings, princes, lords, gentlemen and bourgeois, as to the soldiers, mariners, craftsmen and other lower people, and that there is no higher person, nor cavalier d'honneur, who will travel or go to war without taking a *Chequers* board in his equipment." (my translation). To give a taste of the curious book, here is the full title: Pierre Mallet: Le — iev des dames. — Avec toutes les Maximes — & Régles, tant générales — que particuliéres, qu'il faut — observer an icelui. Et la Méthode d'y bien joüer. — Ortografe nouvéle, & rézonée, sui- — vie par l'ordre de l'Alfabet, par — lequel on se pora aûsi pronte- — mant, que parfétemant instruire — an icêle. — Le tout acompagné de pluzieurs discours,autorités & rézonemans — instructifs, tirés de la Morale, de la Politique, & de l'Istoire. — A Paris, — Par Mr PIERRE MALLET, Ingénieur — ordinére du Roy, & Proféseur aux — Siances Matématiques, rüe de la Hu- — chéte, an l'Académie de Mr. de la Sale, — Mêtre d'Armes. — Et av pale's, — Chés Th. Girard an la grand Sale. 1668. — Avec Privilége de sa Majesté.

[8]Zollinger No. 137.

[9]*The complete gamester* (by Richard Seymour,1734), First part: The Royal Game of Chess, p. 125.

18th century — conclusion

In the 18th century, not much changed. The academie des Jeux contain no draughts. Draughts was (more) well-known and often played than *Chess*, but *Chess* was the noble or royal game. Chessplayers knew *Chequers* as well. The situation is symbolized by the frontis of the 1721 edition of the *Academie de jeux* (see picture), showing gentlemen playing *Chess* — very respectable — in the foreground and gentlemen playing *Draughts*, but kept in the background.

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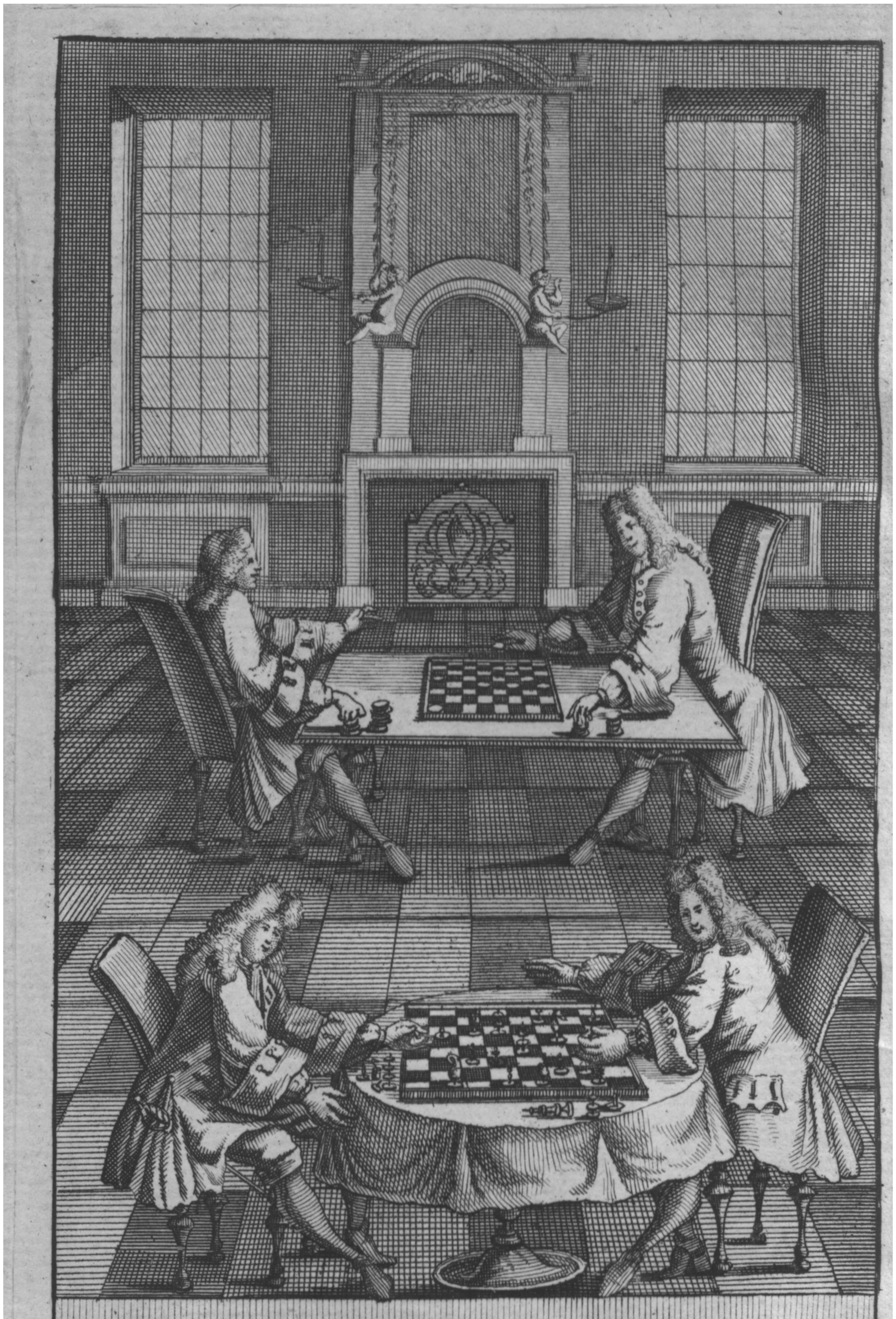
[Kruijswijk 1966] K.W. Kruijswijk, *Algemene bibliografie van het damspel.* Den Haag: van Goor Zonen, 1966.



Proceedings of Board Game Studies Cooloquium XI, pp. 87–91

Origami's Geometry

Liliana Monteiro

Introduction

Origami is the Japanese art of folding paper.

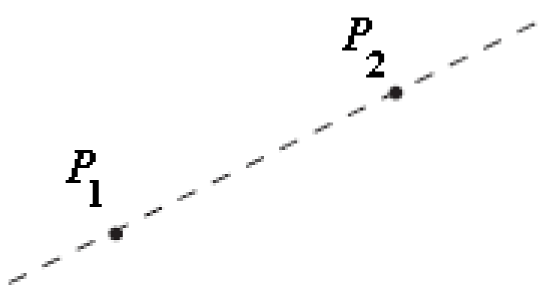
Mathematically, it can be identified with a reflection on a line, named the folding line.

The operations that are possible with points and lines in Origami, with one single fold, are described in seven axioms that are due to the mathematicians Huzita (who is responsible for the first six axioms) and Hatori (responsible for axiom seven). In the following, we will see what these axioms allow.

Huzita-Hatori Axioms

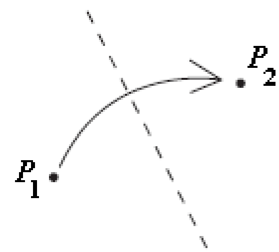
(O1) Given two points P_1 and P_2, we can fold a line connecting them.

[This axiom consists in folding the line that passes through the two initial points.]



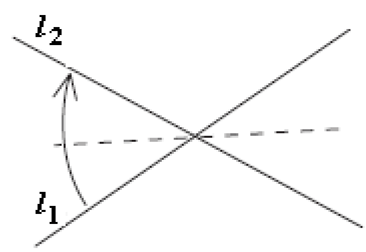
(O2) Given two points P_1 and P_2 we can fold P_1 onto P_2.

[This axiom consists in folding through the perpendicular bisector of the line segment defined by the two initial points.]



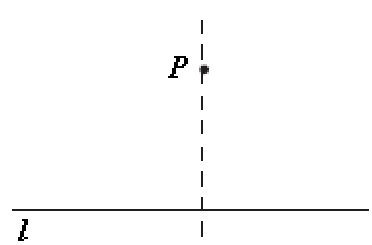
(O3) Given two lines ℓ_1 and ℓ_2 we can fold line ℓ_1 onto ℓ_2.

[If the lines aren't parallel, this axiom consists in folding through the bisector of two vertically opposed angles. If the lines are parallel, this axiom consists in folding through a line that is parallel to the initial ones and is at the same distance from both of them.]



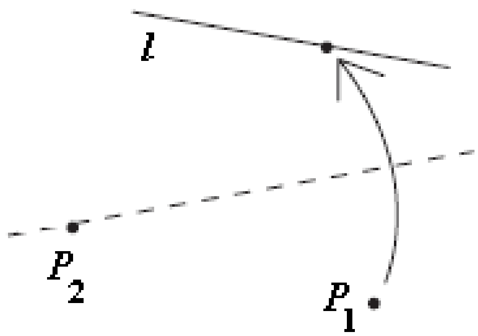
(O4) Given a point P and a line ℓ we can make a fold perpendicular to ℓ passing through P.

[This axiom consists in folding through a perpendicular to the initial line that passes trough the point.]



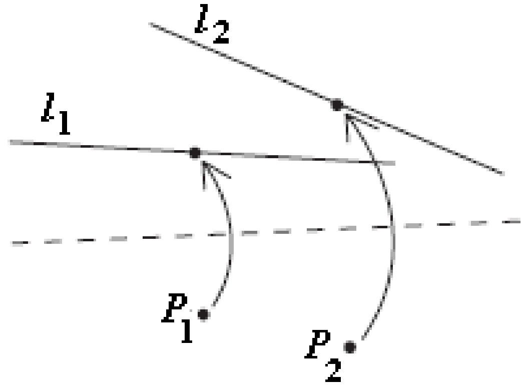
(O5) Given two points P_1 and P_2 and a line ℓ, if the distance between P_1 and P_2 is equal or superior to the distance between P_2 and ℓ, we can make a fold that places P_1 onto ℓ and passes through the point P_2.

[This axiom consists in folding through the line that passes through P_2 and the middle point of the segment defined by P_1 and an intersection point of ℓ with the circle with center P_2 and radius equal to the length of $[P1P2]$. More exactly, this consists in folding through a line tangent to a parabola with focus P_1 and directrix ℓ.]



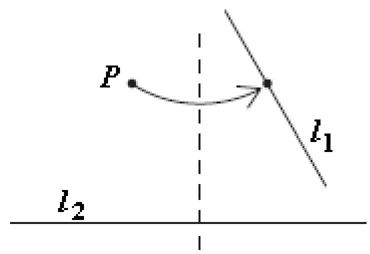
(O6) Given two points P_1 and P_2 and two lines ℓ_1 and ℓ_2, if the lines aren't parallel and if the distance between them isn't larger than the distance between the points, we can make a fold that places P_1 onto line ℓ_1 and places P_2 onto line ℓ_2.

[This axiom consists in folding a line simultaneously tangent to two parabolas.]



(O7) Given a point P and two lines ℓ_1 and ℓ_2, if the lines aren't parallel, we can make a fold perpendicular to ℓ_2 that places P onto line ℓ_1.

[This axiom consists in folding through the perpendicular to ℓ_2 that passes through the middle point of $[PI]$ where I is the point resulting from the intersection of ℓ_1 with the parallel to ℓ_2 that passes thought P.]



Is the list complete?

Now that we have seen all axioms, there is a question left to answer: does this list include all possibilities for one single fold?

The answer to this question is due to Robert Lang, in his paper "Origami and Geometric Constructions" of 2003. Let's see it.

As said before, the basic idea of Origami is of a reflection in a line.

Let's consider lines ℓ, ℓ_1, ℓ_2 and ℓ_F (resulting from a folding); and points P, P_1 and P_2; in order to see the alignments between these elements and find out the degree of the equation we need to solve to perform these operations. Let's pay more attention to nontrivial cases, that is, the cases where we need to create new points or lines:

Symbol	Description	Degree
$r(P_1) \leftrightarrow P_2$	Fold point P_1 to another point P_2	2
$r(P_1) \leftrightarrow \ell$	Fold point P to line ℓ	1
$r(\ell) \leftrightarrow P$	Fold line ℓ to point P	1
$r(\ell_1) \leftrightarrow \ell_2$	Fold line ℓ_1 to different line ℓ_2	2
$r(\ell) \leftrightarrow \ell$	Fold line ℓ onto itself	1
$r(P) \leftrightarrow \ell_F$	Align point P with the fold line	1

The alignment that aligns a line ℓ with the fold line ℓ_F is not considered above because it is trivial, as it doesn't creates a new element. We just need to fold by an already existing line.

To define a line we need to determine the values of two variables. This means we need a degree two equation.

In the cases of alignments of two different points and lines, we already have them. These operations correspond to axioms (O2) and (O3).

$r(P_1) \leftrightarrow P_2$	Axiom 2
$r(\ell_1) \leftrightarrow \ell_2$	Axiom 3

In the cases where the alignments have only one degree we can join them two by two:

	$r(P_2) \leftrightarrow \ell_2$	$r(\ell_2) \leftrightarrow P_2$	$r(\ell_2) \leftrightarrow \ell_2$	$r(P_2) \leftrightarrow \ell_F$
$r(P_1) \leftrightarrow \ell_1$	Axiom 6			
$r(\ell_1) \leftrightarrow P_1$	Axiom 6	Axiom 6		
$r(\ell_1) \leftrightarrow \ell_1$	Axiom 7	Axiom 7	$*$	
$r(P_1) \leftrightarrow \ell_F$	Axiom 5	Axiom 5	Axiom 4	Axiom1

The alignment that places two lines onto themselves ($*$) has no solutions if ℓ_1 and ℓ_2 are nonparallel and infinitely many solutions if they are parallel.

Each of the remaining pairs correspond to one of the Huzita-Hatori axioms. Since these represent all possible alignments that create exactly two degrees of freedom, this shows that the axiom list is complete (and that Hatori's seventh axiom is indeed necessary for completeness).

Solving Quadratic Equations

The quadratic formula tells us that if we know how to perform the operations of addition, subtraction, multiplication, division, and square roots, then we know how to find the roots of any quadratic equations.

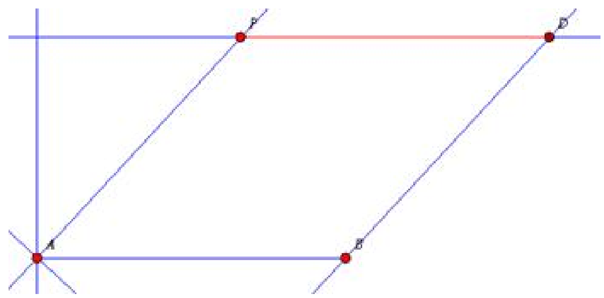
Let's proove that Origami allows all these operations. For that, it will be considered that each number is the length of a line segment.

Addition and Subtraction

To add or subtract one length to another we only need to transfer one length to the point P, the end of the other length.

For that, it's only necessary to fold a perpendicular to AB that passes through A (or B) and a perpendicular to that line that passes through P. Next, it's required to fold a perpendicular to AP that passes through A (or B) and perpendicular to that line that passes through P. All these operations are possible by Axiom 4.

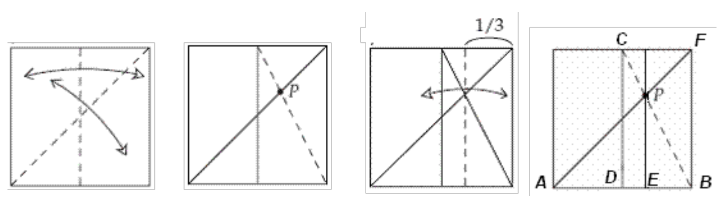
The initial length is equal to the distance between P and the intersection point of the two last lines created in each one of the steps.



Division

To divide a length in two equal parts, it's only necessary to fold it in the middle.

To divide it in thirds, we can follow the instructions below, where A is the origin and the paper's side is one unit long.



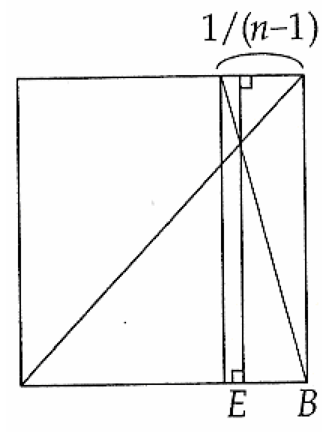
Let's proove that this gives that division. By construction, $AF : y = x$ and $BC : y = -2x + 2$. Intersecting these two lines, we get

$$x = -2x + 2 \Leftrightarrow x = \frac{2}{3} \cdot$$

as we wanted to prove.

To divide in more than three parts, we can follow the previous instructions, but, instead of folding the paper in the middle vertically, we have to fold at $1/(n-1)$ units from the border.

To get this folding it is only necessary to use the method in a recursive way.



Multiplication

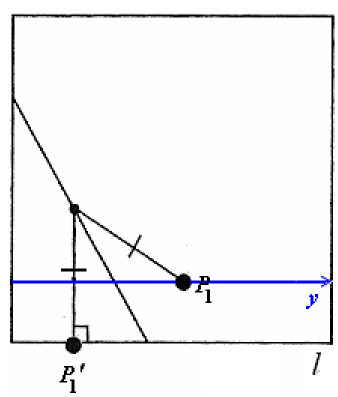
To multiply by a number n, it is only necessary to notice that $n = \lfloor n \rfloor + (n - \lfloor n \rfloor)$, where $\lfloor n \rfloor$ is the integer part of n. This way, multiplication is just an extension of adding and dividing.

Square Root

Let r be the length whose square root we want to find. Let's consider the point $P_1 = (0, 1)$ and the line ℓ be defined by $y = -1$.

We can construct the folding that places P_1 onto ℓ, passing through point $P_2 = (0, -r/4)$ (possible by Axiom 5).

Let $P_1' = (t, -1)$ be the image of P_1 under the folding/reflexion.



The line created by the folding is the bisector of $[P_1P_1']$. Thus, the folding line and P_1P_1' are perpendicular lines, and $M = (t/2, 0)$ is the middle point of the line segment $[P_1P\text{"}_1]$. This way, the equation of the folding line is

$$y = \frac{t}{2}x - \frac{t^2}{4} \cdot$$

As this line passes through P_2,

$$-\frac{r}{4} = \frac{t}{2} \times 0 - \frac{t^2}{4} \Leftrightarrow t = \sqrt{r}\,,$$

witch means that the coordinates of P_1' give the value of the square root.

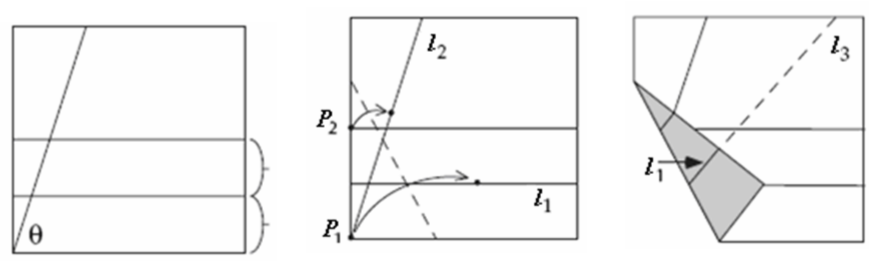
We have now the proof that Origami's Geometry allows solving any second degree equation.

Solving Cubic Equations

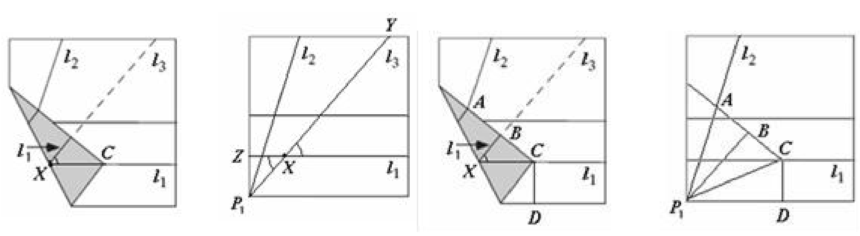
Origami's geometry also allows to solve cubic equations. This is possible by Axiom 5. Let's see some examples.

Angle Trisection

To trisect an angle, we just need to follow this procedure. In this case, we represent an acute angle, but it can be generalized to any angle.



We will prove that this procedure does actually trisect the angle.



The angles YXC and ZXP_1 are equal by construction.

Also by construction, $\overline{AB} = \overline{BC} = \overline{CD}$.

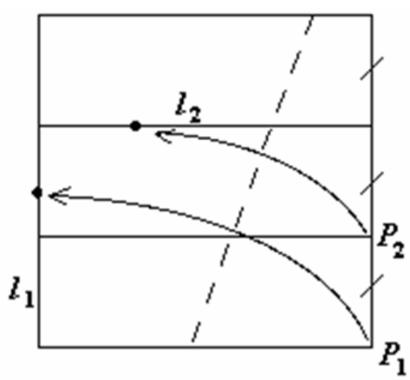
Therefore the triangles ABP_1 and CBP_1 are equal.

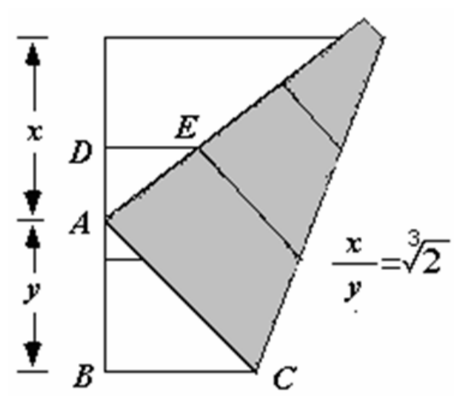
The same happens with the triangles CBP_1 and CDP_1.

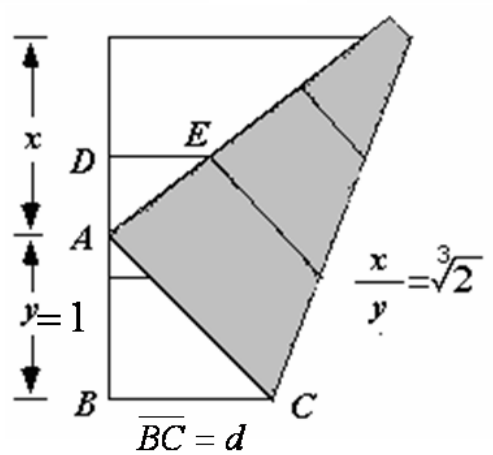
This proves that the angle defined by ℓ_2 and the horizontal is trisected, as we wanted to prove.

Doubling the cube

To double the cube, we just need to follow this procedure, that is possible by Axiom 6.







Let's prove that this procedure actually doubles the cube. For that, let's define $y = 1$. Thus, the length of the side of the paper is $x + 1$;

$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 \Leftrightarrow (x+1-d)^2 = 1^2 + d^2 \Leftrightarrow d = \frac{x^2+2x}{2x+2}\,,$$

$$\overline{AD} = x - \frac{x+1}{3} \Leftrightarrow \overline{AD} = \frac{2x-1}{3}\,.$$

Also, the triangles ABC and ADE are similar, therefore:

$$\frac{\overline{BC}}{\overline{AC}} = \frac{\overline{AD}}{AE} \to \frac{d}{x+1-d} = \frac{\frac{2x-1}{3}}{\frac{x+1}{3}} \to x = \sqrt[3]{2}\,.$$

which finishes the proof.

Comparing

Origami's geometry goes beyond Euclidian geometry, allowing to make all ruler and compass constructions, and solving classic problems, like those considered above.

Actually, it was proved by George E. Martin, in his paper "Geometric Constructions" in 1998, that Origami's geometry is equivalent to *The Marked Ruler Geometry*.

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Puzzles with polyhedra and numbers

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Introduction

1 Introduction

Consider a polyhedron. For example, a platonic, an archimedean, or a dual of an archimedean polyhedron. Construct flat polygonal plates in the same number, shape and size as the faces of the referred polyhedron. Adjacent to each side of each plate draw a number like it is shown in figures 1-5. Some of the plates, or all, can have numbers on both faces. We call these plates, two-faced plates. In this article, the two-faced plates have the same number adjacent to the same side.

Now the game is to put the plates over the polyhedron faces in such a way that the two numbers near each polyhedron edge are equal. If there is at least one solution for this puzzle one says that we have a polyhedron puzzle with numbers.

2 Definitions using combinatorics

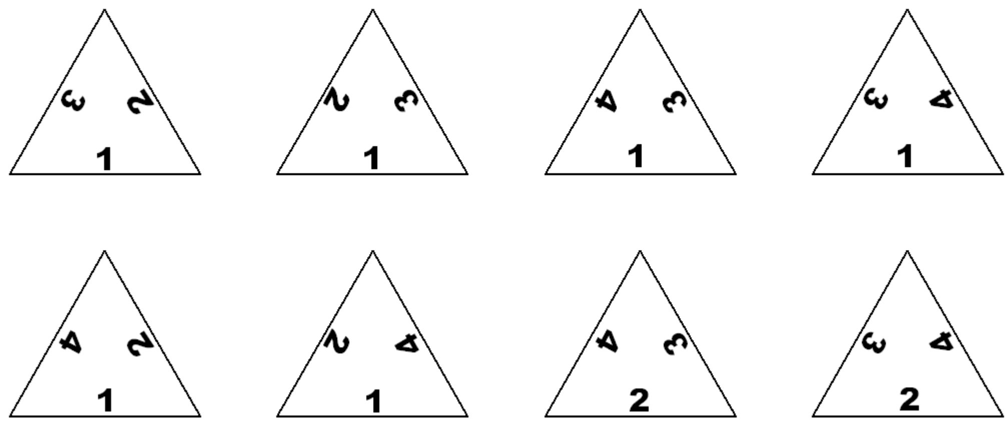
2.1 Platonic and archimedean polyhedra

From now on, assume that the numbers belong to the set $\{1, 2, \ldots, n\}$, and that all the numbers are used.

