O Ábaco de Gerbert

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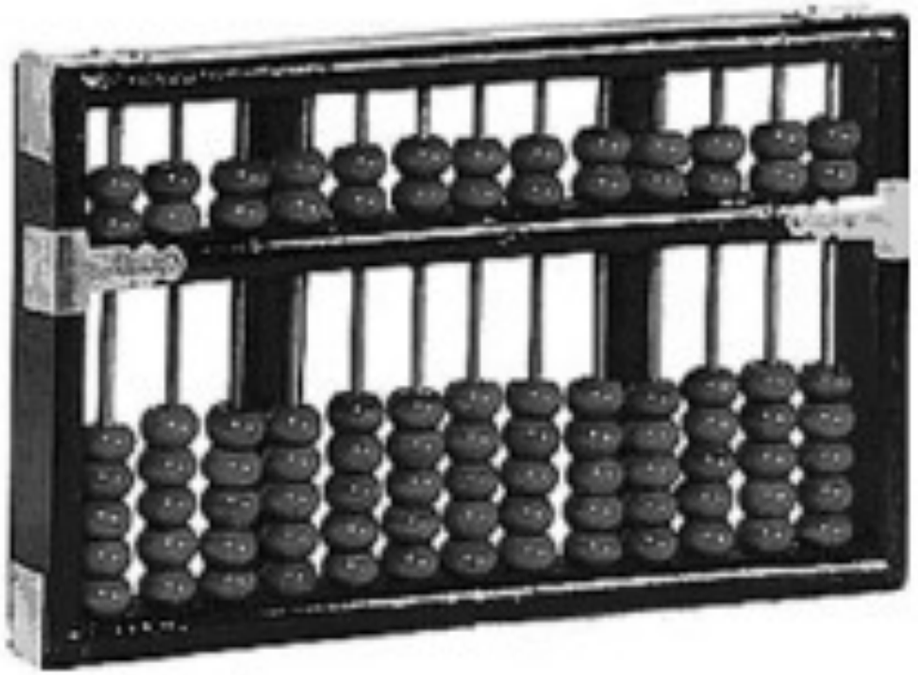
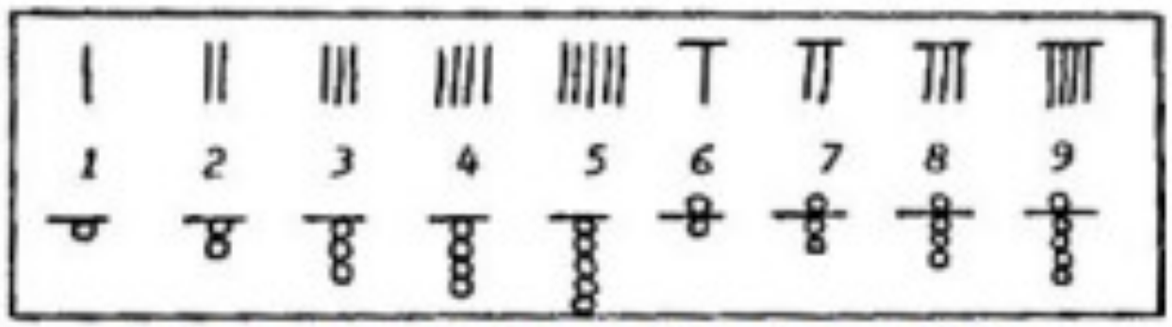