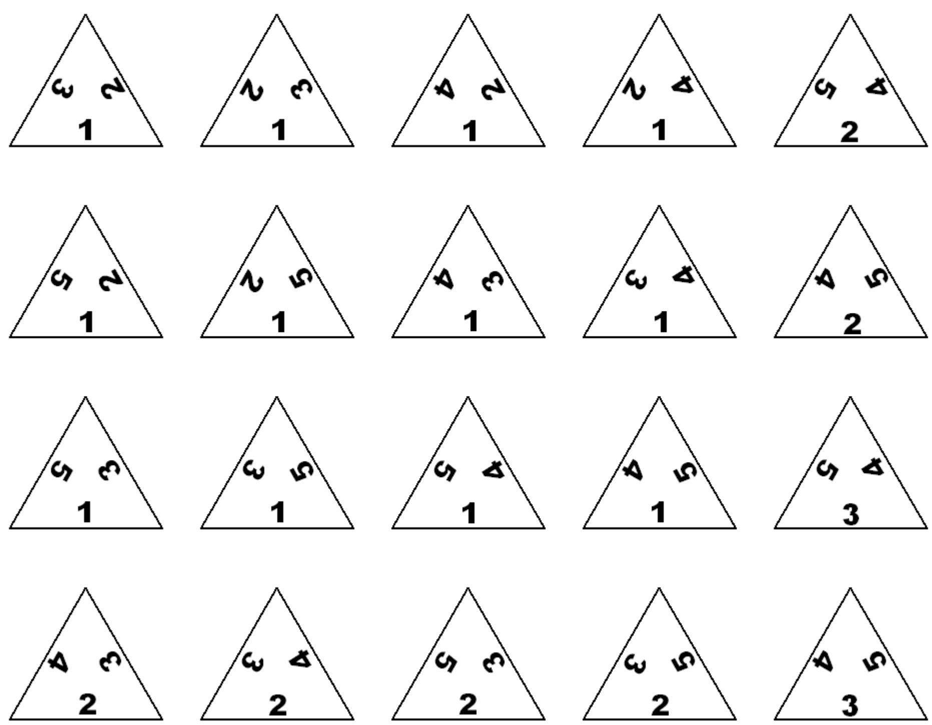
If we have plate faces which have the shape of a regular polygon with j sides, one can ask how many possible ways ν are there to draw the numbers $1, 2, \ldots, n$, without repeating them on each plate face. The answer is

a) For $j = 3$ (equilateral triangle) and $n = 3$, then $\nu = 2$.

b) For $j = 3$ and $n = 4$, then $\nu = 8$ (see figure 1); 8 **is precisely the number of the octahedron faces**. With these 8 plates we make the octahedron puzzle ($\equiv$ the octahedron (1) puzzle).

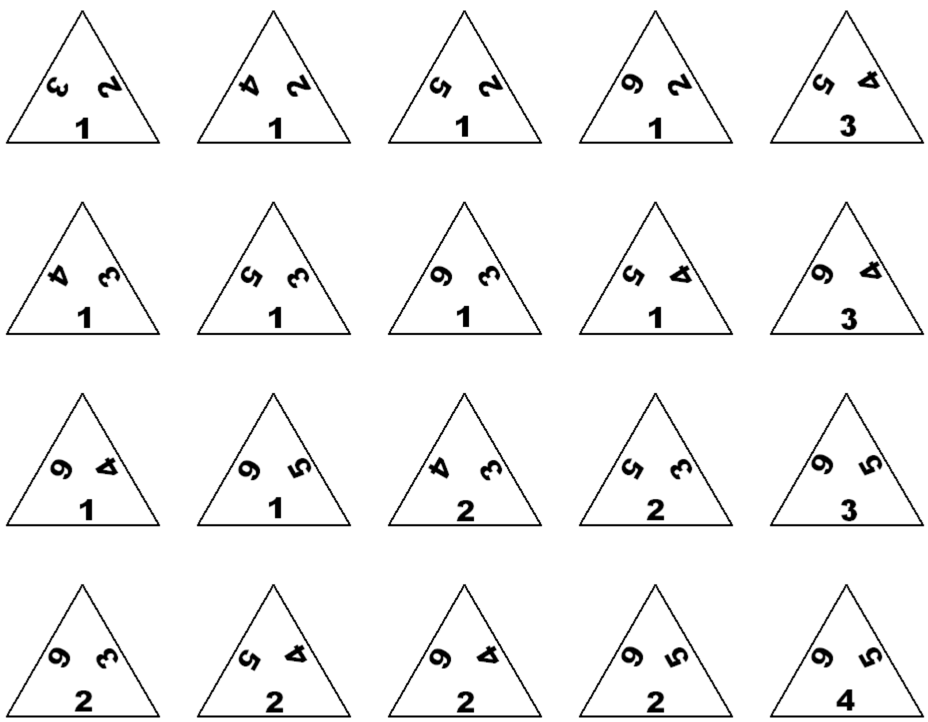


c) For $j = 3$ and $n = 5$, then $\nu = 20$ (see figure 2); 20 **is precisely the number of the icosahedron faces**. With these 20 plates we make the icosahedron (1) puzzle.

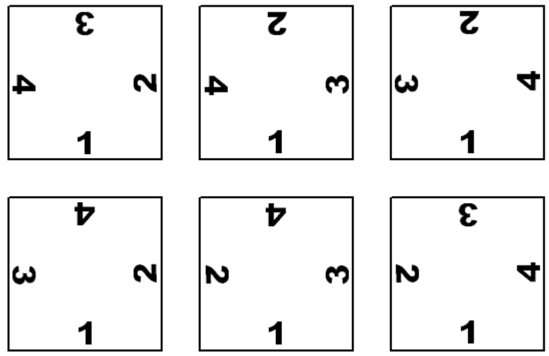


d) For $j = 3$ and $n = 6$, then $\nu = 40$; 40 **is precisely the double of the number of the icosahedron faces**. Construct different plates with the numbers written on both faces. This gives 20 plates. We call the related puzzle, the icosahedron second puzzle (or icosahedron (2)).

e) Consider again $j = 3$ and $n = 6$. Construct different plates with the numbers written only on one face, but in such a way that the numbers grow if we read them, beginning with the minimum, counter clock-wise. This gives 20 plates (see figure 3). We call the related puzzle, the icosahedron third puzzle (or icosahedron (3)).



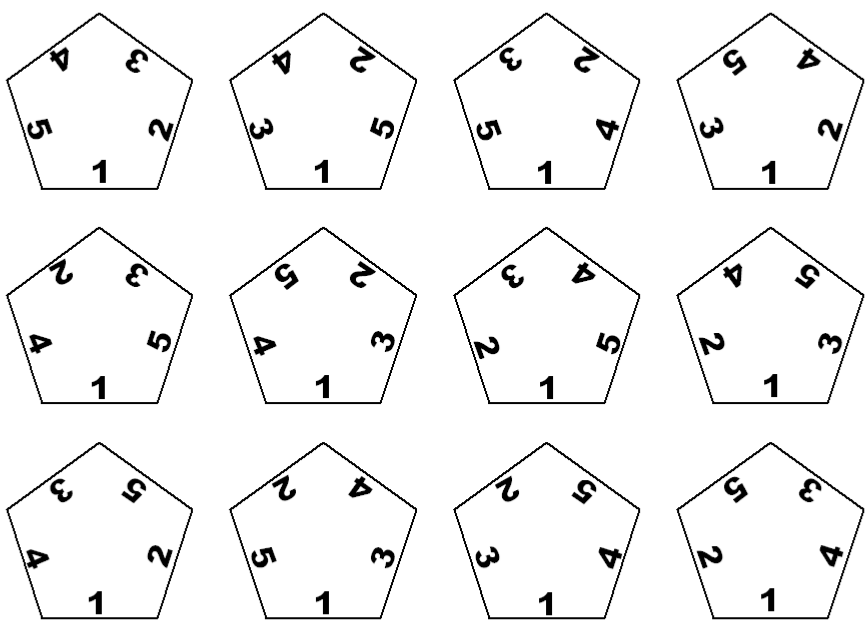
f) For $j = 4$ (square) and $n = 4$, then $\nu = 6$ (see figure 4); 6 **is precisely the number of the cube faces**. With these 6 plates we make the cube puzzle ($\equiv$ the cube (1) puzzle).



g) For $j = 5$ (regular pentagon) and $n = 5$, then $\nu = 24$; 24 **is precisely the double of the number of the dodecahedron faces**.

Construct different plates with the numbers written on both faces. This gives 12 plates. We call the related puzzle, the dodecahedron first puzzle (or dodecahedron (1)).

h) Let again $j = 5$ and $n = 5$. Construct different plates with the numbers written only on one face, but in such a way that the numbers read counter clock-wise, *abcd*5, are such that *abcd* form an even permutation. This gives 12 plates (see figure 5). We call the related puzzle, the dodecahedron second puzzle (or dodecahedron (2)).



i) Consider $n = 4$. With $j = 3$, one has $\nu = 8$ (see figure 1). With $j = 4$, one has $\nu = 6$ (see figure 4). Remark that 8 is precisely the number of the cuboctahedron triangular faces and 6 is precisely the number of its square faces. This is an example of an interesting puzzle using an archimedean polyhedron. We call it the cuboctahedron puzzle ($\equiv$ the cuboctahedron (1) puzzle).

The general formula for ν is

$$\nu = (j-1)! \begin{pmatrix} n \\ j \end{pmatrix} = \frac{n!}{(n-j)!j}.$$

2.2 More puzzles

Take now a deltoidal icositetrahedron. It has 24 deltoidal faces. If we have 24 plates which have the deltoidal shape the number of possible different ways to draw the numbers $1, 2, 3, 4$, without repeating them on each plate is precisely 24. This an example of an interesting puzzle using a dual of an archimedean polyhedron (see Reference [6]).

Consider again the cube. It has 6 faces that are squares. The number of possible different ways to draw the numbers $1, 2, 3, 4$, with two repetitions of the form *aabb* (the numbers are read counter clock-wise) on each square plate is precisely 6. This gives the cube (2) puzzle.

Consider again the icosahedron. It has 20 faces that are equilateral triangles. The number of possible different ways to draw the numbers $1, 2, 3, 4, 5$, with one repetition on each triangular plate is precisely 20. This gives the icosahedron (4) puzzle (see Reference [7]).

These are simple examples of polyhedron puzzles with numbers, which are enough in order to understand the following sections. There are, obviously, others as we shall see. For more examples see Reference [2], which is a development of Reference [1]. Reference [3] is a collection of some of these puzzles paper models.

3 Polyhedron symmetries

Consider a polyhedron in $\mathbb{R}^3$. From now on V denotes the set of the polyhedron vertices, E denotes the set of the polyhedron edges and F denotes the set of the polyhedron faces.

The group of the polyhedron symmetries, Ω, called the polyhedron group, is the set of all isometries ω of $\mathbb{R}^3$, that send vertices to vertices, which implies that they send edges to edges, faces to faces. Every symmetry $\omega \in \Omega$

induces three bijections, that we shall also denote ω, whenever there is no confusion possible: $\omega : V \to V$, $\omega : E \to E$ and $\omega : F \to F$. Denote also $\Omega \equiv \{\omega : V \to V\} \equiv \{\omega : E \to E\} \equiv \{\omega : F \to F\}$, the three sets of these functions. One can say that each one of these three sets Ω is the set of the polyhedron symmetries. Remark that not all one-to-one functions $F \longrightarrow F$, $E \longrightarrow E$, $V \longrightarrow V$ are in Ω. With the composition of functions each one of these three sets Ω forms a group that is isomorphic to the group of the polyhedron symmetries. If $\omega_1, \omega_2 \in \Omega$, we shall denote $\omega_1\omega_2 \equiv \omega_1 \circ \omega_2$.

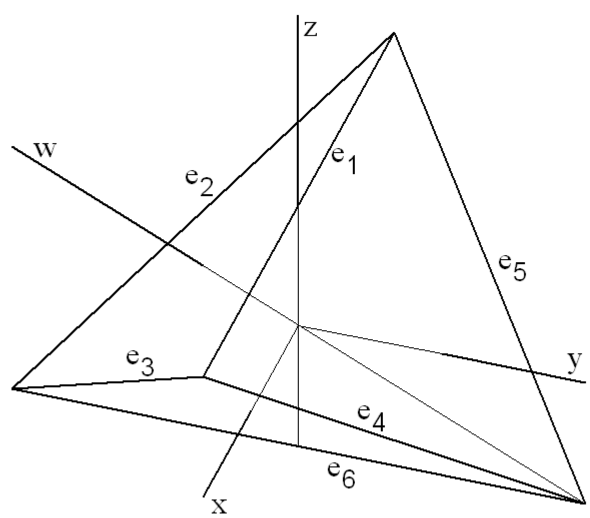
When no confusion is possible, $\omega \in \Omega$ represents also the group isomorphism $\omega : \Omega \to \Omega$, $\omega(\omega_1) = \omega\omega_1\omega^{-1}$, for every $\omega_1 \in \Omega$. Note that ω_1 and $\omega(\omega_1)$ have the same order. Look at the octahedron in figure 7. If ω is a counter clock-wise rotation of 90^o around the z-axis, and ω_1 is a counter clock-wise rotation of 90^o around the x-axis, then $\omega(\omega_1)$ is a counter clock-wise rotation of 90^o around the y-axis. In simple words, ω transports x over y. Here, a counter clock-wise rotation around the z-axis, for example, means that we look from the positive z-semiaxis.

If Ω_1 is a subgroup of Ω, then Ω_1 acts naturally on the face set, F: for $\omega \in \Omega_1$ and $\varphi \in F$, one defines the action $\omega\varphi = \omega(\varphi)$.

In the following we only consider polyhedra centered at the origin, Ω^+ denotes the subgroup of Ω of the symmetries with determinant 1 and Ω^- denotes the subgroup of Ω of the symmetries with determinant -1.

3.1 The tetrahedron group

Consider the tetrahedron (see figure 6) and its group, Ω.



An element of Ω is, for example, the function $\omega(x,y,z) = (-x,y,z)$, that induces the function $\omega : E \longrightarrow E$

$$\begin{array}{lll} \omega(e_1) = e_1 & \omega(e_2) = e_3 & \omega(e_3) = e_2 \\ \omega(e_4) = e_5 & \omega(e_5) = e_4 & \omega(e_6) = e_6 \end{array}.$$

This symmetry has determinant -1 and is one of the twelve elements of Ω with determinant -1. They reverse the orientation. The tetrahedron has no central symmetry.

The symmetries with determinant 1 (Ω^+), can be seen like this: one transports a chosen face in such a way that it goes to one of the four tetrahedron faces; as one has three possibilities of making them coincide (they are equilateral triangles), there are 12 (3×4) symmetries with determinant 1.

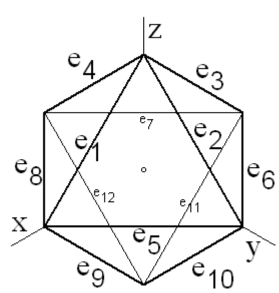
The advantage of describing in this way the symmetries of Ω^+ is that it can be easily adapted to other polyhedra, and used in their computation in a computer program.

Another way of counting the symmetries de Ω^+ is the following: the identity (1); the rotations of 180^o around the three axes defined by the centers of opposite edges (3); the rotations of 120^o and 240^o around the four axes defined by each vertex and the center of the opposite face (8).

The symmetries with determinant -1 (Ω^-) are the compositions of the symmetries de Ω^+ with a symmetry with determinant -1. The cardinal of Ω, the order of Ω, is, therefore, 24.

3.2 The octahedron (cube) group

Consider the octahedron (see figure 7) and its group, Ω. Everything that we say here about the octahedron group can be translated to the cube group interchanging faces with vertices. In other words the group is the same.



An element of Ω is, for example, the central symmetry $\omega(x,y,z) = -(x,y,z)$, that induces the function $\omega : E \longrightarrow E$, $\omega(e_1) = e_{11}$, $\omega(e_2) = e_{12}$, $\omega(e_3) = e_9$ and so on.

The central symmetry has determinant -1. The symmetries with determinant 1 (Ω^+), can be seen like this: one transports a chosen face in such a way that it goes to one of the eight faces of the octahedron; as one has three possibilities of making them coincide (they are equilateral triangles), there are 24 (3×8) symmetries with determinant 1.

Note once more that the advantage of describing in this way the symmetries of Ω^+ is that it can be easily adapted to other polyhedra, and used in their computation in a computer program.

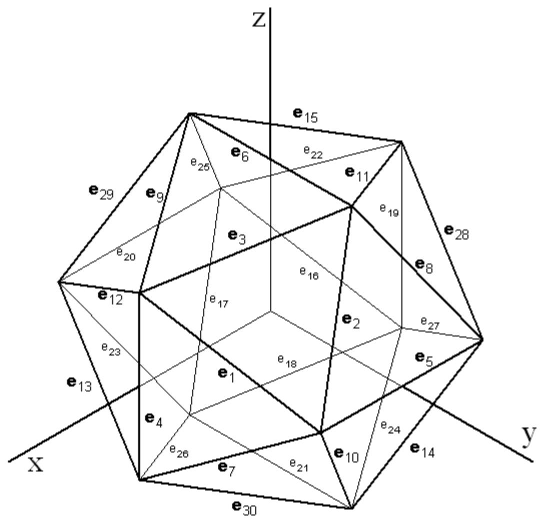
Another way of counting the symmetries de Ω^+ is the following: the identity (1); the rotations of 90^o, 180^o and 270^o around the three axes defined by opposite vertices (9); the rotations of 180^o around the six axes defined by the centers of opposite edges (6); the rotations of 120^o and 240^o around the four axes defined by the centers of opposite faces (8).

The symmetries with determinant -1 (Ω^-) are the compositions of the symmetries de Ω^+ with the central symmetry. The cardinal of Ω, the order of Ω, is, therefore, 48.

In Reference [5] one can see two examples of such symmetries and a detailed description of the octahedron puzzle case.

3.3 The icosahedron (dodecahedron) group

Consider the icosahedron (see figure 8) and its group, Ω. Everything that we say here about the icosahedron group can be translated to the dodecahedron group interchanging faces with vertices. In other words the group is the same.



An element of Ω is, for example, the central symmetry $\omega(x,y,z) = -(x,y,z)$, that induces the function $\omega : E \longrightarrow E$, $\omega(e_1) = e_{16}$, $\omega(e_2) = e_{17}$, $\omega(e_3) = e_{18}$ and so on.

The central symmetry has determinant -1. The symmetries with determinant 1 (Ω^+), can be seen like this: one transports a chosen face in such a way that it goes to one of the twenty faces of the icosahedron; as one has three possibilities of making them coincide (they are equilateral triangles), there are 60 (3×20) symmetries with determinant 1.

Another way of counting the symmetries de Ω^+ is the following: the identity (1); the rotations of 72^o, 144^o , 216^o and 288^o around the six axes defined by opposite vertices (24); the rotations of 180^o around the fifteen axes defined by the centers of opposite edges (15); the rotations of 120^o and 240^o around the ten axes defined by the centers of opposite faces (20).

The symmetries with determinant -1 (Ω^-) are the compositions of the symmetries de Ω^+ with the central symmetry. The cardinal of Ω, the order of Ω, is, therefore, 120.

4 Permutation groups and puzzle solutions

Consider a puzzle with numbers $1, 2, \ldots, n$ drawn on the plates. From now on P denotes the set of its plates which have numbers drawn, and call it the plate set. If no confusion is possible, P will also denote the puzzle itself. S_n denotes the group of all permutations of $\{1, 2, \ldots, n\}$; $\sigma \in S_n$ means that σ is a one-to-one function $\sigma : \{1, 2, \ldots, n\} \rightarrow \{1, 2, \ldots, n\}$. The identity is

σ_0: $\sigma_0(1) = 1, \sigma_0(2) = 2, \ldots, \sigma_0(n) = n$. The alternating group, the S_n subgroup of the even permutations, is denoted by A_n. If $\sigma_1, \sigma_2 \in S_n$, we shall denote $\sigma_1\sigma_2 \equiv \sigma_1 \circ \sigma_2$.

We shall write $\sigma = (\alpha_1\alpha_2 \cdots \alpha_k) \cdots (\beta_1\beta_2 \cdots \beta_l)$, if

$$\sigma(\alpha_1) = \alpha_2, \sigma(\alpha_2) = \alpha_3, \ldots, \sigma(\alpha_k) = \alpha_1, \ldots,$$
$$\sigma(\beta_1) = \beta_2, \sigma(\beta_2) = \beta_3, \ldots, \sigma(\beta_k) = \beta_1$$

where $\alpha_1, \alpha_2, \ldots, \alpha_k, \ldots, \beta_1, \beta_2, \ldots, \beta_l \in \{1, 2, \ldots, n\}$.

If $\gamma \in \{1, 2, \ldots, n\} \setminus \{\alpha_1, \alpha_2, \ldots, \alpha_k, \ldots, \beta_1, \beta_2, \ldots, \beta_l\}$, then $\sigma(\gamma) = \gamma$.

The permutation $(\alpha_1\alpha_2 \cdots \alpha_k)$ is called a cyclic permutation, or a cycle (in this case a k-cycle); k is the length of the cyclic permutation.

We shall use also the group $\{-1, 1\} \times S_n$ denoted by $S_n^{\pm}$. If $\delta_1, \delta_2 \in \{-1, 1\}$ and $\sigma_1, \sigma_2 \in S_n$, then $(\delta_1, \sigma_1)(\delta_2, \sigma_2) = (\delta_1\delta_2, \sigma_1\sigma_2)$. We denote $S_n^+ = \{1\} \times S_n \equiv S_n$, $(1, \sigma) \equiv \sigma$, $(-1, \sigma) \equiv \sigma^-$.

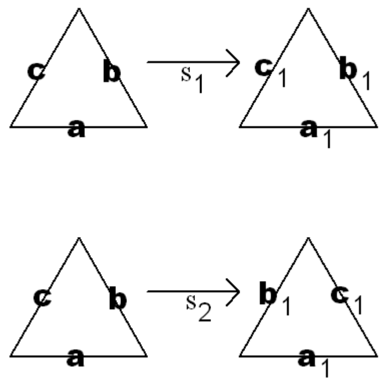
If Λ is a set, then $|\Lambda|$ denotes its cardinal. Hence, if G is group, $|G|$ denotes its order.

As before E denotes the set of the polyhedron edges and F denotes the set of the polyhedron faces. A solution of the puzzle defines a function $\varepsilon : E \to \{1, 2, \ldots, n\}$. Denote $\mathcal{E}$ the set of these functions. One can say that $\mathcal{E}$ is the set of the puzzle solutions.

We shall also consider the group $S_n \times \Omega$. If $(\sigma_1, \omega_1), (\sigma_2, \omega_2) \in S_n \times \Omega$, one defines the product $(\sigma_1, \omega_1)(\sigma_2, \omega_2) = (\sigma_1\sigma_2, \omega_1\omega_2)$. We use here a different definition from the one in Reference [4].

4.1 The plate group

Some S_n subgroups act naturally on P. Let $\pi \in P$ and $\sigma \in S_n$. Assume that $a, b, c, \ldots$ are drawn on π, by this order. Then $\sigma\pi$ is a plate where the numbers $\sigma(a) = a_1, \sigma(b) = b_1, \sigma(c) = c_1, \ldots$ are drawn replacing $a, b, c, \ldots$ (see figure 9).



Let $s \in S_n^{\pm}$ and $\pi \in P$. If $s \equiv s_1 = (1, \sigma) \equiv \sigma$, then $s\pi = \sigma\pi$. If $s \equiv s_2 = (-1, \sigma) \equiv \sigma^{-}$, then $s\pi$ is a reflection of $\sigma\pi$. In this last case, if the numbers $a, b, c, \ldots$ are drawn on π, by this order, then $s\pi$ is a plate where the numbers $\ldots, \sigma(c) = c_1, \sigma(b) = b_1, \sigma(a) = a_1$ are drawn by this order (see figure 9).

The plate group, G_P, is the greatest subgroup of $S_n^{\pm}$ that acts on P. If $s \in S_n^{\pm}$ and $s\pi \in P$, for every $\pi \in P$, then $s \in G_P$.

4.2 The solution group

Let $\varepsilon : E \to \{1, 2, \ldots, n\}$ be a solution of the puzzle. The group of this solution, G_ε, is a subgroup of $S_n \times \Omega$; $(\sigma, \omega) \in G_\varepsilon$ if and only if

$$\sigma \circ \varepsilon = \varepsilon \circ \omega.$$

Denote Ω_ε the following subgroup of Ω: $\omega \in \Omega_\varepsilon$ if and only if there exists $\sigma \in S_n$ such that $(\sigma, \omega) \in G_\varepsilon$. Remark that if $\omega \in \Omega_\varepsilon$ there exists only one $\sigma \in S_n$ such that $(\sigma, \omega) \in G_\varepsilon$. From this one concludes that $\omega \mapsto (\sigma, \omega)$ defines an isomorphism between Ω_ε and G_ε and that $(\det \omega, \sigma) \in G_P$. This defines $g_\varepsilon : \Omega_\varepsilon \to G_P$, $g_\varepsilon(\omega) = (\det \omega, \sigma)$, which is an homomorphism of groups.

For a lot of puzzles $(\det \omega, \sigma)$ defines completely ω. It is the case of all puzzles considered in this article. Hence, when $(\det \omega, \sigma)$ defines completely ω, g_ε establishes an isomorphism between Ω_ε and $g_\varepsilon(\Omega_\varepsilon) \subset G_P$. Denote $G_{P\varepsilon} \equiv g_\varepsilon(\Omega_\varepsilon)$. Finally, G_ε and $G_{P\varepsilon}$ are isomorphic. We can identify (σ, ω) with $(\det \omega, \sigma)$, and G_ε with the subgroup $G_{P\varepsilon}$ of G_P.