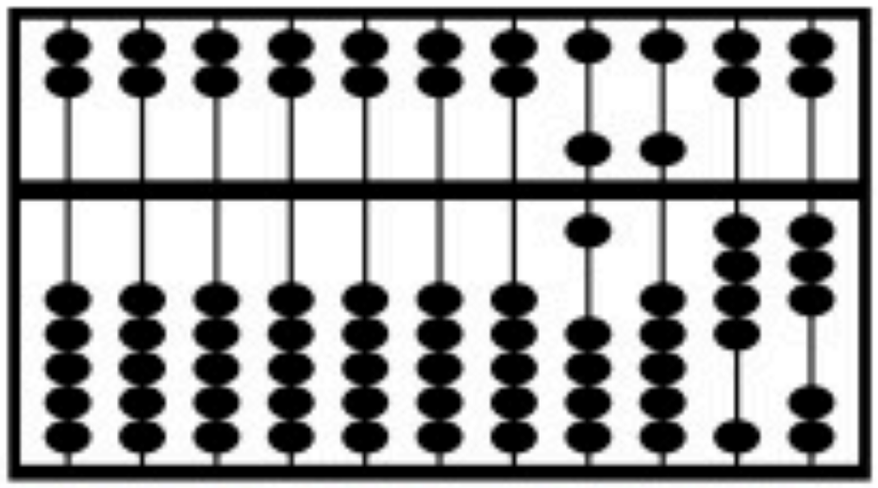
Ábaco chinês (*Suanpan*)

Este tipo de ábaco é o mais conhecido em Portugal, já que era muito popular em Macau. Para representar os números convenciona-se que cada uma das peças abaixo da barra vale uma unidade e cada uma das outras vale cinco. As peças que se utilizam deslocam-se para a barra central. Esquematicamente:



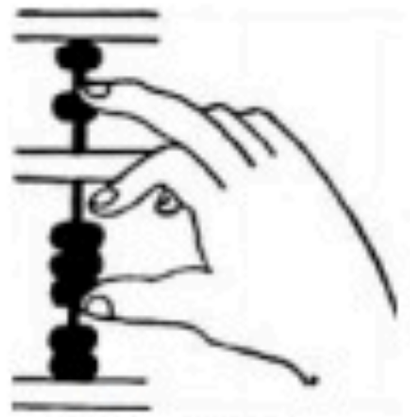
A representação dos números no ábaco chinês

Por exemplo, para representar 6543, deslocavam-se as peças da seguinte forma:



Representação do número 6543 no *Suanpan*

A manipulação desta ferramenta pode ser muito rápida e eficiente. Aconselha-se que se utilizem certos dedos para as peças de baixo e outros para as de cima:



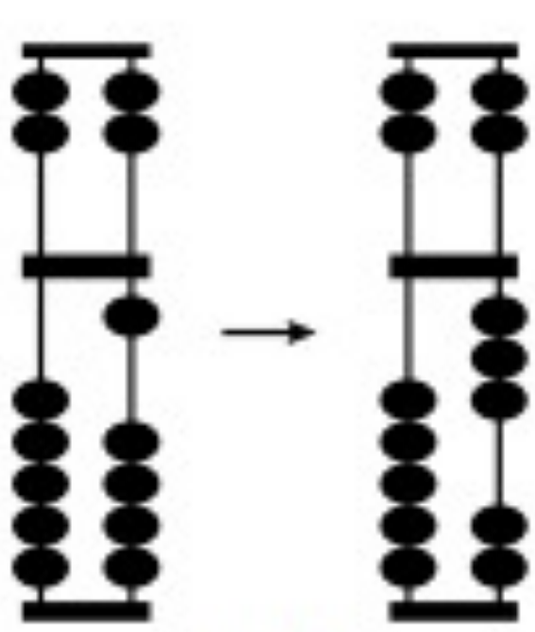
Manipulação do ábaco chinês

Para efectuar as operações havia regras práticas, ensinadas nos manuais aritméticos. Vejamos, as da soma:

1. Um – baixar cinco, anular quatro;
2. Dois – baixar cinco, anular três;
3. Três – baixar cinco, anular dois;
4. Quatro – baixar cinco, anular um;
5. Um – anular nove, adiantar dez;
6. Dois – anular oito, adiantar dez;
7. Três – anular sete, adiantar dez;
8. Quatro – anular seis, adiantar dez;
9. Cinco – anular cinco, adiantar dez;
10. Seis – anular quatro, adiantar dez;
11. Sete – anular três, adiantar dez;
12. Oito – anular dois, adiantar dez;

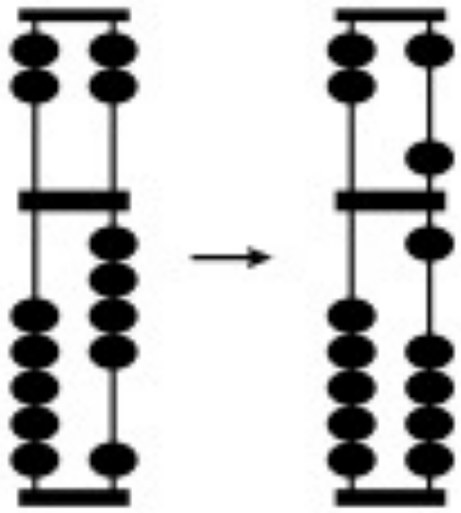
13. Nove – anular um, adiantar dez;
14. Seis – elevar um, anular cinco, adiantar dez;
15. Sete – elevar dois, anular cinco, adiantar dez;
16. Oito – elevar três, anular cinco, adiantar dez;
17. Nove – elevar quatro, anular cinco, adiantar dez.

Consideremos, por exemplo, as regras que dizem respeito a somar dois, isto é, as regras 2 e 6. Se quisermos somar duas unidades a primeira coisa a fazer é tentar subir duas peças inferiores, o que funciona bem ao calcular 1+2 (as próximas figuras contêm somente as varetas das unidades e das dezenas):



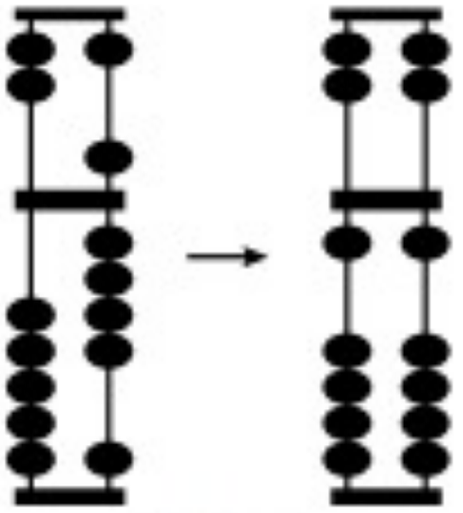
1+2=3

Mas, para somar 2 a 4 já não dispomos de duas peças para subir, temos de usar a regra 2, isto é, juntar 5 e retirar 2 (5-3-2):