4.3 Equivalent solutions

Let $\varepsilon_1, \varepsilon_2 : E \to \{1, 2, \dots, n\}$ be solutions of the puzzle. One says that these solutions are equivalent, $\varepsilon_1 \approx \varepsilon_2$, if there are $\omega \in \Omega$ and $\sigma \in S_n$, such that

$$\sigma \circ \varepsilon_1 = \varepsilon_2 \circ \omega.$$

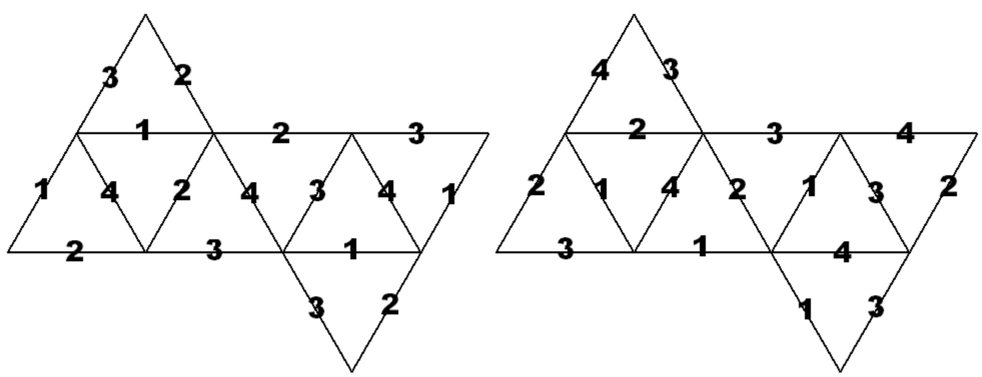
Remark that $(\det \omega, \sigma) \in G_P$.

If $\sigma = \sigma_0$ and $\det \omega = 1$, what distinguishes the solutions ε_1 and ε_2 is only a rotational symmetry. In this case

$$\varepsilon_1 = \varepsilon_2 \circ \omega$$

expresses another equivalence relation, $\varepsilon_1 \sim \varepsilon_2$. When we make a puzzle, in practice, we do not recognize the difference between ε_1 and ε_2. We shall say that they represent the same **natural solution**, an equivalence class of the relation $\sim$.

Figure 10 shows two solutions of the octahedron puzzle that represent the same natural solution.



Let $\varepsilon, \varepsilon_1, \varepsilon_2 \in \mathcal{E}$. As $\varepsilon_1 \sim \varepsilon_2$ and $\varepsilon_1 \approx \varepsilon$, implies $\varepsilon_2 \approx \varepsilon$, one can say that the natural solution represented by ε_1 is equivalent to ε.

This equation involving ε_1 and ε_2 defines an equivalence relation, and a natural solution is an equivalence class of this relation. Remark that if $\varepsilon_1 = \varepsilon_2$, then ω_1 is the identity.

For $\varepsilon \in \mathcal{E}$, represent by $[\varepsilon]$ the set of natural solutions equivalent to ε.

Choose now $\omega_- \in \Omega$, such that $\det \omega_- = -1$. For $\varepsilon \in \mathcal{E}$ and $s = (\delta, \sigma) \in G_P$, denote $\varepsilon_s = \sigma \circ \varepsilon \circ \omega$, where ω is the identity if $\delta = 1$ and $\omega = \omega_-$

if $\delta = -1$. The set $\{\varepsilon_s : s \in G_P\}$ includes representatives of all natural solutions equivalent to ε. Then

$$|[\varepsilon]| = \frac{|G_P|}{|G_{P\varepsilon}|}.$$

The cardinal of all the natural solutions is then given by

$$\sum_{[\varepsilon]} \frac{|G_P|}{|G_{P\varepsilon}|},$$

where the sum is extended to all different equivalence classes $[\varepsilon]$.

5 Examples

Some of the examples we give in this section can easily be studied directly. It is the case of the cube, the octahedron and the dodecahedron (2) puzzles.

We give also results for the dodecahedron (1), icosahedron (1) and (3) puzzles. These results were obtained mostly with a computer. Similar results for the icosahedron (2) puzzle are left to the reader.

There are good reasons for presenting results for these two icosahedron puzzles. The icosahedron (1) puzzle is mathematically rich, has a lot of natural solutions (over one million!). On the other hand, the icosahedron (3) puzzle is also very instructive because it has few natural solutions (”only” 2322), which means that it is difficult to do it without the help of a computer.

5.1 The cube puzzle

In this case there is only one equivalence class. $G_\varepsilon \approx \Omega^+ \approx S_4$. $|G_\varepsilon| = 24$. $|G_P| = 48$.

If one puts a plate on a face then one has 2 different possibilities for the other plates (2 natural solutions): 48/24.

These two different possibilities and the equivalences $G_\varepsilon \approx \Omega^+ \approx S_4$ can be used to translate the cube symmetries into permutations, in the same way as we shall do later with the icosahedron (1) puzzle and its canonical solution.

5.2 The octahedron puzzle

There three equivalence classes. One can distinguish them in the following way. Take a solution. On every vertex of the octahedron, note the numbers that correspond to its four edges. There are two possibilities for a vertex: a) four distinct numbers; b) three distinct numbers, with one of them repeated. In the solution, count the number of vertices where the situation a) happens. They can be 6, 2 or 0, that distinguish the three equivalence classes.

The first class group is of order 24. The second class group is of order 8. The third class group is of order 6. As $|G_P| = 48$, one has that if one puts a plate on a face then one has 16 different possibilities for the other plates (16 natural solutions): $\frac{48}{24} + \frac{48}{8} + \frac{48}{6}$.

Although the cube and the octahedron are dual polyhedra, puzzles are not. Solutions can be dual. The first class of the octahedron puzzle is dual of the cube puzzle class.

5.3 The dodecahedron (1) puzzle

This puzzle has three equivalence classes that we can distinguish in the following way. Consider a solution. On every dodecahedron vertex note the numbers that are on the edges around the vertex. There are 20 possibilities, but not all of them belong to the solution. There are some repetitions: 7 or 3. The solutions that have 7 repetitions on the vertices are equivalent. The solutions that have 3 repetitions on the vertices belong to 2 different equivalence classes. One of these classes has the repetitions on opposite vertices. The other has the repetitions on vertices that belong to the same edges.

The first class group is of order 8. The second class group is of order 24. The third class group is of order 12. As $|G_P| = 240$, one has that if one puts a plate on a face then one has 60 different possibilities for the other plates (120 natural solutions): $\frac{1}{2}\left(\frac{240}{8} + \frac{240}{24} + \frac{240}{12}\right)$.

5.4 The dodecahedron (2) puzzle

This puzzle has 2 equivalence classes that can be distinguished in the following form. Consider two opposite dodecahedron edges. There are other four that are orthogonal to these two. The six edges are over the faces of a virtual cube where the dodecahedron is inscribed. There are five such cubes. The first equivalence class has the same number associated to the edges that belong to the faces of each cube. The second equivalence class has the same number associated to the edges that belong to the faces of one of the fives cubes.

The first class group is of order 120 and the second class group is of order 24. As $|G_P| = 120$, one has that if one puts a plate on a face then one has

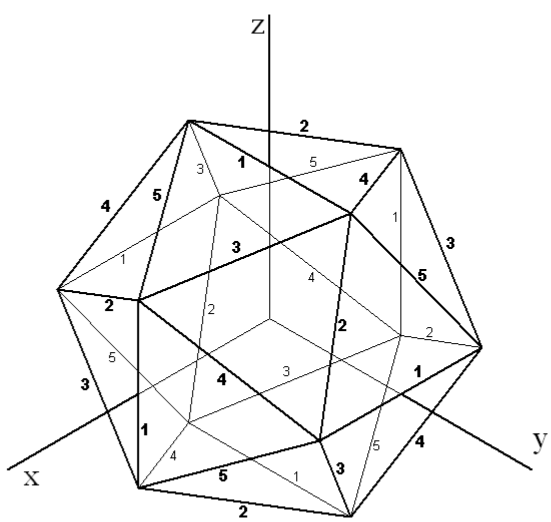
6 different possibilities for the other plates (6 natural solutions): $\frac{120}{120} + \frac{120}{24}$.

5.5 The icosahedron (1) puzzle

The icosahedron (1) puzzle has 5592 equivalence classes: 5366 have groups of order 1, 165 have groups of order 2, 36 have groups of order 3, 1 has a group of order 4, 4 have groups of order 5, 10 have groups of order 6, 1 has a group of order 8, 4 have groups of order 10, 2 have groups of order 12, 2 have groups of order 24, 1 has a group of order 120. As $|G_P| = 240$, one has that once one puts a plate over a face there are 1311360 different possibilities (1311360 natural solutions):

$$240\left(5366 + \frac{165}{2} + \frac{36}{3} + \frac{1}{4} + \frac{4}{5} + \frac{10}{6} + \frac{1}{8} + \frac{4}{10} + \frac{2}{12} + \frac{2}{24} + \frac{1}{120}\right).$$

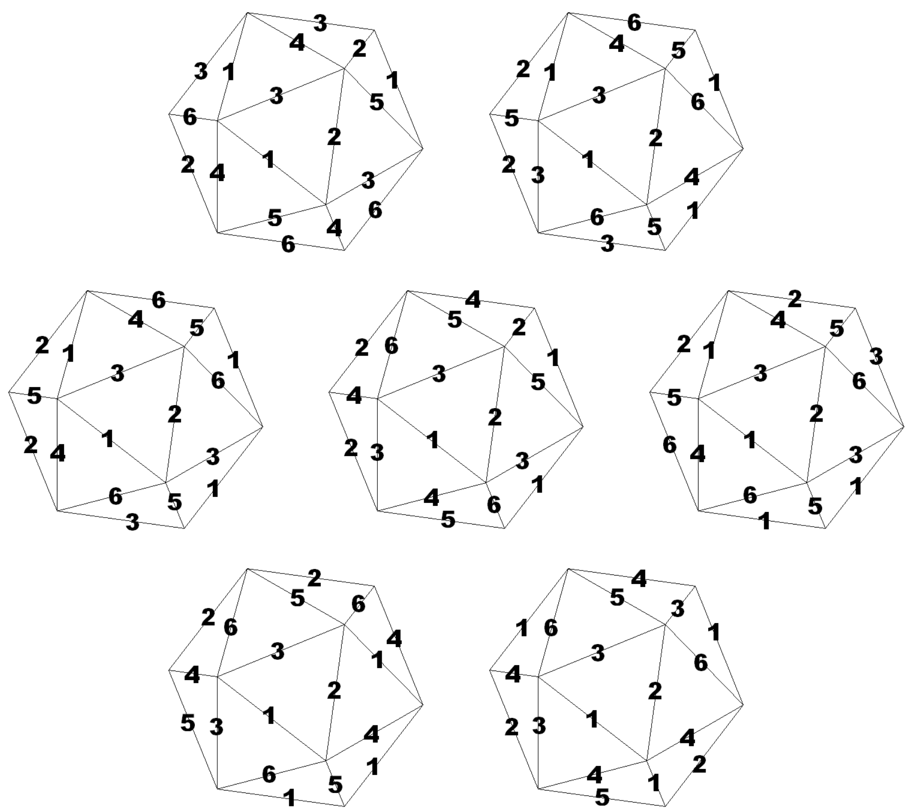
There are two possible natural solutions with a group of order 120. One of them, that we call canonical solution, is dual of the dodecahedron (2) solution with a group of order 120. The group of these two solutions is equivalent to the icosahedron group: $G_\varepsilon \approx \Omega \approx \{-1, 1\} \times A_5$. The canonical solution is represented in Figure 11.



5.6 The icosahedron (3) puzzle

This puzzle has 197 equivalence classes: 190 have groups of order 1, and 7 have groups of order 2. As $|G_P| = 12$, one has that, once one puts a plate over a face there are 2322 different possibilities (2322 natural solutions): $190 \times 12 + 7 \times \frac{12}{2}$.

Remark that the actions of all these 7 groups of order 2 are equivalent. Figure 12 shows representatives of all these 7 equivalence classes.



6 More on icosahedron puzzles

As we have already seen there is a natural solution of the icosahedron (1) puzzle which is dual of the dodecahedron (2) natural solution with a group of order 120 (see Figure 11). This group is equivalent to the icosahedron group Ω: $G_\varepsilon \approx \Omega \approx \{-1, 1\} \times A_5$. As this solution is very easy to construct, one can use it in order to translate in terms of $\{-1, 1\} \times A_5$ everything that happens in Ω. For example, all subgroups of Ω are equivalent to the corresponding subgroups of $\{-1, 1\} \times A_5$.

6.1 Subgroups of the icosahedron group

The icosahedron group Ω has 22 different equivalence classes of subgroups: 1 of order 1 ($\{\sigma_0\}$); 3 of order 2; 1 of order 3; 3 of order 4; 1 of order 5; 3 of order 6; 1 of order 8; 3 of order 10; 2 of order 12; 1 of order 20; 1 of order 24; 1 of order 60; 1 of order 120 ($\Omega \approx \{-1, 1\} \times A_5$). In the following $a, b, c, d, e \in \{1, 2, 3, 4, 5\}$ are different numbers, and in angle brackets we give the number of equivalence classes of solutions in the icosahedron (1) puzzle.

6.1.1 Groups of order 2.

The 3 equivalence classes are generated by $(ab)(cd)$ $\langle 148 \rangle$, $(-1, (ab)(cd))$ $\langle 5 \rangle$ and $(-1, \sigma_0)$ $\langle 12 \rangle$. Look at Figure 11. The rotation around the z-axis by an angle of 180^o, corresponds to the permutation $(23)(45)$. This rotation belongs to the first equivalence class. The reflection using the xy-plane (orthogonal to the z-axis) as a mirror, corresponds to $(-1, (23)(45))$. This reflection belongs to the second equivalence class. The third equivalence class corresponds to the central symmetry $(x, y, z) \mapsto -(x, y, z)$.

6.1.2 Groups of order 3.

The unique equivalence class is generated by (abc) $\langle 36 \rangle$. In Figure 11, the rotations around the axis $x = y = z$ by angles multiple of 120^o correspond to group generated by (234).

6.1.3 Groups of order 4.

Let $\sigma_1 = (ab)(cd)$ and $\sigma_2 = (ac)(bd)$. Remark that $\sigma_1\sigma_2 = \sigma_2\sigma_1 = (ad)(bc)$. The 3 equivalence classes are generated by σ_1 and σ_2 $\langle 0 \rangle$, σ_1 and $(-1, \sigma_0)$ $\langle 1 \rangle$, σ_1 and $(-1, \sigma_2)$ $\langle 0 \rangle$, respectively.

6.1.4 Groups of order 5.

The unique equivalence class is generated by $(abcde)$ $\langle 4 \rangle$. In Figure 11, the rotations around the axis $x = 0$, $z = -\frac{1+\sqrt{5}}{2}y$ (this is the z'-axis in Figure 13), by angles multiple of 72^o, correspond to group generated by (12345).

6.1.5 Groups of order 6.

Let $\sigma_1 = (ab)(cd)$ and $\sigma_2 = (ae)(cd)$. The 3 equivalence classes are generated by σ_1 and σ_2 ($\equiv S_3$) $\langle 3 \rangle$, $(-1, \sigma_1)$ and $(-1, \sigma_2)$ $\langle 0 \rangle$, (abc) and $(-1, \sigma_0)$

$\langle 7\rangle$, respectively.

6.1.6 Groups of order 8.

Let $\sigma_1 = (ab)\,(cd)$ and $\sigma_2 = (ac)\,(bd)$. As before, remark that $\sigma_1\sigma_2 = \sigma_2\sigma_1 = (ad)\,(bc)$. The unique equivalence class is generated by σ_1 and σ_2 and $(-1, \sigma_0)$ $\langle 1\rangle$.

6.1.7 Groups of order 10.

Let $\sigma_1 = (ab)\,(cd)$ and $\sigma_2 = (ac)\,(be)$. The 3 equivalence classes are generated by σ_1 and σ_2 ($\equiv D_5$, the dihedral group of order 10) $\langle 2\rangle$, $(-1, \sigma_1)$ and $(-1, \sigma_2)$ $\langle 0\rangle$, $(abcde)$ and $(-1, \sigma_0)$ $\langle 2\rangle$, respectively.

6.1.8 Groups of order 12.

Let $\sigma_1 = (ab)\,(cd)$ and $\sigma_2 = (acd)$. The 2 equivalence classes are generated by σ_1 and σ_2 ($\equiv A_4$) $\langle 2\rangle$, S_3 and $(-1, \sigma_0)$ ($\equiv \{-1, 1\} \times S_3$) $\langle 0\rangle$, respectively.

6.1.9 Groups of order 20.

The unique equivalence class is generated by D_5 and $(-1, \sigma_0)$ ($\equiv \{-1, 1\} \times D_5$) $\langle 0\rangle$.

6.1.10 Groups of order 24.

The unique equivalence class is generated by A_4 and $(-1, \sigma_0)$ ($\equiv \{-1, 1\} \times A_4$) $\langle 2\rangle$.

6.1.11 Groups of order 60.

The unique equivalence class is A_5 $\langle 0\rangle$.

6.1.12 Groups of order 120.

The unique equivalence class is $\{-1, 1\} \times A_5 \approx \Omega$ $\langle 1\rangle$.

6.2 Subgroups of plate groups

All the subgroups of the icosahedron group have cyclic groups of order 2, 3 or 5 as generators. Let us look at the actions on the faces of such cyclic groups.

The first equivalence class of the order 2 groups has 10 orbits with 2 elements and the determinant of the generator is 1: $(1, 10 \times 2)$. The second equivalence class has 8 orbits with 2 elements, 4 orbits with 1 element and the determinant of the generator is -1: $(-1, 8 \times 2 + 4 \times 1)$. The third equivalence class has 10 orbits with 2 elements and the determinant of the generator is -1: $(-1, 10 \times 2)$.

The equivalence class of the order 3 groups has 6 orbits with 3 elements, 2 orbits with 1 element and the determinant of the generator is 1: $(1, 6 \times 3 + 2 \times 1)$.

The equivalence class of the order 5 groups has 4 orbits with 5 elements and the determinant of the generator is 1: $(1, 4 \times 5)$.

6.2.1 The icosahedron (1) puzzle

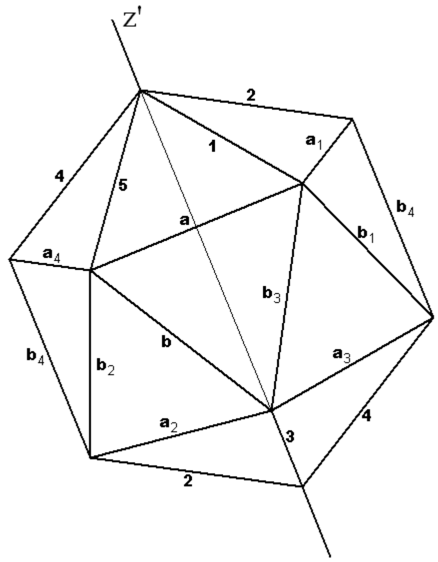
The plate group is $G_P = \{-1, 1\} \times S_5$. The only cyclic groups of G_P whose actions on the plates have orbits of the type $(1, 10 \times 2)$, $(-1, 8 \times 2 + 4 \times 1)$, $(-1, 10 \times 2)$, $(1, 6 \times 3 + 2 \times 1)$ and $(1, 4 \times 5)$ are those generated, precisely, by $(1, (ab)(cd))$, $(-1, (ab)(cd))$, $(-1, \sigma_0)$, $(1, (abc))$ and $(1, (abcde))$.

6.2.2 The icosahedron (3) puzzle

Let $\sigma_1 = (123456)$ and $\sigma_2 = (16)(25)(34)$. The plate group, G_P, is generated by $(1, \sigma_1)$ and $(-1, \sigma_2)$. The only cyclic groups of G_P whose actions on the plates have orbits of the type $(1, 10 \times 2)$, $(-1, 8 \times 2 + 4 \times 1)$, $(-1, 10 \times 2)$ and $(1, 6 \times 3 + 2 \times 1)$ are generated by $\left(1, \sigma_1^3\right)$, $\left(-1, \sigma_1^j \sigma_2\right)$, $\left(-1, \sigma_1^k \sigma_2\right)$, $\left(1, \sigma_1^2\right)$, $j = 1, 3, 5$, $k = 0, 2, 4$. There are no subgroups of order 5. It is easy to see directly that there are no solutions corresponding to the subgroups of order 3. The solutions corresponding to subgroups of order 2, are only those related to the generators $\left(-1, \sigma_1^k \sigma_2\right)$, $k = 0, 2, 4$.

6.3 Finding solutions of a puzzle with a prescribed group

A way to find a solution for a puzzle is to use the group relations between the polyhedron and the plate groups. Take, for example, the icosahedron (1) puzzle and the third equivalence class for groups of order 10.



This class is generated by an element σ of order 5 and $(-1, \sigma_0)$. The numbers on the edges must respect the central symmetry and a rotational symmetry group of order 5. Let $\sigma = (12345)$. In Figure 13, that illustrates this example, $a, b \in \{1, 2, 3, 4, 5\}$, and $a_j = \sigma^j(a)$, $b_j = \sigma^j(b)$, $j = 1, 2, 3, 4$. The numbers a and b must be chosen so that $a \neq b$, $a \neq \sigma^3(b) = b_3$, $b \neq \sigma^2(a) = a_2$. These three conditions give 9 possibilities, but only 5 of them correspond to solutions of the puzzle: $(a, b) = (2, 1), (2, 5), (3, 4), (4, 2), (4, 3)$. The third one is the canonical solution. The first and the fourth ones belong to the same equivalence class. The same happens with the second and the fifth ones. These last two classes are the ones already listed.

7 From groups to polyhedron puzzles

Let S be a subgroup of $S_n^{\pm}$ and $h_1, h_2 : \Omega \to S$ be two isomorphisms.

One says that h_1 and h_2 represent the same natural isomorphism if there exists $\omega \in \Omega^+$ such that

$$h_1 = h_2 \circ \omega.$$

Two natural isomorphisms, represented by h_1 and h_2 are said to be equivalent if there exists $s \in S$ and $\omega \in \Omega$, such that

$$s \circ h_1 = h_2 \circ \omega.$$

Consider a puzzle solution ε and its group $G_{P_\varepsilon} \equiv \Omega_\varepsilon$. Take $\omega_1, \omega_2 \in \Omega_\varepsilon$. If ω_1 is a rotation of order k, and ω_2 is a rotation of order j, then ω_1 transforms ω_2 in another rotation of order j, ω_3, which is $\omega_1\omega_2\omega_1^{-1} \equiv \omega_1(\omega_2)$.

The isomorphism between Ω_ε and G_{P_ε} suggests that if one wants to translate the isometries into elements of $S_n^{\pm}$ the function must be such that if $\omega_1 \longmapsto s_1$, $\omega_2 \longmapsto s_2$, then $\omega_3 \equiv \omega_1(\omega_2) \longmapsto s_1 s_2 s_1^{-1}$.

Note that if $s_1 = (\delta_1, \sigma_1)$, $s_2 = (\delta_2, \sigma_2)$, where

$$\sigma_2 = (\alpha_1 \alpha_2 \cdots \alpha_l) \cdots,$$

then $\omega_3 \equiv \omega_1(\omega_2) \longmapsto s_3 = (\delta_2, \sigma_3)$, with

$$\sigma_3 = (\sigma_1(\alpha_1)\, \sigma_1(\alpha_2) \cdots \sigma_1(\alpha_l)) \cdots.$$

We assign to every semiaxis of order k ($\equiv \omega$), a k-cycle $\sigma = (\alpha_1 \alpha_2 \cdots \alpha_k)$, so that to the counter clock-wise rotation of $\frac{2\pi}{k}$, ω, corresponds the permutation σ. This association must be coherent in the sense that it generates a group isomorphism.

In the cases we are interested in (the tetrahedron, the octahedron and the icosahedron), it is enough to make the association to two neighbor semiaxes. In these cases the semiaxes are defined by the vertices, the edges (the middle point of each edge), the faces (the center of each face), and have their origin at the polyhedron center.

If we assign to ω and ω_1 (two neighbor semiaxes) the cycles σ and σ_1, then to the semiaxis $\omega(\omega_1) = \omega\omega_1\omega^{-1}$ we must assign $\sigma\sigma_1\sigma^{-1}$. When $\sigma_1 = (\alpha_1 \alpha_2 \cdots \alpha_k)$, then $\sigma\sigma_1\sigma^{-1} = (\sigma(\alpha_1)\, \sigma(\alpha_2) \cdots \sigma(\alpha_k))$.

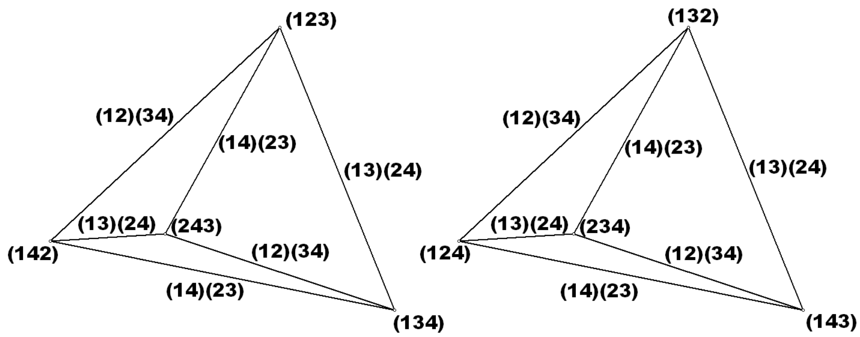
In this section we use group theory in order to find puzzles, for a given polyhedron, such as, for example, maximal puzzles. These are important examples, but others could be given.

To avoid ambiguities, in the puzzles we give in the following, all the edges have numbers, and we use the numbers $1, 2, 3, 4$ in the tetrahedron and octahedron (cube) groups cases, and the numbers $1, 2, 3, 4, 5$ in the icosahedron (dodecahedron) group case.