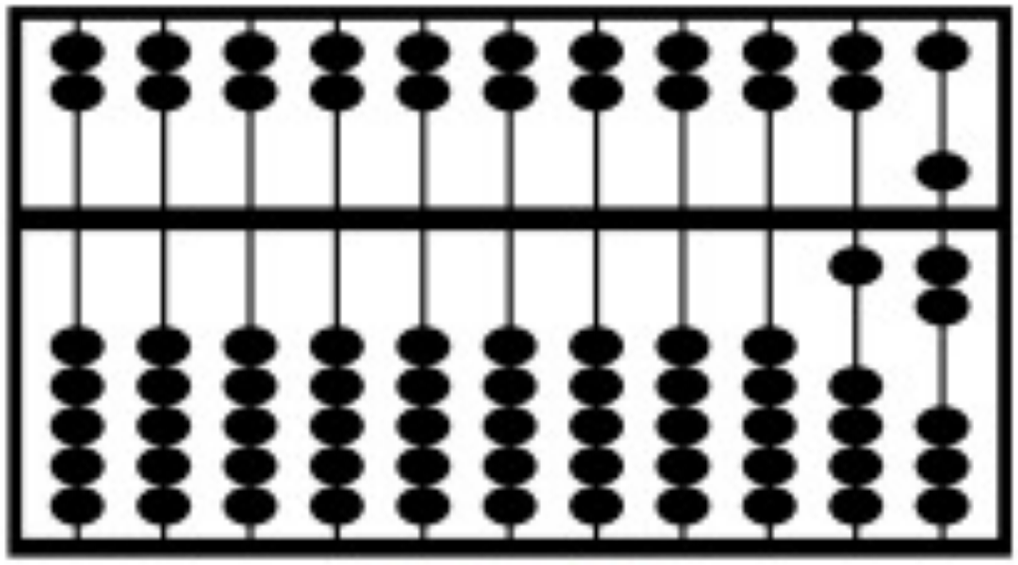
4+2=6

Mas pode ainda dar-se o caso de não dispormos de uma peça de 5, na mesma vareta, que possamos usar. Neste caso temos de transportar para a casa decimal seguinte (o célebre *e vai um*). Temos de recorrer à regra 6. É o que sucede no cálculo de 9+2:



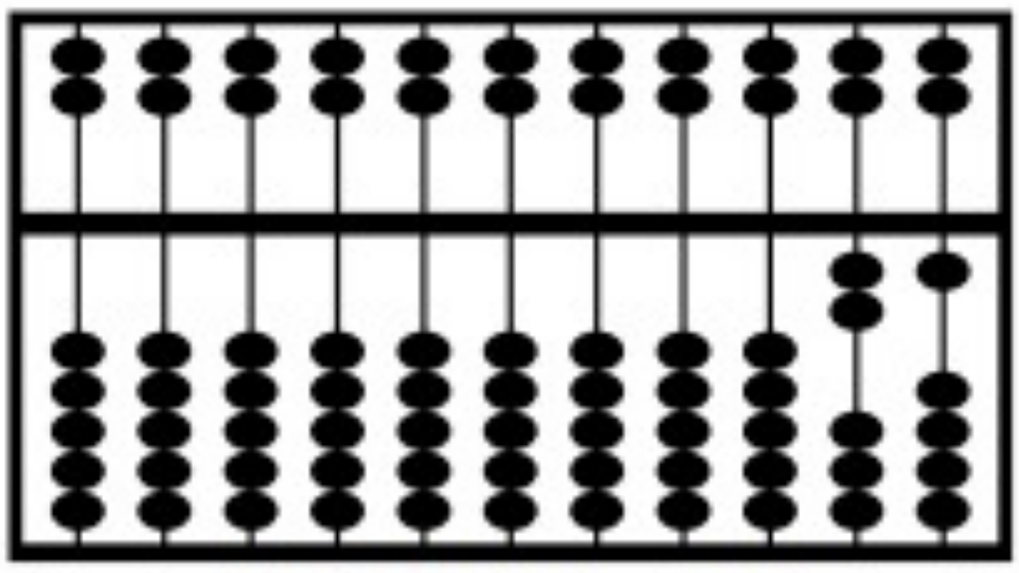
9+2=11

Calculemos agora 17+44. Primeiro, representamos 17:



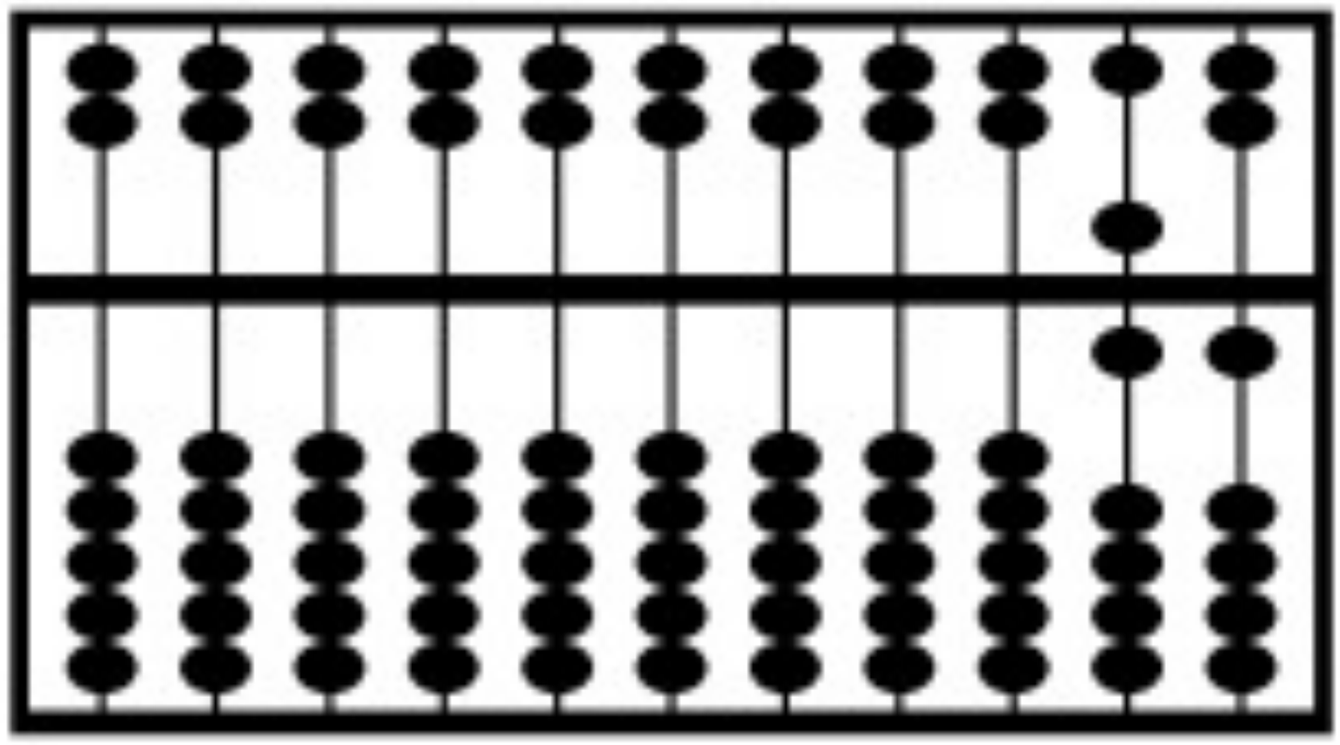
O número 17

Agora adicionemos 4 às unidades. Como o número representado nas unidades é superior a 6, usamos a regra 8. Para tal anulamos 6 e juntamos 10 (isto é, uma unidade na casa decimal seguinte):



O resultado de somar 4 a 17

Agora resta-nos somar 40, isto é, somar 4 à segunda vareta. Como estão lá 2, o resultado é 6.



O resultado de somar 40 a 21

Obtemos assim o resultado final: 17+44=61.

Para as outras operações há regras semelhantes, que constituem um conjunto de automatismos que viabilizam uma manipulação muito rápida.

Grécia

Usavam letras para representar números.

Γ	Δ	Η	Χ	Μ
Pente	Deka	Hekaton	Khilioi	Murioi
Πεντε	Δεκα	Ηεκατον	Χιλιοι	Μυριοι
5	10	100	1000	10000

Ι	ΙΙ	ΙΙΙ	ΙΙΙΙ	Γ	ΓΙ	ΓΙΙ	ΓΙΙΙ	ΓΙΙΙΙ	Δ
1	2	3	4	5	6	7	8	9	10
1 - 10 in Greek acrophonic numbers									

Δ	𐅁	Η	𐅂	Χ	𐅃	Μ	𐅄
10	50	100	500	1000	5000	10000	50000
Higher numbers and combining acrophonic numerals							