7.1 The tetrahedron group

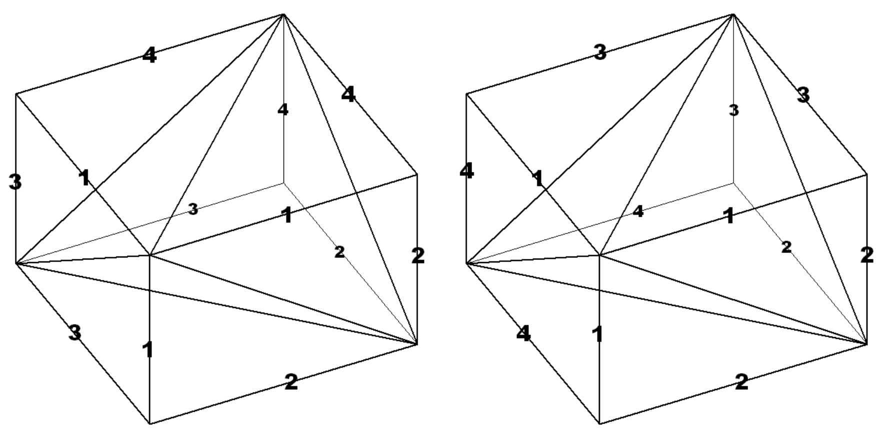
In the tetrahedron a vertex represents a semiaxis of order 3. Associate to a given vertex the permutation (123). It is not difficult to see that the only natural possibility is the one represented in the l.h.s. in figure 14. Something similar happens with permutation (132) and the result is in the r.h.s. of the same figure. Hence, there are two natural isomorphisms for the tetrahedron group (see figure 14) which are equivalent. This equivalence can be done, for example, by any transposition and an element of Ω^{+}. In this case

$$S = (\{1\} \times A_4) \cup (\{-1\} \times (S_4 \setminus A_4)) .$$



7.1.1 Cube puzzles

Figure 15 shows the two solutions of the cube (2) puzzle, which represent even better than figure 14 the tetrahedron group.

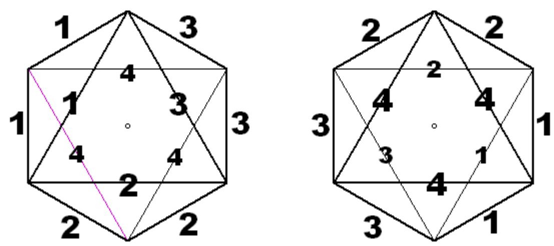


7.1.2 Octahedron puzzles

Figure 16 represents solutions of two equivalent puzzles, that we call the octahedron (2) puzzle. These solutions correspond to the natural tetrahedron isomorphisms. Note that

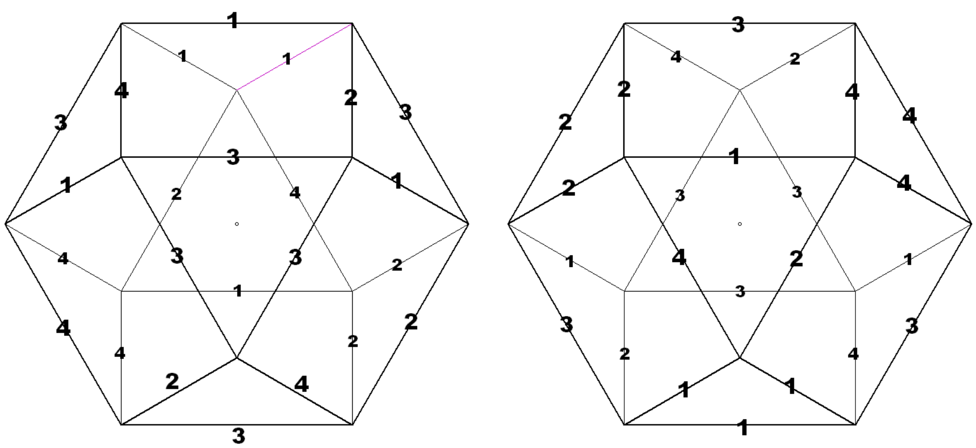
$$S = (\{1\} \times A_4) \cup (\{-1\} \times (S_4 \setminus A_4))$$

is precisely the plate group.



7.1.3 Cuboctahedron puzzles

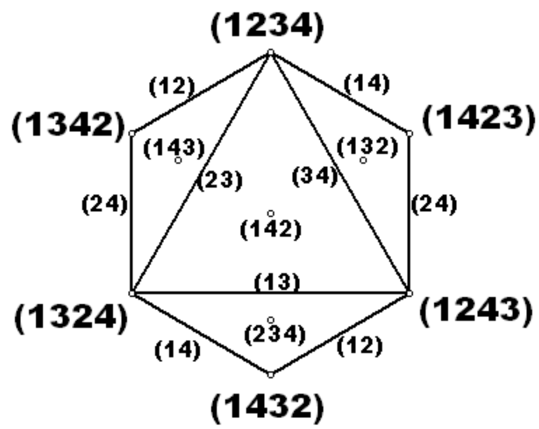
Figure 17 represents solutions of two equivalent puzzles, that we call the cuboctahedron (3) puzzle. These solutions correspond to the natural tetrahedron isomorphisms. Note that, as in the octahedron case, S is precisely the plate group.



7.2 The octahedron (cube) group

There is only one natural isomorphism for the octahedron (cube) group (see figure 18). In this case

$$S = \{-1, 1\} \times S_4.$$

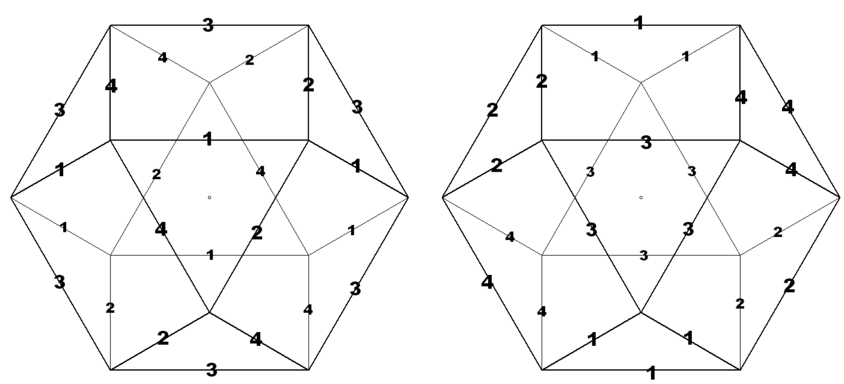


7.2.1 Cube and octahedron puzzles

If one looks for puzzles with solutions that have as their group $\Omega^+ \equiv S_4$, we find the cube (1) and the octahedron (1) puzzles. Those solutions are maximal. This means that there are no puzzles where $S = \{-1, 1\} \times S_4$ is a solution group.

7.2.2 Cuboctahedron puzzles

The cuboctahedron (1) puzzle has only a maximal natural solution which is shown in the l.h.s. of figure 19. As its group is precisely $S = \{-1, 1\} \times S_4$, this solution is excellent in order to represent the octahedron (cube) group. This puzzle and this solution is completely rediscovered using S.

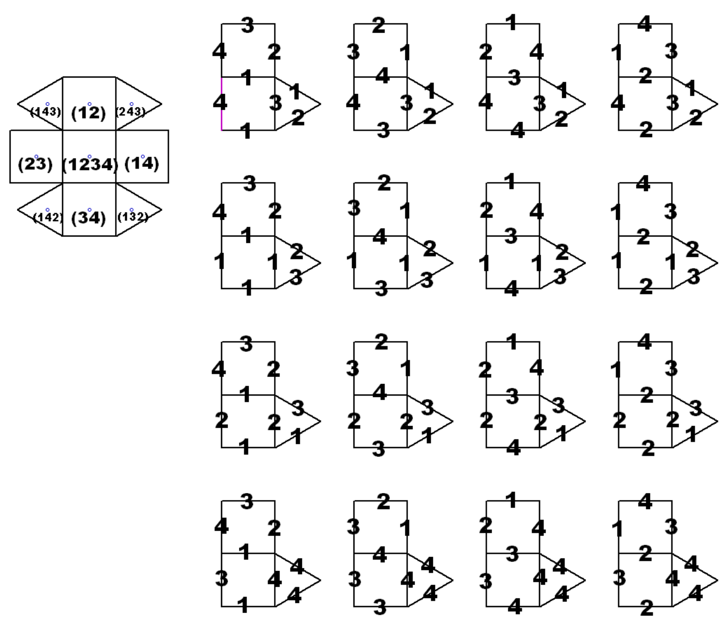


In the r.h.s. of figure 19 is represented the solution of the cuboctahedron (2) puzzle which has also S as group. These two are the only possibilities for cuboctahedron puzzles with S as maximal group.

Note that the rhombic dodecahedron puzzle of Reference [6] has only one natural solution which is dual of the l.h.s. solution in figure 19. This property was crucial in choosing the puzzle.

7.2.3 Rhombicuboctahedron puzzles

Consider a rhombicuboctahedron square face which has no common edge with a triangular face. Associate to this face the permutation (1234). To a neighbor triangular face one must assign the permutation (132), as it is shown in figure 20, if one wants to have a puzzle with a maximal group containing S_4. Figure 20 shows sixteen possibilities and none of them has a central symmetry. All of them are maximal solutions of puzzles with S_4 as a group.

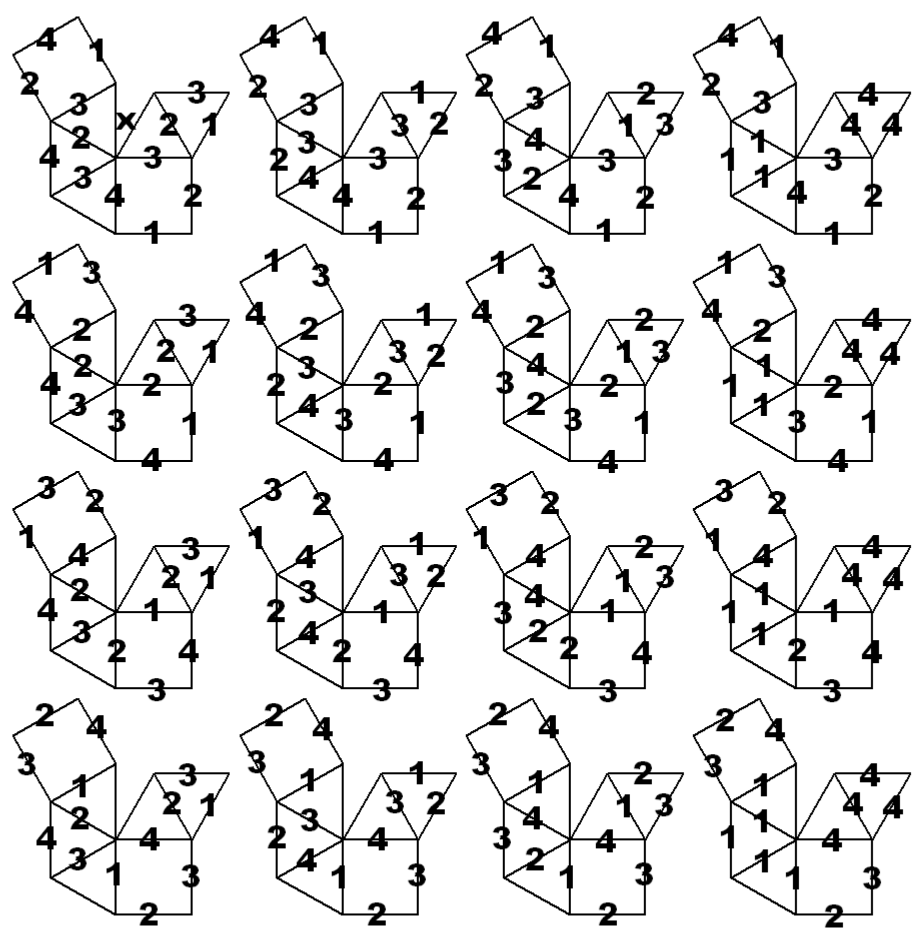


There are six puzzles. All the puzzles have the six plates of figure 4. Four of them have the eight plates represented in figure 1 (see the first three rows in figure 20) and the other two have eight triangular plates of the type aaa (see the last row in figure 20). The first puzzle has the remaining square plates of the type $abac$ (6 solutions). The second puzzle has the remaining square plates of the type $aabb$ (2 solutions). The third puzzle has the remaining square plates of the type $aaaa$ (2 solutions). The forth puzzle has the remaining square plates of the type *abab* (2 solutions). The fifth puzzle has the remaining square plates of the type *abac* (2 solutions). The sixth puzzle has the remaining square plates of the type *aabb* (2 solutions).

The first puzzle is the one presented in Reference [6]. It was chosen because two of its six solutions with S_4 as a maximal group are dual of two maximal solutions of the deltoidal icositetrahedron puzzle presented in the same Reference [6].

7.2.4 Snub cube puzzles

As in the rhombicuboctahedron situation there are sixteen cases, that are shown in figure 21, if one wants to have puzzles with S_4 as a maximal group (the largest one possible, as the snub cube has no central symmetry).



In the figure the edge marked with x must have a 2 or a 3 in each of the sixteen cases. Hence, there are, in fact, thirty two possibilities.

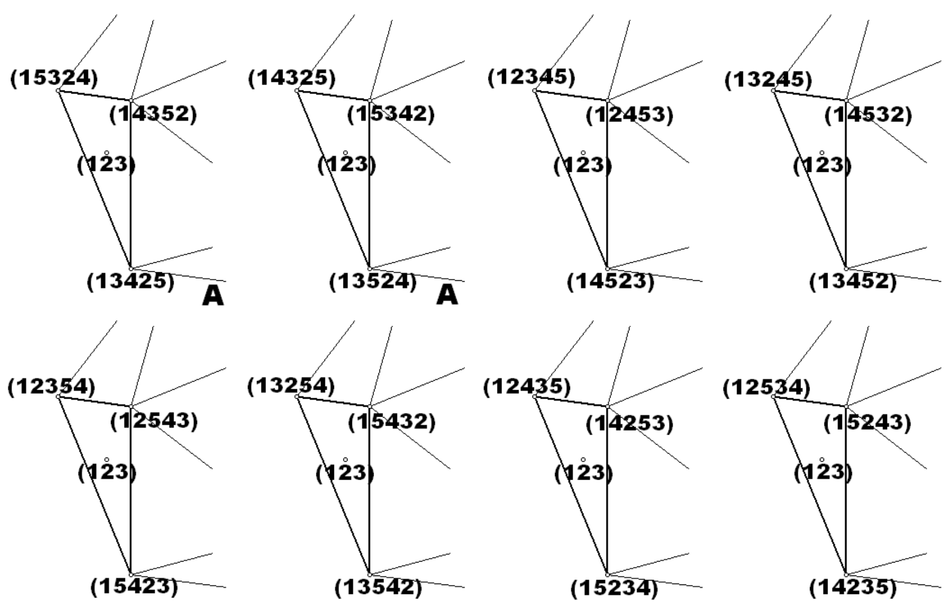
There are five puzzles. All the puzzles have the six plates of figure 4. Three have the eight plates represented in figure 1 and two have eight triangular plates of the type *aaa*.

The first puzzle has the remaining triangular plates of the type *abc* (8 solutions). The second puzzle has the remaining triangular plates of the type *aab* (14 solutions). The third puzzle has the remaining triangular plates of the type *aaa* (2 solutions). These are the first three puzzles (the first three columns in figure 21).

The last two puzzles (the last column in figure 21) are as follows. The fourth puzzle has the remaining triangular plates of the type *abc* (4 solutions). The fifth puzzle has the remaining triangular plates of the type *aab* (4 solutions).

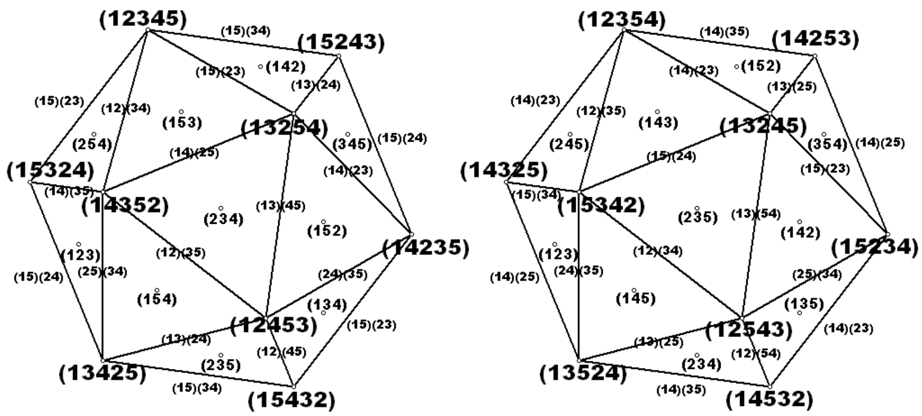
7.3 The icosahedron (dodecahedron) group

In the icosahedron the center of a face represents a semiaxis of order 3. Associate to a given face the permutation (123) as it is shown in figure 22. Then there are 8 possibilities for the vertices of this face, but only two of them, the "A" ones in the figure, are coherent in the sense that they generate a group isomorphism.



There are two natural isomorphisms for the icosahedron (dodecahedron) group (see figure 23) which are equivalent. This equivalence is made, for example, by any transposition and an element of Ω^+. In this case

$$S = \{-1, 1\} \times A_5.$$

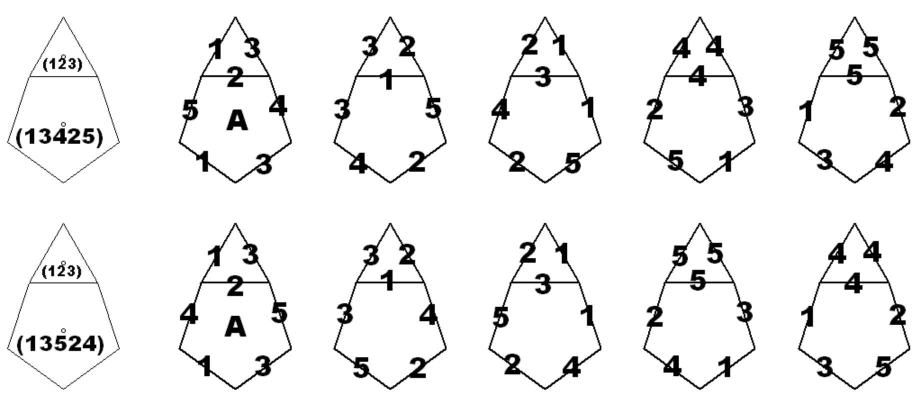


7.3.1 Icosahedron and dodecahedron puzzles

Using this isomorphism one can recover the icosahedron (1) and the dodecahedron (2) puzzles, which have precisely S as maximal group (see Reference [4]).

7.3.2 Icosidodecahedron puzzles

For the icosidodecahedron there is only one puzzle with S as maximal group (in fact, there are two equivalent puzzles). Figure 24 shows the possibilities if one wants to have A_5 as a group (two puzzles). Only one of them, the "A" one, has a central symmetry.



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The History of Combinatorial Game Theory

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Abstract

A brief history of the people and the ideas that have contributed to Combinatorial Game Theory.

1 The Newcomer

Games have been recorded throughout history but the systematic application of mathematics to games is a relatively recent phenomenon. Gambling games gave rise to studies of probability in the 16th and 17th century. What has become known as *Combinatorial Game Theory* or *Combinatorial Game Theory à la Conway*—this to distinguish it from other forms of game theory found in economics and biology, for example—is a babe-in-arms in comparison. It has its roots in the paper [13] written in 1902, but the theory was not 'codified' until 1976-1982 with the publications of *On Numbers and Games* [16] by John H. Conway and *Winning Ways* [11] by Elwyn R. Berlekamp, John H. Conway and Richard K. Guy.

In the subject of *Impartial* games (essentially the theory as known before 1976), the first MSc thesis appears to be in 1967 by Jack C. Kenyon [32] and the first PhD by Yaacov Yesha [63] in 1978. Using the full theory is Laura J. Yedwab's MSc thesis in 1985 [62] and David Wolfe's 1991 PhD [60]. (Note: Richard B. Austin's MSc thesis [3] contains a little of the *Partizan* theory, but Yedwab's thesis is purely Partizan theory.)

In the sequel, there are several Mathematical Interludes that give a peek into the mathematics involved in the theory. Note that the names of games are given in small capitals such as CHESS. Also the notion of game is both general, like CHESS which refers to a a set of rules, and specific as in a specific CHESS position. Whenever I talk mathematically, as in the Interludes, it is this specific notion that should be invoked.

2 What is Combinatorial Game Theory?

This *Combinatorial Game Theory* has several important features that sets it apart. Primarily, these are games of pure strategy with no random elements. Specifically:

1. There are *Two Players* who *Alternate Moves*;
2. There are *No Chance Devices*—hence no dice or shuffling of cards;
3. There is *Perfect Information*—all possible moves are known to both players and, if needed, the whole history of the game as well;
4. *Play Ends, Regardless*—even if the players do not alternate moves, the game must reach a conclusion;
5. The *Last Move* determines the winner—**Normal** play: last player to move wins; **Misère** play last player to move loses!

Proceedings of Board Game Studies Cooloquium XI, pp. 133–145

The players are usually called *Left* and *Right* and the genders are easy to remember —Left for **L**ouise Guy and Right for **R**ichard Guy who is a important 'player' in the development of the subject. More on him later.

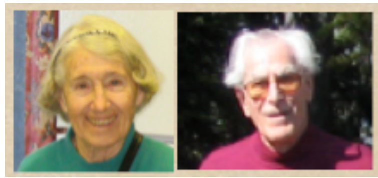

Figure 1: Louise and Richard Guy in Banff

Examples of games[1] NOT covered by these rules are: DOTS-&-BOXES and GO, since these are scoring games, the last person to move is not guaranteed to have either the highest or the lowest score; CHESS, since the game can end in a draw; BACKGAMMON, since there is a chance element (dice); BRIDGE, the only aspect that this game satisfies is that it ends.

Games which are covered by the conditions are: NIM[2]; AMAZONS, CLOBBER, DOMINEERING and HEX. In fact, NIM, AMAZONS, DOMINEERING and also, despite the coments of the previous paragraph, DOTS-&-BOXES and GO have a property that makes the theory extraordinarily useful for the analysis of these games. The board breaks up into separate components, a player has to choose a component in which to play. Moreover, his opponent does not have to reply in the same component. This is why condition (4) is important. The aspect is so important that it has its own name.

The **disjunctive sum** *of games G and H, written $G+H$, is the game where a player must choose to play in exactly one of G and H.*

The game of NIM with heaps of sizes 3, 4 and 5 is the disjunctive sum of three one-heap games of NIM. One could also imagine playing the disjunctive sum of a game of CHESS with a game of CHECKERS and a game of GO. On a move, a player moves in only one of the games but the opponent does not have to reply in the same game. The winner will be the player making the last move over all. As a rule-of-thumb, if a position breaks up into components so that the resulting game is a disjunctive sum then this theory will be useful. If the game does not become a disjunctive sum, HEX for example, then the theory is less useful.

We still need a few more definitions. In an *Impartial* game both players have exactly the same moves—NIM for example. In a *Partizan* game the players have different moves—in CHESS a player can only move his own pieces and not those of his opponent; she would get rather upset if he did.

A game belongs to one of four *outcome* classes. This was first noted, by Ernst Zermelo [64] in 1912, but phrased differently. A game can be won by:

- $\mathcal{L}$eft regardless of moving first or second;
- $\mathcal{R}$ight regardless of moving first or second;
- by the $\mathcal{N}$ext player regardless of whether this is Left or Right;
- or by the $\mathcal{P}$revious player regardless of whether this is Left or Right.

An outcome class is usually referred to by its initial. In an Impartial game such as NIM, since both players have the same moves thus the outcome of a position must be either $\mathcal{N}$ or $\mathcal{P}$.

[1] For rules of these, and other, games see `http://en.wikipedia.org/wiki/` with *Game_of_the_Amazons*; *Clobber*; *Domineering*; *Dots_and_boxes*; or *Hex_(board_game)* at the end of the URL.

[2] NIM **Rules:** On a table there are several heaps of counters. A player chooses any heap and removes any number of counters from that heap. The person taking the last counter wins. For example, suppose there are three heaps with 3, 5 and 7 counters respectively, a player could choose the heap of 7 and remove any number from 1 through 7 counters.

A main aim of the theory is to give a value to each component: essentially how much of an advantage the position is to one of the players—positive value for Left and negative for Right. First, though, we have to deal with *equality*: two games should be the same if both players are indifferent to playing in one or the other. Or

Equality or the 'Axiom of Indistinguishability': *$G = H$ if, for all games X, the outcome for $G+X$ is the same as the outcome for $H+X$.*

Finally, we are ready to talk history! The history breaks up into three main threads and all threads are still very active:

- **Impartial** games under the *Normal* play ending condition which starts with Bouton and NIM [13] through Guy & Smith [27];
- **Partizan** games again under the *Normal* play rule starting with Milnor's [41] and Hanner's [28] work (from GO) through Berlekamp, Conway & Guy [16, 11];
- **Impartial** games under the *Misère* rules starting with Dawson in 1935. (See the Dover collection [19]).

What about the obvious fourth thread?

- **Partizan Misère** games: there are exactly two papers on the subject, [40, 48] both in 2007. This topic is hard!

Mathematical Interlude 1. *How to play and win at* NIM.

If there is one heap, take it all!

If there are two unequal heaps, remove from the larger to leave two the same size.

For three or more heaps, write each heap size as a sum of powers of 2, i.e., as sums of 1, 2, 4, 8, etc; pair off equal powers of 2; if all powers are paired off, invite your opponent to go first. He must disturb the pairings and your winning response is to remove enough counters to re-establish a pairing. Mathematically, write the numbers in binary and add without carrying.

For example: with heaps of size 1, 5 and 7 then as sums of powers of 2, $1 = 1$, $5 = 4+1$ *and* $7 = 4+2+1$*, the 4s pair off but not the 2s or the 1s. The winning move (there could be more than one but not in this situation) is to play to remove 3 (=2+1) from the 7 heap to leave the position 1, 5, 4 where* $1 = 1$, $5 = 4+1$ *and* $4 = 4$*. If the opponent were now to move to 1, 3, 4 then* $1 = 1$, $3 = 2+1$ *and* $4 = 4$ *and only the 1s are paired. No move will ever create another 4 so the 4 has to go but at the same time you should leave a 2 to pair off with the other 2, i.e. move to the position* $1 = 1$, $3 = 2+1$ *and* $2 = 2$.

3 The Winners at Normal Play

3.1 Impartial Normal Play

The game of NIM is *Impartial* since both players have the same moves. Charles L. Bouton [13] analyzed NIM. (See [39] for Bouton's mathematical obituary.) There have been discussions over the origin of the the name *nim* with some references to a Chinese origin. However, Bouton did his PhD in Leipzig so it is likely that the name owes much to the German verb *nimm* meaning ' take'. It took three decades before it was realized, and proved, that each Impartial game is equivalent to a NIM position. Knowing how to win at NIM then allows a player to know how to win all Impartial games. Of course, calculating the equivalent NIM position is non-trivial. The Bouton paper sparked an interest in the area and several important papers resulted. From our vantage point, some of the games that were suggested were very interesting, even important, but went off the track of developing the general theory.

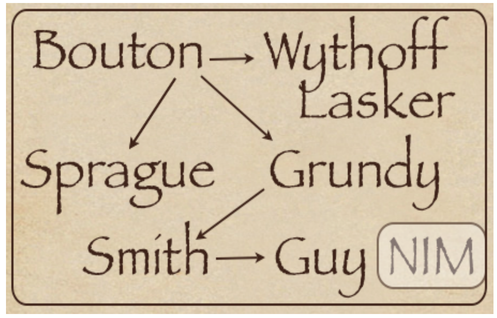

Figure 2: Genealogy of Early Impartial Players

First was WYTHOFF'S GAME, introduced and solved by Willem A. Wythoff [61] in 1907. (See [33] for a very brief biography.) The rules are: *There are two heaps of counters on a table. On a turn, a player either chooses a heap and takes as many counters as they wish; or they may take an equal number from both. The player taking the last counter wins.* The game was also given, independently, by Rufus Issacs, see [6] p. 53: Play with a Chess Queen on a quarter infinite board and a move must move the Queen closer to the corner of the board. The heaps are the coordinates of the Queen's position. Hence the game is sometimes called WYTHOFF QUEENS. The game has interesting connections

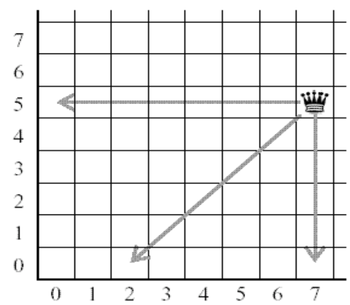

Figure 3: WYTHOFF'S GAME as WYTHOFF'S QUEENS

to the golden ratio and Fibonacci numbers, and has led to some very interesting and beautiful mathematics, see [18]. However, it turned out to be a wrong direction for the theory. The game does not break up into disjoint components. Several other authors followed including Eliakim H. Moore (MOORE'S NIM [42]), where a player make take from up to *k* heaps, *k* being fixed in advance, which is also off in the wrong direction since it merges rather than separates the heaps.

In the right direction, Emanuel Lasker [35] p.183 in 1931, introduced LASKER'S NIM—the same rules as NIM with the extra option of removing no counters but splitting a heap into two (non-empty) heaps. This clearly highlights the disjunctive sum aspect. According to Jörg Bewersdorff [12] (pp.174-176 in the English version), Lasker just missed developing the whole theory of Impartial games. He did understand the outcome classes $\mathcal{P}$ and $\mathcal{N}$ and how they interacted. And, according to Richard K. Guy [1], Michael Goldberg in the 1938 edition of W. W. Rouse Ball's *Mathematical Recreations and Essays* [5] solved much of KAYLES and "was unlucky not to have discovered the complete analysis and the S-G [Sprague-Grundy] theory". The theory was 'in the air' and it was left to Sprague and Grundy to find it.

Roland P. Sprague [55, 56] in 1935 and independently Patrick M. Grundy [21] in 1939 published complete solutions on how to solve Impartial games. (See [53] for a mathematical obituary of Grundy.) This became known as the *Sprague-Grundy Theory* and the value associated with an Impartial game was referred to as the *Grundy-value*. (Guy and Smith didn't learn about Sprague's work until after Grundy's death in 1959.) Since this value is equal to the size of the NIM-heap to which it is equivalent, and since it has been placed inside more encompassing theory, many authors now refer to the *nim-value* of a game and the set of values as the *nimbers*.

In 1949, R. K. Guy, in solving DAWSON'S CHESS[3], also re-discovered the Sprague-Grundy theory and in addition, an infinity of games to which the theory could be applied. (See Mathematical Interlude 2.) R. K. Guy was steered toward Cedric A. B. Smith who had worked with Grundy. (Smith was also a member of *Blanche Descarte*[4]. This led to the 1956 article [27] and to a career that is still active today. In the area of combinatorial game theory, R. K. Guy has: published over 20 articles; published two books (more on those in the Partizan Section); helped organize five major conferences; edited one Conference proceedings; and maintains an 'Unsolved Problems in Combinatorial Game Theory' column. This doesn't count the over two hundred and fifty other publications of his. One interesting aspect, despite all his achievements, Guy got the first solution wrong! Or rather, he solved the wrong problem. Dawson asked for the Misère version and Guy solved the Normal play version believing ([25, 1]), like so many others, that the winning strategy for Misère play is a slight tweak of the strategy for Normal play. Moreover, the original DAWSON'S CHESS is still unsolved today!

WYTHOFF'S GAME, GRUNDY'S GAME and MOORE'S GAME are associated with beautiful and sometimes surprising mathematics. One other game that should also be mentioned in the same vein is WELTER'S GAME [58, 59] from 1952[5]. Welter knew of the work of Sprague and generalized one his NIM games. Despite the apparent 'welter' of confusion about the equivalent nim-values, there is a very pretty way to decide on a good move [16] pp.153–165 and [11] pp.506–515, see also [10, 17].

In terms of the development of the theory the focus now shifts, but before moving on, one other person, *Aviezri Fraenkel* ([20, 26]) should also be noted as the one who has the largest number of publications in the area, with well over sixty papers on mainly, but not restricted to, Impartial games and complexity results. (This count does not include his numerous papers on other areas of mathematics and computer science.)

Figure 4: Aviezri Fraenkel

Mathematical Interlude 2. *The games of R. K. Guy are the innocuous sounding* SUBTRACTION *and* OCTAL *games.*

Given a finite set of numbers, say $S = \{2,3,5\}$, called the subtraction *set, and a heap of n counters, a player may take away 2, 3 or 5 counters. The outcome-sequence for the set S is the sequence of the outcome for a heap: of size 0, 1, 2; .... The outcome sequence for $S = \{1\}$, i.e. where a player is only allowed to take away 1 counter, is: $\mathcal{P},\mathcal{N},\mathcal{P},\mathcal{N},\mathcal{P},\mathcal{N},\ldots$. To see this, a heap of size* 0 *is a $\mathcal{P}$revious player win, since neither player has a move;* 1 *is a $\mathcal{N}$ext since the Next player can reduce the heap to 0 whereupon his opponent has no move and so loses. From a heap of 2, then you, as the next player, must move to a heap of 1 which is an $\mathcal{N}$-position, i.e. you give your opponent a good move, so the outcome of a 2-heap is $\mathcal{P}$. A little more thought gives that a heap of even size is a $\mathcal{P}$-position and an odd-sized heap is a $\mathcal{N}$-position giving the sequence sometimes referred to as* SHE-LOVES-ME-SHE-LOVES-ME-NOT. *The reader is encouraged to find the outcome sequences of: $S = \{1,2\}$; $S = \{1,2,3\}$; etc.*

In general, it is known that for any set S, eventually, the nim-sequence will be periodic, but no-one has discovered a relationship between S and the form of the period. Actually, researchers look for the nim-*sequence where instead*

[3] DAWSON'S CHESS [19] *Given two equal lines of opposing Pawns, White on 3rd rank, Black on 5th, in adjacent files. White to play and capturing is mandatory. The player who makes the last move loses. Who wins?*

[4] See `http://www.squaring.net/history_theory/brooks_smith_stone_tutte.html`.

[5] WELTER'S GAME: is played with coins on a strip of squares numbered 1 through whatever. The coins can be moved to any smaller numbered, unoccupied square but no square can have more than one coin. The coins accumulate at one end of the strip and so the game finishes.

of the outcomes, the size of the equivalent NIM*-heap is recorded. Note that in this case, the $\mathcal{P}$-positions correspond exactly to heaps of size 0.*

OCTAL *games are like* LASKER'S GAME*, where a move, depending on the exact rules, may allow a player to take from a heap and possibly split the remaining heap into two heaps. In* GRUNDY'S *game, heaps are allowed only to be split into two non-equal, non-empty heaps. Despite having been analyzed to heap-sizes of many billions, no periodicity or other regularity has been discovered.*

For an in-depth discussion of these, and other heap games, see [11] Chapter 4.

3.2 Early Partizan Players

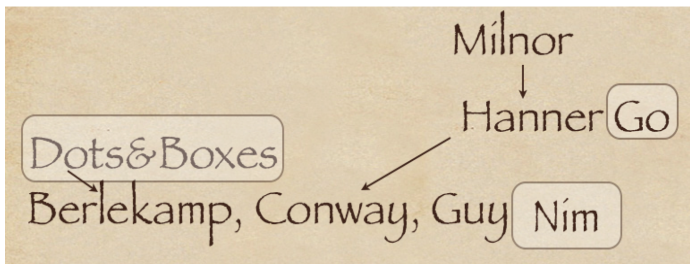

Figure 5: Genealogy of Early Partizan Players

In 1953, arising out of his research in *classical* game theory (the game theory used in economics and biology and other sciences), John Milnor [41] wrote the first theoretical paper on Partizan games. He recognized that there were *hot* positions, positions in which both players are eager to move because of the advantage gained. A mathematical approach to approximating such positions is to see what happens when there are many copies of the position. This gives an idea of the 'mean-value' of the position—on average, what advantage might the position be worth.

Olof Hanner was interested in GO and in 1957 whilst wandering around Stockholm he found a GO book with an annotated game. The author claimed that Black won by one point but Hanner found that Black should win by two points. (See [46] for more details about Hanner.) This led Hanner [28] to define his own version of the 'mean-value' of a game. To quote Yedwab, [62], "*One way to view Hanner's strategy, is that it addresses a basic weakness found in Milnor's strategy, i.e., tempo. In Milnor's strategy, the follower is a wimp that passively responds to the leader's move, even when it is obvious that the leader's move is not sente.*" That is the 'leader' takes advantage of the disjunctive sum by freely choosing which component to play in, but the follower is constrained to playing in the same component as the leader. Note that a move is *sente* if the opponent has to reply in order to prevent a large loss. Also, Milnor's definition of a mean-value is not robust. The approximation to the mean-value could get worse, not better, as more copies are added to the sum, but Hanner had hit upon the right idea.

Richard K. Guy now re-enters the scene as a unifying force. John H. Conway had been interested in games. (Conway has many achievements including the GAME OF LIFE.) Conway knew Mike Guy, Richard's son. They met in 1960[6] in Cambridge when John Conway was a first year graduate student and Mike Guy a new undergraduate. Mike Guy passed along the Impartial theory developed by his father.

Elwyn R. Berlekamp met Richard at a conference in 1966. Berlekamp had just 'solved' the Impartial game of DOTS-&-BOXES[7] with help from the Guy-Smith paper [27]. According to legend and eyewitnesses, Elwyn Berlekamp has not lost a game of DOTS-&-BOXES in over 40 years. (See [8] for more on the game.) He suggested that they write a book and Guy suggested adding Conway. They started work soon after and the two volume set *Winning Ways* appeared in 1982. The present version is the four volume set [11]. During the 16-year production, John H. Conway realized that

[6] Richard and Louise's daughter Anne, the first person to solve *Rubik's Cube*, also started in maths at Cambridge the same year.

[7] John C. Holladay in [30] partially solved DOTS-&-BOXES in the version where a player MUST take a box if one is present. Also, Holladay [29] rediscovered the Sprague-Grundy theory in 1957.

there is a wonderful mathematical theory, based on the 'disjunctive sum' concept, underpinning these games and first published *On Numbers and Games* in 1976 (re-published in 2001 [16]). Conway developed a new number system out of evaluations of games. This system, called *surreal numbers* (by Donald Knuth! [34]) extends the real numbers in a manner similar to that of *Dedekind cuts* which extends the rational numbers to the reals. Later, Elwyn Berlekamp observed that GO games frequently broke up into a disjunctive sum and a new area of games research was born, see [9].

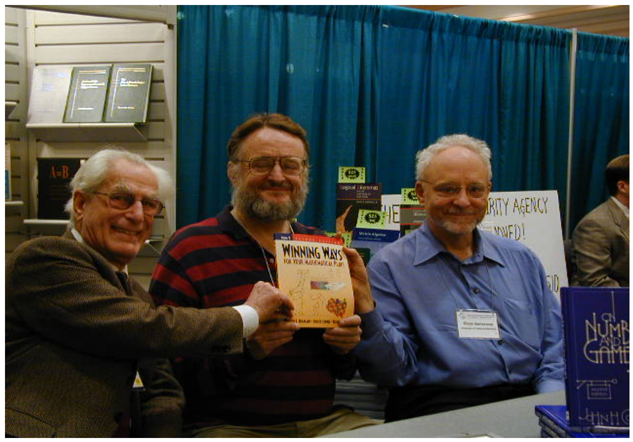

Figure 6: Guy, Conway and Berlekamp

These books still are the standard references and bibles of the subject. Admittedly, *On Numbers and Games* is a graduate level mathematics text but *Winning Ways* is a recreational mathematics book and is very accessible.

Mathematical Interlude 3. *Mathematical Structure of Games—Partially-Ordered Abelian Group.*

1. **Zero:** *If G is a 2nd player win then the outcome of $G+H$ is the same as that of H for all games H. The player who can win H plays this strategy and never plays in G except to respond to his opponent's moves in G. Thus, any 2nd player win game acts like 0 in that it changes nothing when added to another game.*

2. **Negative:** *Given a game G, $-G$ is G with the roles reversed. For example, in* CHESS *this is the same as turning the board around.*

3. **Equality:** *$G=H$ if $G+(-H)$ is a 2nd player win; i.e. neither player has an advantage when playing first. Note this is a 'definition' of equality and mathematically we can say $G+(-H)=0$ is the same as $G=H$. [Note that this really defines an equivalence relation and the 'equality' is for the equivalence classes.]*

4. **Associativity:** *$G+(H+K)=(G+H)+K$ is straightforward from the definition of the disjunctive sum;*

5. **Commutativity:** *$G+H=H+G$, again straightforward;*

6. **Inverses:** *For any game G, $G+-(G)$ is a 2nd player win, (i.e. $G+(-G)=0$) by 'Tweedledum-Tweedledee'—whatever you play in one, I play exactly the same in the other.*

7. **Inequality:** *$G \geq H$ if Left wins $G+(-H)$; i.e. there is a bigger advantage to Left in G than in H.*

The structure really is a partial order. For example, when playing NIM*, a heap of size 1 and a heap of size 2 are incomparable. Let's call these games $*1$ and $*2$ for easy reference. Note that $-(*2)$ is the same game as $*2$ since the Left moves are the same as the moves available to Right so interchanging them has no effect on the play of the game. We already know that $*1+*2$ is a first player win—the winning move is to $*1+*1$. By the definition of 'equality' then $*1 \neq *2$. Moreover, by the definition of inequality $*1 \not> *2$ and $*1 \not< *2$.*

3.3 Very Modern Normal History

With the publishing of *On Numbers and Games* and *Winning Ways* the full framework of the theory of Combinatorial Games was laid down. However, much work remains to be done within that framework. Indeed, the activity can be classified (roughly) into five main areas.

- **Algorithmic Game Theory:** Theory is fine, but most people want to know how to win in an actual game. When analyzing games, hand calculation can only go so far[8]. In the early 1990s, David Wolfe developed the software *Gamesman's Toolkit* which has been superseded in 2000 by Aaron Siegel's *CGSuite* [49].
- **Complexity:** long an interest of Computer scientists as well as mathematicians. Look to researchers such as Aviezri Fraenkel and Erik Demaine.
- **Hot Games:** Games in which there is a large advantage in moving first. These games are perhaps of the only ones of interest to real world games players. This area has the most overlap of computer scientists and mathematicians. The mathematicians want to know the exact values and the very best strategies by working backwards from the end. The computer scientist wants good heuristics that will allow good play from the beginning.
- **Impartial games:** Even though these games, such as NIM, started the area, much remains to be discovered.
- **All-small games:** Games like CLOBBER[9] in which either both players have a move or neither does. It is not possible to build a large advantage as in Hot games.

But, as is the wont of mathematicians, almost immediately, if not sooner, research started into pushing the envelope, relaxing the conditions that define combinatorial game theory. Briefly, the highlights are:

- **Scoring Games** such as GO and DOTS-&-BOXES. The Chinese scoring convention almost makes GO into a combinatorial game and the last player to move in DOTS-&-BOXES is usually the winner so the theory is applicable to these games. Indeed, research into GO is pushing the limits of the theory. A new avenue of research has been started by Elwyn R. Berlekamp [7]. He introduced the idea of an *enriched* environment—a stack of coupons in decreasing order. A player may make a move or take the top coupon of the stack, the value of which is added to the player's score. This is a useful analysis tool in all hot games and has even made its appearance in International GO events.
- **Loopy Games:** Games in which the play is not guaranteed to end. In 1966, C. A. B. Smith [54] first extended the Impartial theory (of Sprague and Grundy) to games with cycles (see also [4]) and, in 1978, John H. Conway [15] showed that canonical forms could be defined for some loopy games. A mistake in the analysis of FOX-&-GEESE in *Winning Ways* led to more, very recent, advances obtained by Aaron Siegel [50, 51, 52].
- Allowing a **random** element. In Richman games [37, 36], players bid for the right to play next in an otherwise combinatorial game.
- Allowing **three** or more players. The problem is that there could be off-the-board strategies; the players could form coalitions for all or some of the game. Also, player A on his last turn could make either of players B and C the winner but not himself. Li [38] (1977) and Straffin [57] (1985) considered the formation and behavior of coalitions. Propp [47] (2000) and Cincotti [14] (2000) considered the situations when one player has a winning strategy against a coalition of the other two.

[8]Having said that, around 1950, R. K. Guy was calculating the nim-sequences for SUBTRACTION and OCTAL games up to heaps of size 600, whilst C. B. Haselgrove wrote a program that ran EDSAC out of memory at size 400. P. M. Grundy managed to get the nim-sequence, from a computer, for his game (i.e. GRUNDY'S GAME) up to heap size 1100 many of which were wrong because of overflow errors! E. R. Berlekamp discovered a structure within the nim-sequence of this game, called the *sparse space* phenomenon, that has allowed dedicated machines to extend the nim-sequence to roughly 17 billion.

[9]CLOBBER is played on a rectangular board, 8×6 for example, starting with alternating black (Left) and white (Right) pieces. A piece is moved one square horizontally or vertically provide the new square is occupied by an opponent's piece which is then removed from the board.

4 Misère Play; or the Best Losers

Thomas R. Dawson ([31]) was a composer of chess problems (which is how he and R. K. Guy met). The solution to his 1935 Misère problem of DAWSON'S CHESS, of course, depends on the width of the board. This is the problem that Guy solved for Normal play. Many researchers believe that the strategy for a Misère game is to take the Normal play strategy and tweak it at the end of the game. While this is true for NIM, it is not true in general.

Results in Misère play have been few and far between. From 1935 up to 2001, there are only thirteen papers on Misère games, although they are also considered in both *On Numbers and Games* and *Winning Ways*. In *On Numbers and Games*, Conway shows that the theory for Normal play will not translate to Misère play. In Normal play, many games are equal to each other which means that a strategy for one works for all the others. In Misère play almost all games are equal to themselves and few, if any, others: the strategy for one game does not help with the strategy for any other game.

First, Patrick M. Grundy and Cedric A. B. Smith [22] considered Impartial games under Misère play rules. Essentially, all but two of these eleven papers, only add to human knowledge by dealing with specific games. However, in the last few years, Thane Plambeck, and now Aaron Siegel, have started a new and exciting chapter in this area. The history of Misère games is only now being written.

Figure 7: Thane Plambeck on the left and Aaron Siegel on the right

Mathematical Interlude 4. *Recall that, having decided which universe (Normal or Misère) of games we are playing in, the definition of equality of games is:*

$G = H$ if for all games X in this universe, the outcome for $G+X$ is the same as the outcome of $H+X$.

Plambeck's approach is to limit the size of the universe. Given a game position G, Plambeck's universe is restricted to only those games that can be reached from G. This universe is called the closure *of G. Equality is now defined as:*

Given a game G then for H and K in the closure of G, $H = K$ if for all games X in the closure, the outcome of $H+X$ is the same as the outcome of $K+X$. *Games can now be equal in one universe but unequal in another.*

5 Sources

Not everyone can have the good fortune of having talked to Richard Guy, and also Elwyn Berlekamp, John Conway, Aviezri Fraenkel, Thane Plambeck, Aaron Siegel and David Wolfe. For the less fortunate, some biographical details can be found on Wikipedia and the St. Andrews history website, These early 'players' were, and still are, proficient mathematicians who accomplished much across many fields. It is well worth reading the more personal recollections, stories and biographies that can be found in [1, 20, 25, 33, 39, 46, 53].

More mathematical papers are listed in the bibliography. Another great resource is Aviezri Fraenkel's bibliography of papers in [43, 44, 45] which also appear as a dynamic survey in the *Electronic Journal of Combinatorics* at http://www.combinatorics.org/.

For some mathematical survey papers see [24, 43, 44, 45]—the last will appear later this year. For an introduction to Impartial games see *Fair Game* [23], to the full theory see *Winning Ways* or *Lessons in Play* [2].

Acknowledgements

Figure 8: Richard J. Nowakowski

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The use of the game of chess to represent famous battles

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Abstract

The game of chess has attracted great passions throughout its history. Some of its enthusiasts have pushed chess in order to represent different kind of situations. Lewis Carroll used a chess game to be the central element of one of Alice's stories. The struggle between good and evil (the latter represented by the devil himself) has been the theme of a series of problems. There were also some attempts to use chess to represent battles. This has occurred in the end of the 19th century and beginning of the 20th century in a period where the first wargames were in fact appearing (although apparently were yet too expensive to become really popular). In this paper I will look into two examples of these trials, examining the way the main problems linked with this representation were solved.

Introduction

> "The fundamental law of war", says Napoleon, "is this, — the greater force always overcome the lesser." (Young, 1912, pp. 3–4)

I have a strong affect and interest to chess. But my interest in chess being twofold, is not exactly about its warrior nature: In fact I am a chess player and I love playing (except if I lose too many games in a row); I am also interested in the cultural images connected with chess, being the military just one aspect really.

Chess is sometimes presented as a game that represents a battle between two sides with equal forces. And it certainly has attracted the attention of many war strategists. But it also has been theorized as a war-like phenomenon at the beginning of the twentieth century. However, both the chess and the military community in Portugal dubiously received that approach.

Just when the twentieth century was beginning, one book showed up, where its author tries to conceptualize chess as war-like activity. One of the interesting features of this book is a set of 17 principles or laws concerning the art of chess play. These laws were written in a military kind of discourse, and could be, at least theoretically, interchangeable between chess and war, if just we changed a few words, like piece for instance:

> 1st law of the art of chess play Whenever two undefended kindred pieces having no line of communication are simultaneously attacked by an adverse force, then one of the given kindred pieces is lost. (Young, 1912, p.18) 2nd law of the art of chess play
>
> At every turn to play and no line of operations existing, always act simultaneously with the column of attack in topographical zone, with the column of support in the kindred hypothetical zone, and with the column of manoeuvre in the adverse hypothetical zone, and always reject every move which violates those principles governing the processes incident to these prime strategic factors. (Ibidem, p.70) 3rd law of the art of chess play
>
> I. The column of attack ceases to exist whenever the net value of the kindred determinate force is less than the mobility of the objective plane.
>
> II. The column of support ceases to exist whenever the last kindred promotable factor is eliminated.
>
> III. The column of manoeuvre ceases to exist whenever the last kindred point of impenetrability is eliminated (Ibidem, p.71).

The first laws are somehow understandable from the point of view of modern chess. But some of the laws from there tend to become increasingly far away from chess and more close to the military discourse. One good example is this:

> 6th law of the art of chess play
>
> Having located two tactical keys, two points of command, or one tactical key and one point of command, then connect these points by logistic radii, and those points at which the given logistic radii intersect will be points of communication, and that point of communication common to both will be the topographical centre (Ibidem, p.95)

Curiously, the last of these laws is again close to chess and its concepts, namely the concept of initiative which is highly valued today:

> 17th law of the art of chess play
>
> In short, the initiative is a condition — in fact, it is the only condition — in which the perfect application of strategic knowledge to warfare and to chessplay by means of the processes of their respective arts, is possible. In other words, it is the bridge which unites the principles and formulas of strategic science with the processes of the strategic art. In every situation the initiative is governed by the following law:
>
> At every turn to play dictate the opponent's reply, either:
>
> Strategically, i.e., by occupying a topographical key, and threatening on the next move to occupy another topographical key; or,
>
> Tactically, i.e., by occupying, or by threatening on the next move to occupy, an inadequately defended tactical key (Ibidem, p. 248)

Young's book also contained a battle represented in game. That was the battle of Waterloo, between Napoleon forces and Wellington and Blucher forces. In this specific case, it is not a known game, rather a game constructed from the description of the battle.

It has to be said that perhaps due to the specific method of construction, the game is not a plausible one (against good strategy in chess, also with strange moves in the end describing Napoleon forces retreating). It is also a very long game, with too many moves representing the forces being directed to their place in the battlefield.

Inspired by this attempt, one Portuguese player attempted to represent the Battle of Chryssus with a chess game. Even then, by what is presented in Ansur (1907), many Portuguese officers were not at all convinced that chess could really represent battles.

This game was adapted by the frigate captain Baldaque da Silva from the game played by Morphy in 1858.

The description of the battle where Mr. Baldaque based his own is from Alexandre Herculano, a Portuguese writer, in his work 'Eurico the presbyter'.

The first and second lines of the board represent the Muslim field; the 5th and 6th lines the plane margin; the 8th line the left side of the Chryssus (which is called Guadalete today and is in Spain). The Arab forces are white, commanded by Tarik. The Visigoth forces are black, commanded by King Roderick.

We are in the year 711, Arabs are beginning their conquest of the Iberian Peninsula.

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The Battle of Chryssus (in Ansur (1907), by the frigate captain Baldaque da Silva)


1. e4 The Arabian esculcas move forward
1... e5 The mountaineers of Hermínio occupy the plains
2. Nf3 The squadrons of Juliano move to face the mountaineers
2... d6 The veterans of Narbone sustain the mountaineers


5. Qxf3 The squadrons of Tarik retake the position
5...dxe5 The veterans of Narbone defeat the Berbers
6. Bc4 The cavalry of Mugueiz (El-Rumi) attack the left flank
6... Nf6 The Black Knight comes in defence of Ruderick

 

7. Qb3 The Saracen squadrons reinforce the African cavalry

7... Qe7 Ruderick sends the Cantabrian cavalry to defend

8. Nc3 The Berber tribes of Mazmuda advance

8...c6 The Herminian and the Vasconian continue the attack

 

9. Bg5 The Saracen squadrons run towards the black knight

9...b5 The Herminian and the Vasconian continue the attack

10. Nxb5 The Berbers take their position

10...cxb5 The mountaineers retake the position


11. Bxb5+ The African cavalry disperse them and attack Ruderick

11... Nb8d7 The troops from Lusitania and Galicia cover Ruderick

12.o-o-o The emir Tarik regroup with the traitor Goth forces of Sisebafo, Ebbas and Oppas.

12...Rd8 Ruderick reinforces the back with the Carthaginian cavalry


13. Rxd7 The forces of Oppas take the position of the Lusitanian troops

13... Rxd7 The Carthaginian cavalry retake it

14. Rd1 The Arabs attack again with Juliano forces

14... Qe6 The Cantabrian cavalry make a last attempt


15. Bxd7+ The Arabs attack Ruderick closely

15... Nxd7 The black knight, alone, defends him

16. Qb8+ The Saracen squadrons charge over Ruderick (King of the Goth)

16... Nxb8 The black knight desperately defends him


17. Rd8++ Ruderick dies at the hands of the traitors of the count of Septum.

Mathematical Games

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Portugal has a National Championship of Mathematical Games since 2004.

This championship involves students from 1st to 12th grades and six mathematical games, three games for each level of the Portuguese school system. Figure 1 illustrates the six games used in the 4th edition of the National Championship of Mathematical Games. In the last edition the final occurred on April, 29 in Braga, at the University of Minho, with 1100 participants.

Games (2008)	**Grades**			
	1st to 4th	5th and 6th	7th to 9th	10th to 12th
Dots and boxes	✓			
Traffic lights	✓	✓		
Wari	✓	✓	✓	
Hex		✓	✓	✓
Amazons			✓	✓
Slimtrail				✓

Figure 1: Games used in the 4th edition of the National Championship of Mathematical Games

But, what are mathematical games?

For Neto & Silva (2004), mathematical games are strategic or abstract games without chance and without hidden information. *Chess*, *Draughts*, and *Mancala* games are examples of mathematical games.

Recently, a study with students from 3rd to 6th grades (8 to 11 years old) investigated the relationship between *Chess* and problem solving involving geometric and numeric patterns (Ferreira & Palhares, 2007). The goal of this study was to find whether there is a relationship between two distinctive abilities: the ability to play *Chess* and the ability to find patterns.

The methodology of the *Chess* study was quantitative, with a correlational design. In fact, correlational studies are appropriate in educational research when there is a need to discover or clarify relationships and little or no previous research has been undertaken (Cohen & Manion, 1989). The sample was constituted by 437 students from 3rd to 6th year of schooling and interesting conclusions were found. These are the main conclusions:

- The strength of chess play is positively related to problem solving involving patterns, with a coefficient of correlation of 0.46. A detailed analysis reveals that school year affects this relationship. However, excluding its effects, the relationship is still above 0.38;
- The strength of chess play is positively related to numeric patterns ($r = 0.46$);
- Playing or not playing chess has no relation with problem solving involving patterns ($r = 0.13$).

In 2007, the final of the National Championship of Mathematical Games occurred in Évora and it was the background of a new study. In this study we were curious to find whether there is a relationship between the ability to find patterns and the ability to play mathematical games other then *Chess*, as for example *Traffic Lights* and *Amazons*.

As instruments to collect data we used a survey and a test (constructed and validated during the chess study mentioned before). The test had 24 questions, from wich we present, in Figure 2, two examples of a geometric (on the left side) and a numeric question (on the right).

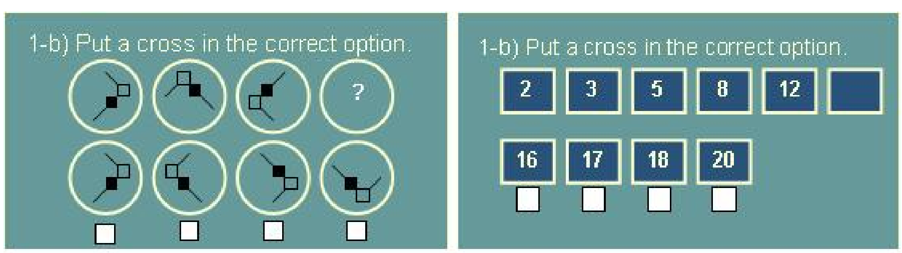
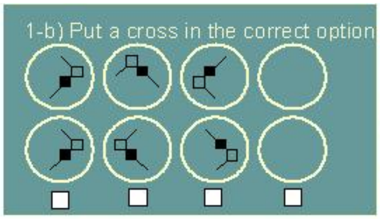


Figure 2: Two examples of the test questions

The elaboration of the test correction criteria was based on the principles reported by Charles, Lester and O'Daffer (1992). The 24 questions have different correction criteria, appropriate for each one. In Figure 3 we present an example of the correction criteria for a geometric question.



Don't answer.	0 Points
Fills in the 3rd square (left to right). Partial identifications of the final position.	4 Points
Fills in all the incorrect options.	10 Points
Correct answer.	12 Points

Figure 3: Correction criteria for geometric question 1-b

The statistical treatment was done using SPSS for Windows, version 13.0 and in the analysis, different statistical procedures have been used. For example, Cronbach's Alpha was used to measure internal consistency.

As we can see in Figure 4, Cronbach's Alfa from 3rd to 6th grades was 0.763. Fraenkel and Wallen (1990) claim that Cronbach's Alpha must be greater than 0.70. Thus we consider this coefficient as a sufficiently good result. But from 7th to 9th grades de Cronbach's Alfa was 0.678 witch is close to the mark mentioned above.

From 3rd to 6th grades

Cronbach's Alpha	N of Items
,763	24

From 7th to 9th grades

Cronbach's Alpha	N of Items
,678	24

Figure 4: Cronbach's Alfa for 3rd to 6th and 7th to 9th grades

To interpret the correlation coefficient we followed the intervals general interpretation of Cohen and Manion (1989) and Fraenkel and Wallen (1990):

- Correlation between 0.2 and 0.35 reveals a small relationship between variables, too small to make predictions;
- Correlation between 0.35 and 0.65 are often found in educational research. They may have theoretical and practical importance depending on the context. They allow for group predictions.
- Correlations above 0.65 indicate a very strong relationship hard to find in Education or in sciences dealing with human beings.

The statistical analysis reveals some interesting results. First, for the game *Traffic Lights*, played by students from 3rd to 4th grades, the coefficient of correlation was -0.757 with a level of significance of 0.05. Actually, this coefficient is considerably higher than the results obtained by chess players in the previous study.

Traffic lights 3rd and 4th grades		Ranking	Test
Ranking	Pearson Correlation	1	-,757(*)
	Sig. (2-tailed)		,011
	N	10	10
Test	Pearson Correlation	**-,757(*)**	1
	Sig. (2-tailed)	,011	
	N	10	10

Figure 5: Correlation between test scores and *Traffic Lights* players ranking

Finally, for the game *Amazons*, played by students from 7th to 9th grades, the coefficient of correlation was -0.587. But in this case the level of significance is only close to the minimum score which is 0.05.

Amazons 7th to 9th grades		Ranking	Test	
Ranking	Pearson Correlation	1	-,580	*)
	Sig. (2-tailed)		,061	1
	N	11	11	0
Test	Pearson Correlation	**-,580**	1	1
	Sig. (2-tailed)	,061		
	N	11	11	0

* Correlation is significant at the 0.05 level (2-tailed).

Figure 6: Correlation between test scores and Amazons players ranking

We must clarify that the reason for a negative coefficient is due to the fact that players ranking and test scores have an opposite direction (best players had a lower number but better test results had a bigger number).

The results presented show us that in 3rd and 4th grades, there's a strong relationship between the ability to solve problems involving patterns and the ability to play the game *Traffic Lights*. However, from the results presented emerges also the need to implement further research to clarify some aspects and to get stronger results.

Researching strategic games

This year we have organized tournaments with a grand total of 271 students from 3rd and 4th grades (these students are 8 and 9 years old) in four primary schools. The games used in these championships were: *Dots & Boxes*, *Traffic Lights* and *Wari*. At the end, the students took the test, but we haven't yet analyzed the data.

In the next years we intend to organize tournaments with students from 5th to 6th grades, in one or two schools, with about 60 students per school. The games that we intend to use are *Traffic Lights*, *Wari* and *Hex*, or other strategic games. At the end we have to apply the test to collect the data. To analyze the data we intend to use statistical tools, as for example Kolmogorov-Smirnov, Pearson, Spearman, Kendall's Tau's test, or others tests that we'll find convenient for our research. The goal of this research is to find whether playing mathematical games is related to identifying patterns, as well as ascertain if some games are more mathematical then others in the sense that some games may be more related to identifying patterns than others.

There are also some possible steps that we can take in our research, as for example:

- Check the test variability for 10th to 12th grades to find if we can use this grades in the study;
- Research physical games as, for example, Tennis (because Tennis has an individual ranking) and use the Tennis ranking as tool for statistic study;
- Research strategic games with hidden information, as cards or dominoes.

These possible steps are new suggestions that we have to plan and reflect about the reasonably of their application to our research. But we are also open to new ideas that can help us find answers to questions around mathematical games and their use with educational purposes.

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Goths, Vikings and Hanseatic Town. Gaming pieces from the Museum of Archeology and History Elbag collection

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The Museum of Archeology and History in Elblag, Poland, is one of the most interesting museums in Poland. In its walls we can find one of the richest archaeological collections in Poland and many other interesting objects from the past centuries.
Elblag's Museum is concentrated on the history of Zulawy Region, Elblag and its nearest places. Archaeologists from Elblag conduct and supervise many field work on various archaeological sites. First is a Gothic cemetery in Weklice, situated about 15 km from Elblag. Excavations are held in co-operation with The Polish Academy of Sciences. Goths (we suppose that in this place lived the tribe called Gepedoios) lived in our region about 80–300 a.D., before they moved southeast. Cemetery in Weklice is one of the richest gothic burial sites in Poland. So far there had been more than 500 graves excavated and we think that there are still more to be found. Interesting thing is that untill now, we haven't found any Gothic settlement in this region connected with this cemetery.
This colloquium has a specified topic — board games. All around the world, and of course in Poland, in places influenced by the Romans there are findings of game boards, gamin pieces etc. Goths in Weklice also had many trade contacts with Romans, especially through famous amber route, which started in Italy and ended at the Baltic coast. There are many findings of roman origin in Weklice (imports from Gaul, Italy such as complete wine set or from princess grave). Most of the objects had been founded in women's graves, there are only few iron or glass findings in men's graves. What is very strange — there is only one gaming piece!

Figure 1 Glass gaming piece from Wekline[1]

Probably there was also an amber dice, due to the excavations report from the 80's, but it has been missing. Special research has to be made to find whether it was written as a mistake, has been stolen or has this dice been moved unfortunately to some other boxes. Do the Goths/Gepedoios know board games? Or was it a man privilege to play in that specific culture, and that's why there are no such things in graves in Weklice? This burial site is, in that way, different from other Gohtic burial sites in Poland, where tens of gaming pieces were found.

Classification: Type of a game:

Board Game Type	**Number of gaming pieces**
Backgammon	4
Tafl	37
Total:	41

Viking settlement in Truso

In the end if IXth century a.D. English King Alfred the Great had sent two voyagers with an order to create an account for him about their travel. One of them, named Wulfstan, traveled from Danish Hedeby (now Germany) to Truso, located in the region of the Vistula Rivermouth. His travel accounts, written circa 890 a.D., as well as those of another trader, Ohthere, were included in Alfred the Great's translation of Orosius' Histories. That is the

[1]Gothic burial site inWeklice, grave no 452, inventory number 2275.

only written source for Truso. For many centuries Truso had been searched in many different locations and finally in 1982 has been discovered by dr. Marek F. Jagodzinski. Truso was probably founded by Danish Vikings in very specific area: on the Slavonic an Prussian border as a trade center, trade emporium.

More than 25 years of excavations brought us many interesting artifacts. Unfortunately, there are no wooden artifacts due to the specific environmental location of Truso (most of the culture layer lies below sea level and just below the surface, terrain is very wet). One of the most important, if not the most important one, branch of the Viking trade in Truso was the amber and the items made of it.

At first we have to mention, that there are no findings of gaming boards in Truso, but there are many gaming pieces, mostly for tafl-type games and mostly made of amber. There are 4 tafl gaming pieces made of animal bones and 2 beautiful gaming pieces made from animal teeth. Also, we have found 3 stone tafl gaming pieces and 4 backgammon checkers: 3 made of animal bone and the fourth is made of amber. Those statistic data below has to read as incomplete – excavations from the 80's had to be checked again and probably there can be more unfinished gaming pieces or gaming pieces classified as an amber material.

The most interesting ones are those specified as King's gaming pieces. First is made of amber and probably could be used as a king pawn in a tafl-type games.

Figure 2 Amber King gaming piece from Truso[2]

Second one, made of bronze, has avery interesting provenance. This object has been recognized as a part of a Charlemagne governor's scepter.

[2]Amber gaming piece recognized as a king pawn in tafl-type games. Found in Truso, excavation area 1/2005, inventory number 82/05. Size: diameter 5,4 cm., height: 2,7 cm.

Among the other Carolingian, Scandinavian and Arabic objects this also the proof of Truso's Vikings rich contacts (maybe not only trade contacts ...) with all known world in that era.

Figure 3 Bronze tafl gaming piece – part of a Charlemagne governor's spepter[3]

Most of the gaming pieces found so far in Truso are made of amber (65,85 %). Also, 100 % of semi-finished gaming pieces is made of amber.

Classification: Material(complete gaming pieces):

Material	Number of gaming pieces	Percent
Bone or horns	5	12,20
Amber	27	65,85
Stone	5	12,20
Bronze	1	2,44
Glass	1	2,44
Animal Teeth	2	4,88
Total:	41	100,00

Classification: Shape (finished, fragments and semi-finished gaming pieces):

[3]Charlemagne governor's scepter. Location: Truso, excavation area XXVIII/25 B, inventory number 2123/03. Size: diameter 2,3 cm., height 4 cm.

Shape	**Number of gaming pieces**	**Fragment**	**Semi-finished**	**Total**	***Percent***
Cube	9	0	9	**18**	*22,22*
Sempispherical	5	1	3	**9**	*11,11*
Cone	4	0	5	**9**	*11,11*
Pyramid	2	0	0	**2**	*2,47*
Cylinder	1	0	1	**2**	*2,47*
Disk	4	0	0	**4**	*4,94*
Tooth	2	0	0	**2**	*2,47*
Shoe	0	0	3	**3**	*3,70*
Cuboid	0	0	3	**3**	*3,70*
Meniscus	12	5	4	**21**	*25,93*
Prism	0	0	1	**1**	*1,23*
Irregular	2	2	3	**7**	*8,64*
Total	41	8	32	**81**	*100,00*

Hanseatic Town Elblag

Elblag, founded by the Teutonic Knights in 1237, very soon become one of the most important ports at southern Baltic coast and being a member of the Hanseatic League played a leading part in it till the end of the XIV c. Since the end of the Thirteen Year War in 1466 till first partition of Poland in 1772 was within boundaries of Polish Kingdom and came back to Poland after the 2^{nd} World War. The Old Town had been completely destroyed in 1945 – from over 600 buildings only 6 remained. Archaeological excavations in the area of the old town in Elblag started in 1980. Those systematic, multi-branch archaeological-architectonic surveys are still completely innovative and are, in sphere of merit, obligatory till now.
Archaeological excavations were conducted at the area of about 13.500 m^2 (2004) what places Elblag among best surveyed European towns. Elblag's museum obtains the richest (in Poland) set of archeological artifacts. Till now, there is about 1.500.000 of mass artifacts and at least 250.000 allowed archaeological artifacts. There are many unique objects, such as green glasses in horn rim from XV century (the oldest complete glasses in Europe), the only one in Europe musical instrument called gittern, rich collection of other musical instruments and wax tablets.
Among those objects, archaeologist have founded also some gaming pieces. And there is on e connotation with Viking settlement in Truso: after almost 30 years of excavations, among so many objects — there is no game board

finding.
24 gaming pieces founded so far in Elblag can be divided into three main groups: for backgammon or alquerque, for chess and dominoes. Three unknown square gaming pieces can be possibly classified as a backgammon type.

Classification: Game type

Game Type	**Number of gaming pieces**
Backgammon or checkers	12
Chess	2
Dominoes	7
Unknown	3
Total:	24

The most important one is the chess gaming piece made of ivory or walrus tusk, dated for the half of the XV century. Second chess pawn is made of wood and is dated for the XVIII century.

Figure 4 Chess gaming piece from Elblag[4]

Dice

Dice	**Number of dice**
Amber	5
Bone	4
Total:	9

During many years of excavations museum's archeologists have founded only 9 dice. 5 are made of amber and 4 are made of bones or horns. Those

[4]Ivory or walrus tuskchess gaming piece. Old Town Elblag: excavation area XXV, Inventory No.1353. Date: half of the XV century. Height 4,7 cm., base diameter 2,7 cm.

three shown on the picture below have typical western configuration of sides: 11 / 7 / 3. The last one is a faked. Two sides are a little bit longer: after the throw, this dice often shows 1 or 2.

Figure 5 Bone dice from the Old Town Elblag. The last one is faked

Summary

Years of excavations on the unique archaeological sites such as Weklice, Truso and Elblag brought many interesting findings, and among them gaming pieces and dice. Only one gaming piece from the Gothic burial site might suggest us, that the gothic tribe called Gepedoios which probably lived there didn't liked board games.
On the other hand, few hundreds years later, Vikings in this region have enjoyed playing board games. Truso emporium can be seen as a main amber trade center on the south Baltic coast. Mostly amber gaming fafl pieces and almost the same number of semi-finished gaming pieces might suggest that this was a place where amber gaming pieces were made. Amber gaming pieces of the same material as in Truso were found in Birka and in Hedeby. But, we have to say that clearly, statistic data for the gaming pieces in Truso is still incomplete — many archaeological reports from the 80's had to be checked whether they are correct and complete.
Hanseatic town Elblag. Findings of dominoes, backgammon gaming counters and chess pawns are typical to the medieval town. But in almost 1,5 million of artifacts founded in Elblag there is only a small number of gaming pieces and no gaming boards. What is strange, till now the archaeologists in Elblag haven't found any knucklebones. It is simply the evidence that they weren't looking it – bones were generally gathered together an counted as, for example, 150 pig bones. The excavations need to be started in the

museum's magazines – and this research may probably bring many other interesting findings. All mentioned artifacts can be seen in The Museum of Archaeology and History in Elblag, Poland.

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their own five lines and that the line between these was the sacred line. Its significance he explains by adding that "the beaten player goes to it last" (Od. 1397, 28; Il. 633, 59). This seems to imply that the player who first manages to place his pieces on the sacred line wins the game.

Our earliest reference to the game is a verse by Alkaios (c. 600 BC), implying that moving a piece from the sacred line can lead to final victory — in a sense similar to "playing the trump card" nowadays (Bergk 1884: 177 no.82; Voigt 1971: 320 no. 351). But generally it was regarded a bad idea to move a piece once it had arrived there. This is why the 3rd century poet Theokritos writes (Idylls, VI 18): "and from the line she moves the piece, because to love's desire often appears beautiful what is not beautiful".

Of course, without any further information these passages are difficult to understand. The reason is that the authors cited above presented their information in a very condensed and abbreviated style sufficient to explain or to allude to the proverb, whereas it was not their intention to give precise rules of an ancient Greek board game. Moreover it is very likely that at least some of them did not even know the game, which is certainly true at least for Eustathios who reports what he had read in the ancient literature. Austin's conclusion, however, that "the obscurity of all this evidence is impenetrable" (Austin 1940: 267-271) was due to the fact that he completely ignored archaeological finds that can convincingly be connected with these references and add much to their understanding. Some of the early finds had already been taken into consideration by Lamer in his important article "lusoria tabula" from 1927, who also checked the literary evidence completely (Lamer 1927: cols. 1970–1973, 1992–1998). But even such an eminent board game historian as Murray went over the game rather superficially, wrongly stating that we knew "nothing more than that it was played on a board of five lines" and that Pollux described the five-lined board as a board of 5 by 5 cells, which is not the case at all (Murray 1952: 28). Pollux tells us not only that the game board consisted of five lines (and only five lines) but also that the game was for two players who had five counters to play with. Moreover we learn that one of the lines on the board was particularly significant in the play of the game. Finally, both Pollux and Eustathios include *Five Lines* among the Greek board games played with dice. All this is a lot more of information than what Murray wanted to consider. Murray, who nearly exclusively relied on Austin, did not pay any attention to such finds as the tables from Epidauros interpreted as gaming boards by Blinkenberg half a century before. He suggested instead that the five-lined board might have had the form of a pentagram. Apart from the simple fact that such a form cannot explain how there should be one line of special importance, as a support to his surprising proposal Murray referred to the designs on the roofing slabs of the Sethi temple at Gourna in Egypt, taking it for granted that the five rayed stars on the roof are gaming boards and earlier in date than the Greek game. However, most of the designs there can be identified as mason's marks and magic symbols (Parker 1909: 643–44 fig. 273). This holds particularly true for the five rayed star — a Coptic magic symbol until today (Viaud 1978: 47) — and various Coptic crosses. The existence of designs dating to the Christian era makes any attempt to date the drawings on the roof more precisely impossible.

In 1968 Pritchett catalogued the material known until then from mainland Greece, Delos and Cyprus (Pritchett 1968:189—198), but included in his list a number of objects which more convincingly can be identified as abaci. Despite of all this material at disposal, May (Jouer dans l'Antiquité 1991: 172–73) still based his account on Becq de Fouquières' mostly outdated speculations from 1869 (Becq de Fouquières 1869: 397–405).

Let us now consider the most important finds of gaming tables that can be connected with the game of *Five Lines*. The earliest example seems to be a painted terracotta miniature gaming table (fig. 1) found together with a cubic die at Anagyros (Vari) in Attica in a grave dating to the middle of the 7th century BC (Kallipolitis 1963: 123–124, 172, pl. 53–55 ?–?). The board measures 18.3 by 24.8 cm and has on its surface five incised parallel lines ending in a circular cavity on both sides, thus forming two rows of five holes along the longer edges of the board. The faces of the die, which has small holes as points, are painted with geometric ornaments, a horse, a woman and perhaps the goddess Athena, comparable to a roughly contemporary die from the Athenian acropolis (Karusu 1973; Schädler 1999).

Probably the same game was depicted on another small gaming table, found together with a die in the necropolis of the Kerameikos at Athens and dating to the early 6th century, but unfortunately the surface of the table is not preserved (Kübler 1970: 394–95, 512 cat. no 129, p. 102). Both tables were adorned with terracotta statuettes of mourning women to show that these tables have been properly made to be used as grave goods. An explanation for this tradition is given a little later, in the first half of the 5th century, by the Greek poet Pindar (frg. 129) who described the idea of a happy existence in the netherworld, where "some enjoy horses and wrestling, others board games, and yet others the music of the lyre".

Figure 1: Clay model of a gaming table from Anagyros, Athens, National Museum

When Pindar wrote down these lines the Athenians had abandoned the custom of offering terracotta gaming tables to their dead. Now they used to offer black-figured vases decorated with a depiction of the two heroes Ajax and Achilles playing a board game with dice. The most elaborate and probably the earliest of these scenes was painted around 540 BC by the Athenian vase painter Exekias on an amphora now preserved in the Vatican Museums (Brommer 1974: no. 9; Woodford 1982: 173–74, 183 F1, pl. IIIa; Buchholz 1987: 144–45 no. 21; Mommsen 1988: 445–454). Here not only the names are written beside the heroes, but also the results of their throws: Achilles on the left calls out that he has got a 4, while Ajax on the right only has thrown a 3. Inscribed numbers appear also on some other representations, among which in two cases a 2, implying that the vase painters thought of a cubic die and not of an astragal with the numeration 1-3-4-6 (Woodford 1982: 185). The use of a die together with the gesture of their hands and the presence of gaming pieces, sometimes differentiated in black and white, on top of the block between the two that can be observed on most of the extant examples clearly shows that they are playing a board game with dice.

Unfortunately in this case as well as in practically all the other paintings of the kind — Buchholz has listed 168 representations — the game board is seen from the side, so that only the counters can be observed in some cases. But there is one vase painting on a *kyathos* in the Musées Royaux d'Art et d'Histoire in Brussels dating to the beginning of the 5th century (Brommer 1974: no. 48; Buchholz 1987: 168 no. 139; Vanhove 1992: 186 no. 44) which astonishingly offers a view of the board seen from above (fig. 2): the game board consists of five parallel lines, and both ends of these lines are occupied by one counter. There can be hardly any doubt: this is the Greek game of

Five Lines. The hypothesis that the heroes were thought to play exactly this game has already been forwarded by several authors (such as Becq de Fouquières 1869, Beazley 1963: 2–3, May 1991: 173, and others; see also Schädler 1999: 41, Pfisterer-Haas 2004: 383), but only the vase painting in Brussels proves it.

Figure 2: Ajax and Achilles playing *Five Lines*. Black-figure vase painting on a kyathos, early 5th century BC (Musées Royaux d'Art et d'Histoire, Brussels, inv.no. R2512)

As a preliminary essence of what both the written and the archaeological sources reveal we can therefore conclude that already around 600 BC in ancient Greece there existed a game played on a board showing five parallel lines. The game was for two players, who had as many counters as lines at their disposal, i.e. five each, which were placed on or at each end of these lines. The game was played with the help of a die. Only from the written sources we learn that the central line was called the "sacred line" which must have had a particular importance in the game, since the players tried to avoid moving a piece from that line which had already arrived there.

Possible enlarged variants

Achilles and a companion (probably Ajax) playing a board game are also depicted on an Etruscan mirror (Körte 1897: 144–146 pl. 109; Mansuelli 1945: 58), where the board they are keeping on their knees shows seven parallel lines (fig. 3a and 3b).

On both sides each line ends in a circle representing a counter or a depression to keep one. Two rectangular objects are depicted between the lines that can be taken as dice.

Figure 3a: Achilles and Ajax (?) playing a board game, Etruscan mirror (after Körte 1897, pl. 109)

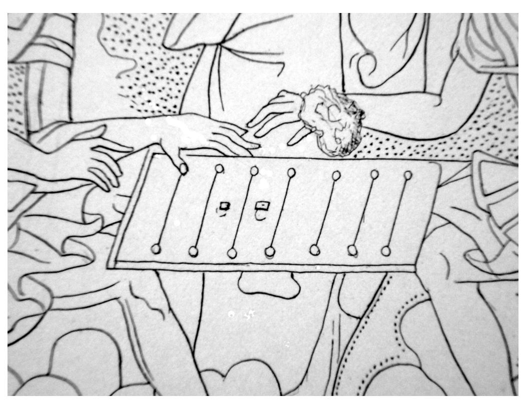

Figure 3b: Achilles and Ajax (?) playing a board game, detail. Etruscan mirror (after Körte 1897, pl. 109)

A Praenestine mirror in the British Museum (Körte 1897: 191–193 pl. 146; Walters 1899: 377 no. 3213) dating to the 3rd century BC (compare the mirrors Mansuelli 1943: 517–518 pl. 40 no. 13 [2nd half of the 3rd cent. BC], Liepmann 1988: 43–45 no. 17 [early 3rd cent. BC], and de Puma 1987: 38–39 no. 21 [early 3rd cent. BC]), should also be added to the representations of the game (fig. 4).

Figure 4: Couple playing a board game. Praenestine mirror, 3rd century BC. British Museum, London (after Körte 1897, pl. 146)

The gaming table used by the couple shows twelve or perhaps thirteen parallel lines ending in small circles, which corresponds to the game boards discussed above, but differs completely from boards for XII *Scripta* to which the mirror has wrongly been attributed (Walters 1899: 377; Bell 1979: 30 fig. 25; May 1991: 179 fig. 174, who wrongly dates it to Roman times). Roman XII *Scripta* boards consist of three rows of twelve points (variously fashioned like squares, circles, points, lines, letters or other symbols) divided — like with *Backgammon* boards — by a bar in the middle (Schädler 1995).

Another important find is a miniature terracotta gaming table from Athens in Copenhagen (fig. 5), dating to the early 6th century BC, the lifetime of Alkaios, which probably also served as a grave good (Ussing 1884: 149–151, 172; Breitenstein 1941: 19 no. 171 pl. 19; Pritchett 1968: 197 pl. 7,1; Lund/Rasmussen 1995: 67). On its surface measuring 37 by 12 cm are drawn nine parallel lines occupied by oval knobs at each end, obviously representing gaming stones.

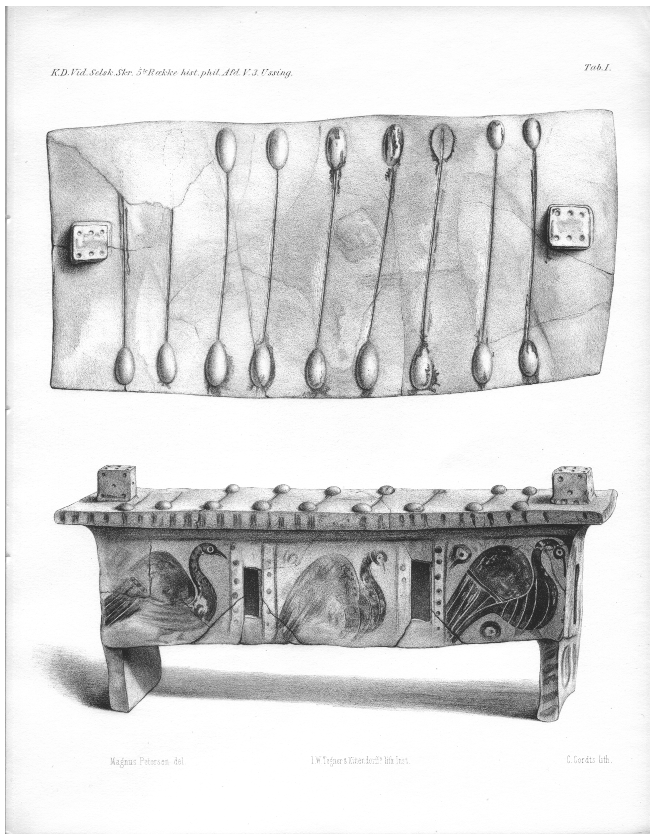

Figure 5: Miniature terracotta gaming table, early 6th century BC. National Museum Copenhagen (after Ussing 1884, pl. 1. The Royal Danish Academy of Sciences and Letters)

At both ends of the board a cubic die with the upper face showing 6 is preserved. As with the mirrors mentioned above there are more than five lines on the board, so that these representations have been taken as free copies of the game intending that the artisans did not pay much attention to the exact number of lines.

This may be the case with the Praenestine mirror, where perhaps a board with 11 lines was intended (see below), but nine lines as on the table in Copenhagen differs too much from five and requires completely different proportions of the whole object as to assume a simple error. Therefore we have to reckon with the existence of enlarged versions of *Five Lines*. This assumption may be corroborated by the existence of a series of boards with 11 lines. One of the stone gaming tables dedicated possibly during the 4th century BC in the sanctuary of Asklepios at Epidauros shows six shallow lines added clearly at a later date to the existing two groups of five lines in order to create a gaming area with eleven lines next to one with five (Blinkenberg 1898: 3–4 no. 2 fig.3–4; Pritchett 1968: 190–191 no. 2 pl. 1,2–3). Boards with eleven lines have been found at several sites, sometimes with the third, sixth and ninth lines cross-cut, pointing to a special significance of these lines.

Thus the boards with eleven lines appear to be boards where two groups of five lines with their sacred lines in the middle have been joined by adding a central "sacred" line between the two groups. In this way Claude Saumaise understood Eustathios' text already in the 17th century (Salmasius 1671: 748–49). In fact, this 5+1+5-layout corresponds to Pollux' (IX 98) and Eustathios' (Il. 633, 58) peculiar expression Lamer (Lamer 1927: col. 1971) came across, that "a line in the middle was called the sacred line" instead of "the line in the middle...". From the extant gaming boards this expression seems to refer to both possibilities, i.e. that there was precisely one sacred line only in the standard version with five lines, whereas there were more than one on the 11-lined board. It should also be mentioned that at Roman sites in Asia Minor such as Ephesus, Smyrna, and Aphrodisias (Schädler 1998; Rouché 2007) a great number of game boards showing two rows of five or two rows of eleven squares can be seen while other numbers of squares are extremely rare. It seems therefore that in Roman times *Five Lines* and its larger variant were played on squares instead of lines (Schädler 1998: 18–19; Schädler forthcoming).

So it seems that there existed several larger versions of *Five Lines* as well as a double version of the game with eleven lines. This doubling of an existing game reminds us of the Egyptian game of 'twice 20', a double version of the famous game of 20 squares, created simply by joining two boards (Pusch 1977). Recently Irving Finkel suggested that perhaps even the Indian game of *Pachisi* "might owe its ultimate origin to the doubling of a simpler game" (Finkel 2006: 61).

Game boards or abaci?

It must be mentioned that not all the boards with parallel lines should be recognized as gaming boards. This interpretation holds certainly true for boards associated with dice and/or counters. But there are other boards, mainly stone boards, which more likely are to be explained as abaci, i.e. calculation boards. While Pritchett took all the lined boards as gaming boards (Pritchett 1968: 200–201), recently Schärlig considered all these boards to be abaci (Schärlig 2002: 80, 179–80). On the other hand those boards with the ends of the lines hollowed out to form a circle, like for example a board from Eretria (Schärlig 2001: fig. 5) and the boards cut into the pediment of the Letoon on the island of Delos (Gallet de Santerre 1959: 38 with n. 2, pl. IV; Pritchett 1968: 195–96 no. 13 pl. 5, 2–4), are best explained as game boards. Among the patterns incised into the pediment running around the little temple the excavators have identified several *Five Lines* boards (Deonna 1938: 337; Gallet de Santerre 1959: 38 with n. 2, pl. IV; Pritchett 1968: 195-96 no. 13 pl. 5, 2-4).

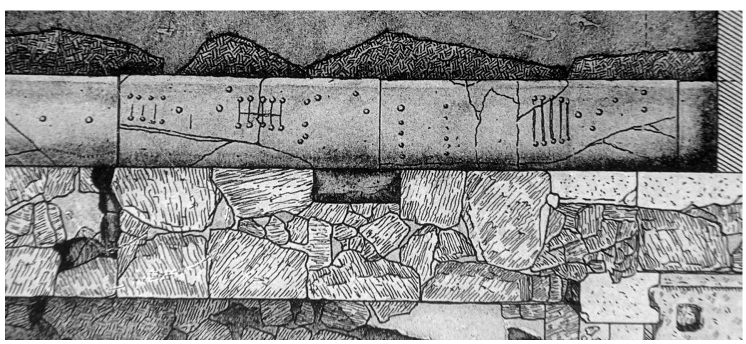

Figure 6: Game boards on the northern pediment of the temple of Leto, Delos (after Gallet de Santerre 1959, pl. IV)

On the northern pediment (fig. 6) there are one board consisting of five parallel lines ending in circular holes crossed by a perpendicular line, one similar board but without the perpendicular line, and one board consisting of two parallel rows of five holes. A forth pattern shows three parallel lines ending in small holes and two extra holes without a line, and is probably simply an unfinished board.

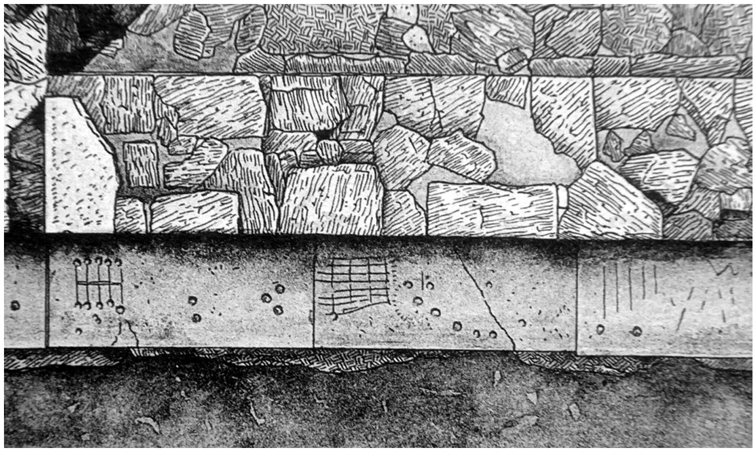

Figure 7: Game boards on the eastern pediment of the temple of Leto, Delos (after Gallet de Santerre 1959, pl. IV)

The eastern pediment at the rear of the temple (fig. 7) has a probably unfinished gaming board in the shape of a grid of 3 by 6 squares measuring about 27 by 27 cm (Deonna 1938: 337), a complete *Five Lines* board with perpendicular line (fig. 8) as well as a pattern of five parallel lines of different length.

Figure 8: *Five Lines* board on the eastern pediment of the temple of Leto, Delos (photography by the author)

While one cannot determine whether the rows of five holes are unfinished boards or just a different type of *Five Lines* board (compare the boards of 2 by 5 squares of Roman times so frequent at Ephesos and Aphrodisias: Schädler 1998), it is difficult to interpret these patterns as abaci. Two main arguments may support this assumption: First of all, as Schärlig convincingly argued (Schärlig 2001: 82, 180), the counters on the abaci were placed not on the lines but in the columns between the lines, so that depressions at the ends of the lines would not make sense. Secondly, even if the counters would have been moved along the lines, one depression at both ends of the lines would not make sense either, since in calculation procedures five counters are needed on each line.

On the other hand the boards showing numerals besides the lines like a board from Salamis (Pritchett 1968: 193–95 no.11 pl. 4,1; Schärlig 2001: 66–67, fig. 1), one from the Amphiareion at Oropos (Pritchett 1968: 191 no.4 pl. 2,1; Schärlig 2001: 77–78), two further stones from Oropos (Schärlig 2001: 67–69) as well as one from the sanctuary of Asklepios at Epidauros (Blinkenberg 1898: 2–3 no. 1; Pritchett 1968: 189–90 no. 1, pl. 1,1) are very likely to have been used for calculation purposes. This does not exclude that they were also used to play a game from time to time. Especially the fact that some of these boards have the third, central and ninth lines marked by crosses cannot be explained by them being used for calculations (Schärlig 2001: 190), but by their use as game boards with the sacred lines marked by a cross as has been explained above.

How to play

As far as the modes of playing *Five Lines* are concerned, we may draw the following conclusions. The standard game of *Five Lines* was played on a board with five parallel lines. Larger versions could have more lines, but always an odd number, simply because there had to be a central line. From the written sources we learn that this central line had a special significance. It seems that the aim of the game was to move all one's counters onto this "sacred" line. The term "sacred" reminds one of the ancient Greek concept of asylum and *hiketeia*, i.e. the inviolable right of persons in search of aid to take refuge in a sanctuary (Sinn 1993) where nobody had the right to remove a suppliant by force, and describes pretty well the function of this line. Apparently the number of counters used corresponded to the number of lines, each player having as many counters as lines on the board. The points, holes or circles at both ends of the lines on some of the boards demonstrate that normally one counter only was placed at the ends of the lines.

This arrangement is represented on the vase in Brussels, but also on the terracotta model of a gaming table in Copenhagen (9 lines) as well as on the Etruscan and the Praenestine mirrors (11 and 13? lines respectively). Probably this was the starting position. Some of the boards at the Letoon of Delos do not even have lines but just two rows of five holes showing that these points were important (fig. 6). Since, as Pollux points out, each player had five counters on *Five Lines*, it seems obvious to conclude that one player placed his counters at one end of the lines and the second player on the opposite ends. It seems that the transversal line running perpendicularly through the middle of the board that can be found on some *Five Lines* boards was introduced to distinguish the two sides. In my view it is this arrangement Pollux' and Eustathios' referred to with their expression "from both sides" (ἤκατέρωθεν).

Recently Kurke (Kurke 1999: 263–64) hypothesized that there was a special "king piece" in the game of *Five Lines.* The idea is based on an unusual reading of a passage in the scholia to Theokritos (Schol.ad Theokr.6.18.19a), where we are told that the piece moved from the sacred line was called "the king" (*basileus*). The passage has never been taken seriously, because the scholiast himself says that the game he refers to is *Chess* (ζατρίκιον). Moreover, would Pollux or Eustathios who try to explain a famous proverb about a piece moved from the sacred line not have mentioned the fact that this piece was a special one? Even more important as an argument against Kurke's suggestion is the fact that not one of the representations described above shows such a special piece. With the intention to corroborate his understanding of the scholion, he advances another unusual interpretation. One of Herakleitos' most enigmatic sentences (Diels 1922: 88 no. B 52) which has nourished debate ever since reads 'time is a child at play (παίζων), playing on a game board (πεσσεύων). Royal power is in the hands of a child'. The fact that a board game (*Petteia*) and royal power (*basileia*) are alluded to here in the same context makes Kurke conclude that a "piece called the king" was involved and moreover that the board game intended by Herakleitos was played with dice and thus must have been *Five Lines* (Kurke 1999: 263–64 with reference to Kahn 1979: 227–28). Because of the far-reaching conclusions concerning the symbolic meaning of *Five Lines* and its role in ancient Greek society drawn from this daring construction, it is necessary to have a closer look, without digging deeper than necessary into philosophical questions raised by Herakleitos.

First of all it should be noted that according to the written sources *Five Lines* was obviously not considered a form of *Petteia* (games with counters) which Herakleitos alludes to, but of *Kubeia* (games of chance). But there is also — as far as I know — not a single reliable and convincing trace of a special piece in any ancient Greek board game.

As far as the understanding of Herakleitos' statement is concerned I would briefly like to introduce a few ideas from the perspective of the history of games. Herakleitos speaks of a child playing (παίζων) and then explains more precisely that he is playing on a game board (πεσσεύων). Astonishing as this rendering is the picture of a child playing a board game. Normally Greek children played with all kinds of toys and with knucklebones (see *Jouer dans l'Antiquité* 1991: 50–81, 100–105, 166–173), but board games, especially strategic games such as "polis" were an adults' domain (see *Jouer dans l'Antiquité 1991: 166–173*). Greek children appear not to have played board games at all, an important aspect in my opinion, which has hardly ever been explored in this context. The question arises therefore why it was important for Herakleitos to stress that the child is amusing himself by moving pieces on the board of a game for adults? If his point would have been to compare the arbitrary nature of events in time with the randomness of outcomes in children's plays, the image of the child at play without any further explanation would have sufficed the purpose perfectly, since people would practically automatically have thought of a child playing with knucklebones, the favourite pastime of Greek children at the time. In another anecdote about Herakleitos' life for example Diogenes Laertius (IX 3; Robinson: 166; Musaviev 2003: 27 M 22a; 159) reports that once Herakleitos himself was playing at knucklebones with children in the sanctuary of Artemis. As some Ephesians criticized him for passing his time in that way, he replied: "is it not better to do this than to 'politeuesthai' with you?". This is certainly to be taken as one of his famous wordplays, in that the verb would normally mean "to make politics", but posed here in contrast to 'astragalizein' ("to play with knucklebones") one cannot help thinking of a secondary meaning in the sense of "to play polis", i.e. the game "city" (Kurke 1999: 268). It seems that he wanted to criticise his fellow citizens for not taking politics as seriously as children do with regard to their knucklebones — not the only instance the philosopher polemicized against Ephesian politicians. In this anecdote Herakleitos contrasts a children's game (knucklebones) with an adults' strategic board game ("polis"). Therefore it must be significant that in the statement discussed here he states more precisely that the child is not just playing a children's game such as knucklebones but that he moves pieces on a game board. Obviously his point is neither about randomness nor about a child's game.

What Herakleitos' child is doing is not to play a board game — as Kahn (1979: 227) supposed — but to play as if he was playing a board game, since he does not know the rules or the aim of the game. This is why I preferred to translate $\pi\varepsilon\sigma\sigma\varepsilon\acute{\upsilon}\omega\nu$ as "playing on a game board" instead of "playing a board game" as most translators have it (Diels' "Die Zeit ist ein Knabe, der spielt, hin und her die Brettsteine setzt" is also very close to my understanding). He moves pieces on a game board, which is not just a piece of wood, but has a geometrical structure consisting of lines and perhaps squares with certain measures and in determined numbers, thus implying an order which predetermines such movements, even to a human being who does not know the rules of the game. Evidently Herakleitos introduced the game board as an element of measure and order. But the movements of the pieces on it are not in keeping with the rules of the game, which are unknown to the child, who consequently neither has an aim nor an overview or a reasonable plan.

The analogy between the child moving pieces on a game board and time proposed by Herakleitos seems to me to point to the fact that time is understood as the ordered change of the world (Plutarch, Qu. Plat. viii. 4, p. 1007, discussing Herakleitos: "time is ... motion in an order") operating in a measured framework which is the cosmos (compare Herakleitos, frg. 30: the cosmos changing in due measure). But time, as the child, is not aiming at something, thus has neither a specific intention nor a strategy; although governing change it has no plan, an idea different to Aristotle's teleological doctrine.

The fact that Herakleitos did not choose for his figure a game of chance such as dicing with knucklebones, which would have been far more appropriate for a child, but a board game with its geometrically structured surface, does in my opinion rule out the idea that he intended to refer to a board game with some element of chance (a random generator such as a die), an idea forwarded by Marcovich (2001: 493–95 no. 93) and Kahn (1979: 227) and supported by Kurke (1999: 264). It seems on the contrary that he wanted to exclude the idea of a game of chance: why should he otherwise add "$\pi\varepsilon\sigma\sigma\varepsilon\acute{\upsilon}\omega\nu$"? Not even is the unsystematic disposal of the pieces on the board due to the child's lack of acquaintance with the rules of the game identical to the purposely creation of chance by inventing and introducing a random generator. It is evident from all this that Herakleitos did not have *Five Lines* in mind, when he made his famous statement.

To return now to the elements of *Five Lines*, the use of dice is attested by both the literary and archaeological sources. Judging from the find from Anagyros, one die was used when playing on five lines, whereas two dice belonged to the larger boards. Not only can two dice be identified on the Etruscan mirror mentioned above, but two dice are also placed on the nine-lined board from Athens in Copenhagen. On the gaming table in Copenhagen a square trace can be seen in the centre of the board, which has been taken as the trace of a third die now lost. There are, however, several arguments speaking against this hypothesis. First of all the object once placed here was turned 45° with regard to the two dice. Secondly, three dice are neither mentioned in the written sources nor do they appear in the archaeological record discussed here. Moreover must the hypothetical situation that a winning move is represented with all eighteen points occupied by one player's counters after one lucky throw of three sixes (Blinkenberg 1898: 9) be discarded. Apart from the fact that the important role of the "sacred line" is not taken into consideration, the corresponding numbers of eighteen points on nine lines and on three cubic dice are merely coincidental, while the normal number of lines is five with ten points respectively. Finally the underlying hypothetical rule that the players had to place a number of pieces on the points according to the result of the throw of dice simply does not correspond to the fact that the pieces were moved from one line to the other as is clearly indicated by the proverb "moving the piece from the sacred line" to which the literary sources refer. Therefore it is more plausible to think that here in the centre of the board a statuette of a mourning woman similar to those on the board from Anagyros was once placed.

As depicted on the vase painting in Brussels the two players sat at the short sides of the board, so that the lines came to lie horizontally before them. As the depictions including all the Athenian vase paintings depicting the scene show, they used their right hands to move the pieces, so it is likely that the players' counters were those placed on the players' right hand sides. The pieces were moved from line to line according to the spots on the dice. Presumably a counter having reached the last line on one side of the board was shifted along the line to its other end, where it moved in the opposite direction along the other side back to the first line, where the same manoeuvre was repeated and so forth. It is likely that movement was in a counter clockwise direction implying that on their own side of the board the players moved their pieces forward. This presumed circular movement around a board with two rows of points reminds one of *Mancala* games and backgammon games. If the interpretation of the sources is correct, that the aim of the game was to place all or as many pieces as possible on the "sacred line(s)", then probably the pieces had to move around the board several times, for just one turn was surely not sufficient.

In the light of the written sources and the archaeological record *Five Lines* must have been very popular from at least the late 7th until the 3rd century BC. As the vase paintings demonstrate, the Athenians during the decades around 500 BC imagined Ajax and Achilles, the two greatest heroes in the Troian war, playing the game. Later in the 5th century "five lined boards and the throws of dice" are mentioned by Sophokles in a verse which was part of his tragedy "Nauplios" (Pollux IX 97; Pearson 1917: 85 frg. 429). Nauplios was the father of Palamedes, who was thought to have invented the game during the siege of Troy. Therefore it is likely that *Five Lines* is also meant in Euripides' "phigenia in Aulis" (192–199), where we find both Palamedes and Protesilaos "sitting and amusing themselves with intricate figures at a board game", while Diomedes and Achilles trained themselves in athletic disciplines. About the same time even Plato referred to the game to explain certain ideas to his pupils (Laws 739a). *Five Lines*, the game of the heroes, was regarded a noble game for centuries.

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Five Lines — written sources

Alkaios, frg.82 Bergk III[4]
... νῦν δ' οὖτος ἐπικρέτει
κινήσαις τὸν ἀπ' ἴρας πυκίνως λίθον

Now he is the master,
he has moved the small stone from the sacred (line).

Pollux, Onomastikon VII 206
κυβείας δὲ εἴδη κὰι ἡ πλειστοβόλινδα παιδιά, καὶ τὸ ἀρτιάζειν, καὶ διαγραμμίζειν [καὶ διαγραμμισμός][FS] *[καὶ χαλκίζειν]*[BC] *[καὶ χαλκισμός]*[A] *[καὶ ἱματελιγμός]*[FS] *καὶ ναυμαχία. ἐνῆ τις καὶ ἱερὰ γραμμή, ἀφ' ἧς ἡ παροιμία «κινήσω τὸν ἀφ' ἱερᾶς».*

Among the dice games (better "games of chance") are known the game of high score, "odd and even", *diagrammismos*, ["heads and tails" (?), *himateligmos*,] and battleship. And an old (game) "sacred line", whence the proverb "I move the piece from the sacred (line)".

Pollux, Onomastikon IX 97
ἐπειδὴ ψῆφοί μέν εἰσιν οἱ πεττοί, πέντε δ'ἑκάτερος τῶν παιζόντων εἶχεν ἐπὶ πέντε γραμμῶν, εἰκότως εἴρηται Σοφοκλεῖ «καὶ πεσσὰ πεντέγραμμα κὰι κυβων βολαί».
τῶν δὲ πέντε τῶν ἑκατέρωθεν γραμμῶν μέση τις ἦν ἶερὰ γραμμή. καὶ ὁ τὸν ἐχεῖθεν κινῶν πεττὸν ἐποίει παροιμίαν «κινεῖ τὸν ἀφ' ἶερας».

Since on the one hand the counters are stones, and on the other hand each of the two players had five upon five lines, it is suitably said in Sophokles: „five lined boards and throws of dice".
Of these five lines on both sides a one in the middle was the sacred line. And moving a piece already arrived there gave rise to the proverb „he moves the piece from the sacred (line)".

Eusthatios of Thessalonica, Commentary to Homer's Odyssey 1396, 61
ἐπὶ πέντε γραμμαῖς τὰς ψήφους ἐτίθουν, ὧν ἡ μέση ἱερὰ ἐκαλεῖτο.

Onto five lines they placed the counters, of which the one in the middle is called the sacred.

Eusthatios of Thessalonica, Commentary to Homer's Odyssey 1397, 28
τοὺς δὲ πεσσοὺς λέγει (ὁ τὰ περὶ Ἑλληνικῆς παιδιᾶς γράψας) ψήφους εἶναι πέντε, αἷς ἐπὶ πέντε γραμμῶν ἔπαιζον ἑκατέρωθεν, ἵνα ἕκαστος τῶν πεττευόντων ἔχῃ τὰς καθ' ἑαυτόν ... παρατείνετο δέ φησι δι' αὐτῶν κὰι μέση γραμμή, ἣν ἱερὰν ὠνόμαζον ὡς ἀνωτέρω δηλοῦνται, ἐπεὶ ὁ νικώμενος ἐπ' ἐσχάτην αὐτὴν ἵεται. ὅθεν κὰι παροιμὶα «κινεῖν τὸν ἀφ' ἱερᾶς», λίθον δηλαδὴ, ἐπὶ τῶν ἀπεγνωσμένων καὶ ἐσχάτης βοηθείας δεομένων.

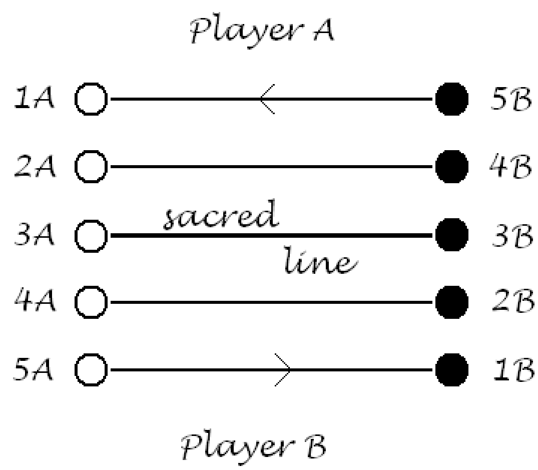
Concerning board games the one who has written about the games of the Greeks [i.e. Suetonius] says that there are five counters, which are played on five lines on both sides, while both the players have them for themselves ... and moreover he says that in between expands the central line, they call the „sacred" as has been said above, because the beaten player goes to it last. Whence the proverb „to move it (i.e. the counter) from the sacred (line)", when people are desperate and need final help.

Eusthatios of Thessalonica, Commentary to Homer's Iliad 633, 58-60
καὶ ὅτι, ὡς ξύσμα ξύσμή, ὄυτω καὶ γράμμα γράμμή, οὐ μονον ἡ ἐν μαθήμασιν, ἀλλὰ καὶ ὁποὶαις ἔπαιζον οἱ κυβεύοντες. ὧν μία τις μέση γραμμὴ ὠνομάζετο ἱερά, ἐπειδὴ ὁ ἡττώμενος ἐπ' ἐσχάτην αὐτὴν ἵετο. ὅθεν καὶ παροιμία «κινεῖν τὸν ἀφ' ἱερᾶς», ἐπὶ τῶν ἐν ἀπογνώσει δεομένων βοηθείας ἐσχάτης.

A line in the middle is called „sacred", because the beaten (player) goes to it last. Whence the proverb „to move it from the sacred (line)", after those in desperation who need final help".

How to play *Five Lines* — a suggestion for rules

Principally it is impossible to reconstruct the rules of a lost board game precisely. First of all because the few allusions in texts and the archaeological finds do not clarify all the necessary details. Secondly for most popular traditional games many variants of rules develop in time and space so that THE rule has never existed and will never exist.



Nevertheless, a suggestion for a rule can give an idea of the basic principles of the game and check the interpretation of the sources. Moreover, teachers and museum people during didactic programs about ancient games can propose to play the game instead of only talking about it. But it must be kept in mind that the only thing we can say for sure is that the rule suggested here was certainly not the one played by the ancient Greeks.

1. The game *Five Lines* is for two players.

2. The game board consists of 5 parallel lines. The line in the middle (the 3rd line) is called "sacred line". It is possible to draw a transverse line to cut the board into two halves.

3. The players sit at the short ends of the board with the five lines horizontally before them.

4. Each player has five counters. At the beginning of the game they place their counters on the ends of the lines (from now on called "points") at their right hand side of the board so that all the ten points are occupied.

5. The aim of the game is to move all the five counters on the opposite half of the sacred line, i.e. at the left hand side of the players (a more simple possibility would be to try to place all one's five counters anywhere on the sacred line).

6. The players take turns in tossing the die and moving one of their pieces according to the result of the throw.

7. The pieces are moved from line to line, i.e. from point to point, in an anti-clockwise direction. A counter having reached the last point on one side of the board is shifted along the line to the opposite point, where it is moved down until it reaches the first line, when the same manoeuvre is repeated and so forth.

8. Counters can move or to a vacant point or to the sacred line. This means that only one counter can be placed on any point, except for the sacred line where more counters (even from both players at the same time) can be placed. If in the first move a 5 is thrown the only possible move is from one side of the sacred line to the other, since all the other points are occupied.

9. *Zugzwang*: if possible a move has to be executed, even if a counter must be drawn from the sacred line. In case a move is not possible the player looses his turn.

10. The player who first reaches the goal, i.e. has moved all his five counters onto the (left half of the) sacred line, wins the game.

Double version

1. When playing on a board with eleven lines, the third, sixth (central) and ninth lines are sacred lines.

2. Two dice are used, the numbers of which are considered individually, so that a player may move with two different counters or add the two results and move one counter only.

Education via a Board Game - Understanding forecasting

Rozainun Haji Abdul Aziz[*] *& D.F. Percy*[†]

Abstract

The purpose of this presentation is to put forward a "learning and teaching" strategy through a board game, which we call "nun-forecaster" . The whole idea is to introduce forecasting to students. Nun-forecaster is a metaphor to camouflage learning and teaching, through an activity, about forecasting at the first impression. Afterwards, a mathematical expression is provided to measure under and over-forecast, in an attempt to provide an advanced insight into forecasting.

This game symbolizes the significance of planning, then we go deeper into forecasting in a business or even in our everyday life. The game takes us through a journey of ups and downs and uncertainties where we are not sure of what lies ahead so we are forced to accept circumstances here. Nevertheless, we must proceed till we finish the whole journey.

In the events, given by nun-forecaster board game, each of which we can formulate through mathematics and explained behaviour, we offer insights and solutions.The game is like a path game as it tests perseverance, patience and learning.

Players may comprise of even school children, besides college and university students, academicians and researchers; each group with different levels of understanding. Companies can use this as ice breaker for their training sessions. Schools can use this to stimulate the class and adopt a new approach of learning and teaching – i.e. using the board game as a platform to develop and build knowledge from.

We hope education can also be acquired through a board game like this and that it should not be played just at home with family members and friends

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leisurely, but bring it out in a formal class in a school, in a university and at work. Hence, teaching and learning can be made more interesting and more stimulating.
To the high level readers, an insight into a mathematical model proposed is given in concept with the hope that both academicians and practitioners will progress in achieving forecast accuracy. The model explains the use of probability distribution against point forecasts, the cost function and fundamentals of Bayesian methodology in approach.
Previous observations through pilot study, postal survey, case study and a follow-up survey form a basis in formulating the mathematical model explained. In writing the paper, we attempt to give explanations for and cost effects of imperfect forecasts, an oversight which frequently occurs to management.

Keywords: board game, forecasting, mathematical model.

Introduction

This paper extends the findings of a postal survey and case study on practices and perceptions of forecasting (Aziz-Khairulfazi, 2004), which addresses modelling issues for forecasting scenarios. Its intention is to raise awareness of various modelling approaches that can be used to enhance the quality of forecasting processes, rather than to identify specific models, which tend to be user-specific.
However, the subtle introduction to this area of study is through a board game, "nun-forecaster" where it attempts to develop interest from the players by first treat the whole scenario via "ups and downs" of a business so that critical thinking towards planning and expectations of a business are slowly inculcated. Once this is achieved, we offer an advanced level of understanding in the extension of planning i.e. forecasting which further indulges into a mathematical explanation. An innovative learning and teaching strategy, the whole idea is to set off on the "right foot" and bring "board game" into the picture at the onset. d'Astous and Gagnon (2007) examined factors influencing board game and appreciation of the players or consumers. The paper presented how players might be able to learn while enjoying a particular board game. Five-category typology of a typical board game were proposed by Day (1981) namely exploratory, creative, entertaining, mimetic and cathartic games; these of which are taken to form the basis of "nun-forecaster".

The rest of this paper will bring in issues in forecasting for the benefit of understanding at a higher level.
It has been noted that organisations make forecasts and that forecasting accurately is rarely achieved. As many business decisions involve forecasting, successful forecasting practice is crucial to reduce or close the gaps in this process (Drury, 1990; Moon et al., 2003). This failure is dueto the behaviour of forecasters. Three reasons are offered here, namely, the process of interpreting data, forecaster bias and forecaster preferences (Stekler, 2003).
Using a Bayesian approach to understand and interpret the above, subjective probabilities for the likelihood of an event are elicited and revised as new information is received. In support of this approach, there is also a need to emphasise to consider the individual's role in the forecasting process (Stekler, 2003).
Observing the practice, and learning about the perceptions, of forecasting from the study samples are not complete if the practice and perceptions are not represented by models. Ultimately, an organisation or a unit could forecasts for profits, sales, investments, cash flow surplus, student numbers, teaching loads and other resources using such models and, depending on the nature of its activities.
Forecasts are prepared based on estimates, which, in practice, correspond with point predictions. Typically, a single estimate is obtained as a result of group decision-making in predicting future performance. This group decision-making is done through members offering their expert opinions with regard to a particular issue. Forecasts are said to be imperfect when actual performances do not turn out as predicted. This paper offers some mathematical modelling and consideration of cost implications for this forecasting scenario (Armstrong,2001; Clemen et al., 1996; Aziz-Khairulfazi and Percy, 2003).

The Issue of Modelling

How and why modelling comes into play for forecasting functions in commercial and service industries were highlighted in the literature (Aziz-Khairulfazi, 2004; Stekler, 2003; Moon, 2005). One particular situation identified is where there actions of the forecasting team towards a set of available information can affect the initial forecast predictions, which are usually inaccurate.
A case study observation was conducted that uses Fisher's exact test to delineate significant associations in order to identify important variables

(Aziz-Khairulfaziand Percy, 2003). We observe the weakness in estimating forecasts using single point predictions, and our study should offer possible and reliable solutions to overcome this weakness. What interests us are issues relating to the outcome of the forecasting teamwork and what forecast estimates are involved. Three parts contribute to our analysis, namely:

1. mathematical modelling involving establishing a suitable probability distribution and loss function in order to apply Bayesian decision theory;

2. cost implications with respect to imperfect forecasts;

3. differential equations involving rates of change among variables, to describe and explain the underlying structural behaviour.

Bayesian Approach for Enhancing Point Predictions

From the investigations carried out, we observed that targets or single point predictions determined by an organisation, or particular unit within an organisation, become the platform towards which actual performances are inclined (Goodwin,2002). Even at the setting stage of targets and forecasts, the process of decision-making can be demanding to ensure crucial factors are not excluded. Single point predictions also add to the mood and motivation of people involved with the forecasts, be they preparers or users. These single point predictions do not allow for variations in case the outcomes of the actual performances turn out different from planned due to uncontrollable factors. Once the actual results are noted, the management will look back at their forecasts to identify what and why are the differences. By looking at just one figure, any deviation may incur costs and thereafter affect the people involved.
A previous study indicated that an essential aspect of decision-making involves consulting experts, who usually give differing opinions of information (Aziz-Khairulfaziand Percy, 2003; Edwards and Aziz, 2000). A considerable volume of literature is available to provide solutions addressing this problem. It is recommended that expert opinions be treated as data for further analysis in arriving at more reliable point predictions. In this analytical part of the research, three aspects of modelling, namely a probability distribution, cost function and Bayesiandecision analysis are described.

Probability distribution

At a university in United Kingdom, the current forecasting situation is that point predictions are prepared and then passed on to users (Aziz-Khairulfazi, 2004). As these are invariably inaccurate, we regard this as a flaw and now propose that forecasts should consist of probability distributions rather than point predictions to allow for this in accuracy. Our emphasis is on the outcome from the interaction of people, not only on the results achieved. We believe that there must be a build up of managerial structures and communication networks to increase and improve stability in the forecasting function. On the basis of extensions to the central limit theorem, the normal distribution is deemed appropriate here.This choice is supported by general theory relating to the laws of error (Eisenhart,1983).
Adopting the normal distribution, we assume $X \mid \mu,\, \sigma \backsim N(\mu, \sigma^2)$ where X is the actual profit, which is an unknown random variable at the time of preparing a forecast $\mu = \widehat{x}$, is a point forecast for the value of X and σ is the standard deviation which measures the uncertainty of our point forecast.
The benefits of establishing variations from point predictions and assigning normal distributions to these point predictions are now given. Firstly, as forecast accuracy is unexpected, the variation will improve motivation and drive. As such, management is better prepared in all kinds of possible situations and this does not affect forecasters' capability as a measure of improving the accuracy of forecasts.

Cost function

The element of costs is introduced and illustrated here as funding and money are important sources of running the business. When actual performance conflicts against forecasts, there is a loss involved and this results in a cost to the organisation (Goodwin, 2002). This aspect of loss may take the form of functional relationships which, in their simplest but most common form, are bilinear. The following illustration explains this situation:

Let the forecast be $\widehat{x}$ and the actual be x; when the actual conflicts with the forecast, there is a difference and an element of cost is involved. Therefore, for example,

if $\widehat{x} = RM1000$; $x = RM500$	cost is 5 units
if break-even i.e. $\widehat{x} = RM1000$ and $x = RM1000$	cost is 0 units
if $\widehat{x} = RM1000$; $x = RM1200$	cost is 2 units or less

Figure 1 shows a graph depicting the above effects. We measure cost in units to indicate that the costs involved are not just monetary, but include time and effort wasted. Therefore, a measurement for these must be devised collectively by the people involved. This may mean that the cost involved is less when actual is more than forecast rather than when actual is less than forecast. This difference may be due to intangibles and may represent the hidden costs. As long as the difference between actual and forecast results is material, further breakdown of the costs involved must be scrutinised and addressed to find solutions to improve future forecasts. For example, when $\widehat{x} = RM1000$ and $x = RM500$, this is a situation of over-forecasting. Among the consequences of this condition are:

1. employees will be demotivated as their high expectation of the company to perform is diminished. As a result, this might lead to a high turnover of employees;

2. resources will be over–utilised as unrealised provisions are used;

3. the reliability of forecasts will be in question;.

4. the forecasting exercise will not be cost–effective.

Similarly, when $\widehat{x} = RM1000$ and $x = RM1200$, this is a situation of under–forecasting. The consequences of this condition are:

1. under–utilisation of resources;

2. potential investments will be withdrawn;

3. doubts about the reliability and cost effectiveness and cost–effectiveness of forecasting will arise.

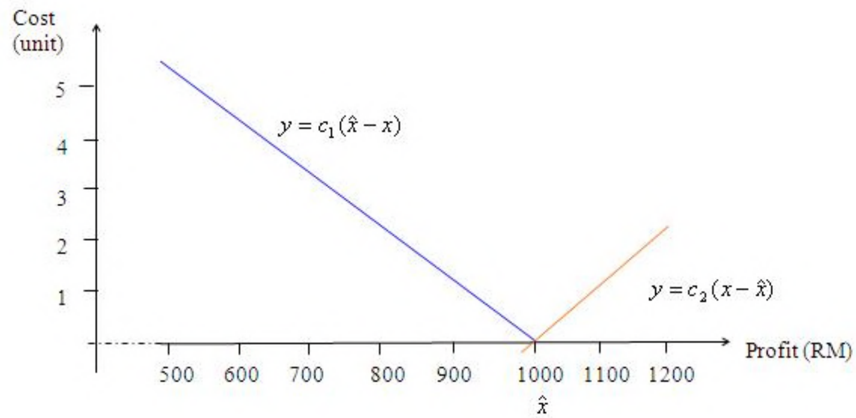

Figure 1 Graph Showing the Cost of under and over–forecast of profits

Bayesian methodology

The classical, or frequentist, approach to estimation corresponds here to the generation of point predictions enhanced by prediction intervals, though managerial decisions are usually based on the point predictions only. Regarding the observed profit as arising from a normal distribution, however one can establish a subjective predictive distribution by looking at the chances or likelihoods of achieving various targets away from this point prediction. This variation provides an indication of how the actual outcome evolves around its forecast.This explains and allows for the differences between the actual and forecast values.
For example, we might present forecasts in terms of relative likelihoods like this: it is twice as likely to achieve a profit of $RM10,000$ than a profit of $RM15,000$. Better still, we could present quantiles or even the full distribution for profit. Bayesian decision theory allows distributions of predictions to model possible departures from point forecasts like this to make sure that the uncertainty of achieving them is considered. This uncertainty is here expressed using a normal distribution of relative likelihoods for the probability density function of profits. As for any density, the area under the normal curve is one. For a simplified analysis, one could consider a two-phased outcome, or binary response, so that if there is two-thirds of a chance that the profit is at least $RM10,000$, then the chance of not making that amount of profit is one third. This enhances the quality of forecasts but ignores system feedback, which we consider shortly.

The distribution for the variation of profits can be obtained in two ways: subjectively or objectively. For example, we might establish a normal distribution with associated loss function objectively. Using an ARIMA model requires no subjective devising, revising and adjusting. At this point, the expected cost of a poor forecast can be calculated. If profits are more than $RM2500$, for example, the cost involved is proportional to the difference between the point prediction and the actual profit achieved. Applying there commendation given by Moon (2005), the mathematical functions involved in this modelling of imperfect forecasts take the following forms for this application, where $\widehat{x}$ is a point prediction and x is the actual profit:

1. Normal distribution function for profits

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}; -\infty < x < \infty \tag{1}$$

2. Cost function forthis application is the bilinear form

$$\text{where } c(x) = \begin{cases} c_1(\mu - x); & x < \mu \\ c_2(x - \mu); & x > \mu \end{cases} \tag{2}$$

which is illustrated in Figure 1.

This means that there is a cost involved when the actual profit is more or less than the forecast profit. This cost refers to the cost associated with imperfect forecasting. The costs in this study may include time, effort wasted, opportunity loss, penalty loss, and also not being able to invest in fixed assets, projects and profitable contracts.
Then, decision analysis is based on minimising the expected cost

$$\begin{aligned} E(c(X)) &= \int_{-\infty}^{\infty} c(x)f(x)dx \qquad (3) \\ &= \int_{-\infty}^{\mu} c_1(\mu - x)\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}dx + \int_{\mu}^{\infty} c_2(\mu - x)\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}dx \end{aligned}$$

The loss function $c(x)$ can be bilinear, as in our analysis,or of some other unspecified form. The bilinear cost function shows aproportionate increase in cost with the difference between actual and forecast performances. This is true for both sides of the relationship, $x > \mu$ and $\mu > x$. However, it does not assume symmetry unless $c_1 = c_2$ above.
To evaluate equation (3), we make the substitution

$$y = \left(\frac{x - \mu}{\sigma}\right)^2 \Rightarrow dy = \frac{2}{\sigma^2}(x - \mu)dx \tag{4}$$

in both integrals, so that

$$\begin{aligned}
E(c(X)) &= \int_{\infty}^{0} -c_1 \frac{\sigma^2}{2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y}{2}} dy + \int_{0}^{\infty} c_2 \frac{\sigma^2}{2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y}{2}} dy \\
&= \frac{(c_1+c_2)\sigma}{2\sqrt{2\pi}} \int_0^{\infty} e^{-\frac{y}{2}} dy \\
&= \frac{(c_1+c_2)\sigma}{2\sqrt{2\pi}} \left[-2e^{-\frac{y}{2}}\right]_0^{\infty} \\
&= \frac{(c_1+c_2)\sigma}{\sqrt{2\pi}} \qquad (5)
\end{aligned}$$

$$\text{where } c(x) = \begin{cases} c_1(\mu - x); & x < \mu \\ \\ c_2(x-\mu); & x > \mu \end{cases} \text{ and } X \mid \mu, \sigma \sim N(\mu, \sigma^2) \qquad (6)$$

This clearly illustrates how, under the assumption of a normal distribution and bilinear loss function, the expected cost of inaccurate forecasting is directly proportional to the standard deviation of the predictive distribution.
Since forecasting considers the future, which is usually unpredictable, any incidences of unexpected outcomes should be precautioned and any remedial actions should be recommended. These initiatives are taken so that organisations will be ready to face the future. Any strong form of information, available at the last minute, may force the organisation to change forecasts abruptly. It is at this point that top management intervenes to allow forecasts to reflect reality. As events like this may be difficult to measure, the use of modelling will be a helpful support tool for guiding calculations.

Explanations for and Cost Effects of Imperfect Forecast

To explain the cost implications of imperfect forecasts, we now consider these in the context of service industries. There are various indicators that can be used to measure performance, such as patients per day for hospitals, customers per hour of service utilities and passengers per destination for the flight industry, to name a few. In our case, we consider the university scenario in terms of student numbers as a performance measure. If the actual number of students is more than the forecast number of students, there is a need for extra logistics, including space, rooms, lecturers, time-tabling, accommodation, computer facilities and administration. The quality of teaching and success of graduates might be compromised because of mass production. There will be more drop-outs and a higher failure rate which will affect the image of the university.
While universities commit themselves to provide facilities for the extra students, it

may be for the short-term only. There will be insufficient budget available to sustain over-capacity as a result of inefficiency on the part of management not being able to forecast and cater for extra students.
However, if the actual number of students is less than the forecast number, these results in under-capacity, as facilities are under-utilised or idle. The university over-pays the lecturers in terms of salary per student and so the marginal cost per student is higher.
The whole idea of this modelling is to arrive at not just effective and efficient solutions to account for and minimise the total loss, but also to be aware of situations and consequences arising from inaccurate forecasting.

Conclusion

Modelling in our case attempts to describe the mechanism of relationships between variables that operate in practice; an extension we offer to integrate with management accounting. In demarking the selected variables, we use the law of parsimony or Occam's Razor in that the model includes only required and important variables and does not include all reasonable predictor variables automatically. It should also be noted that parsimony is a principle in science where the simplest answer is always preferred.
Several aspects constitute the modelling process. We first saw how single point estimates or predictions can be improved by assigning probability distributions to describe variations that may be possible, hence increasing the reliability and credibility of the forecasts.Then, we saw the measure of loss functions as a result of imperfect forecasts and how it can be quantified, using Bayesian decision theory, according to whether actual results are less than forecast or vice-versa (Drury, 1990; Stekler,2003; Aziz-Khairulfazi and Percy, 2003).
The effects of imperfect forecasts were also explained for both service industries, and manufacturing and trading industries. The cost factor was included and differential equations were introduced to render the whole modelling aspect complete. They give a clearer perspective of empirical evidence cultured with mathematics and functional relationships objectively. It can be seen that outcomes of improved teamwork and decision making, for example, are related inthis way.
Last but not least, in order to get a total picture of the whole research implication onto practice, future study to reflect impact is recommended.

Management Proposition

It should be in the interest of the board game designer that lessons should be learned from playing the game itself as it simulates the practice. The intention here is to make learning as easy and as fun. As interaction is apparent while playing among two-four players, they are forced to communicateand exchange related information, besides entertaining themselves. Aspects of marketing also play role in promoting board games; usually they are played as leisure but they should now be a part of teaching and learning strategy.

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