Grécia

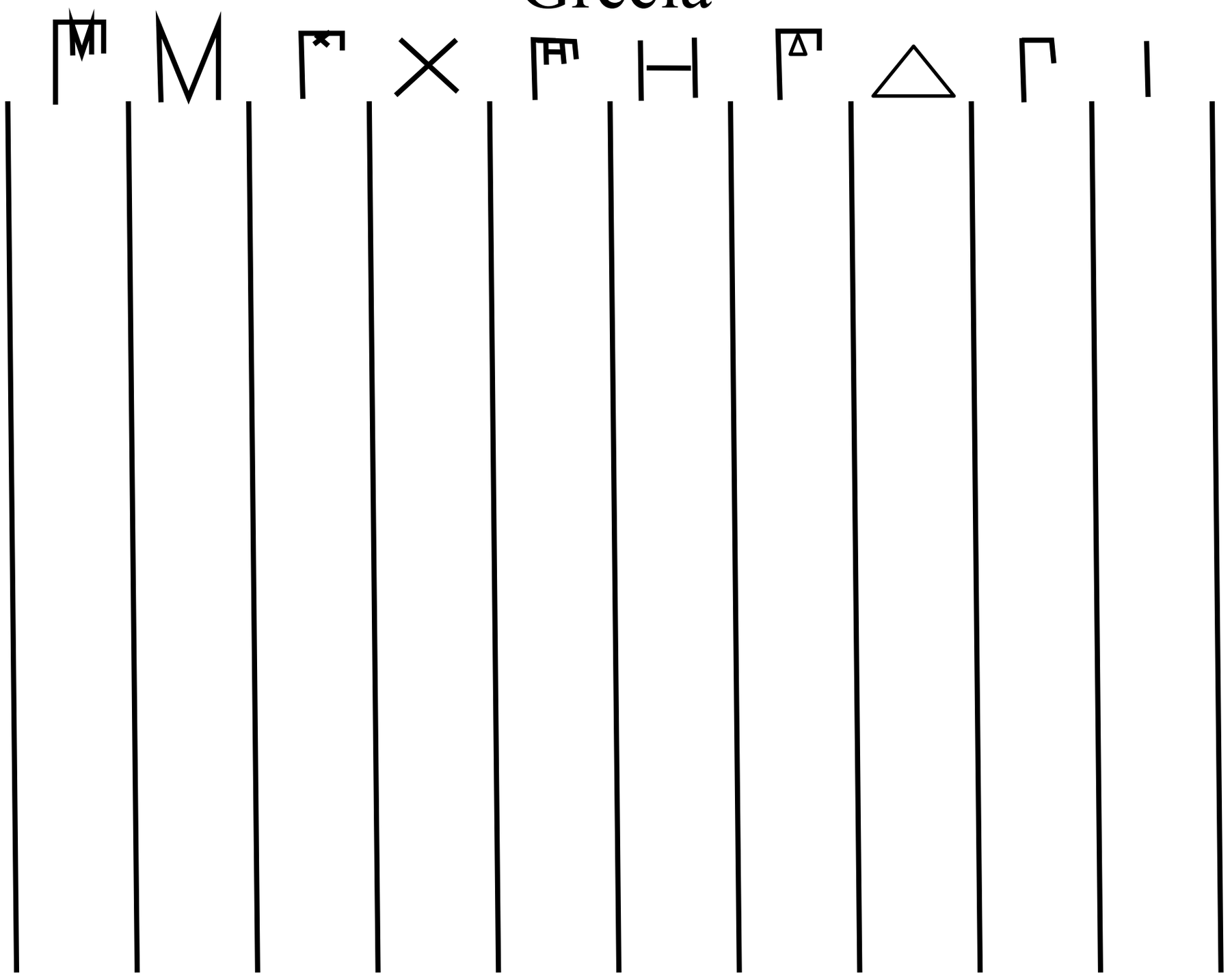
Neste sistema, as letras eram atribuídas aos números por ordem alfabética.

Α	Β	Γ	Δ	Ε	Ϛ	Ζ	Η	Θ
α	β	γ	δ	ε	ϛ	ζ	η	θ
1	2	3	4	5	6	7	8	9

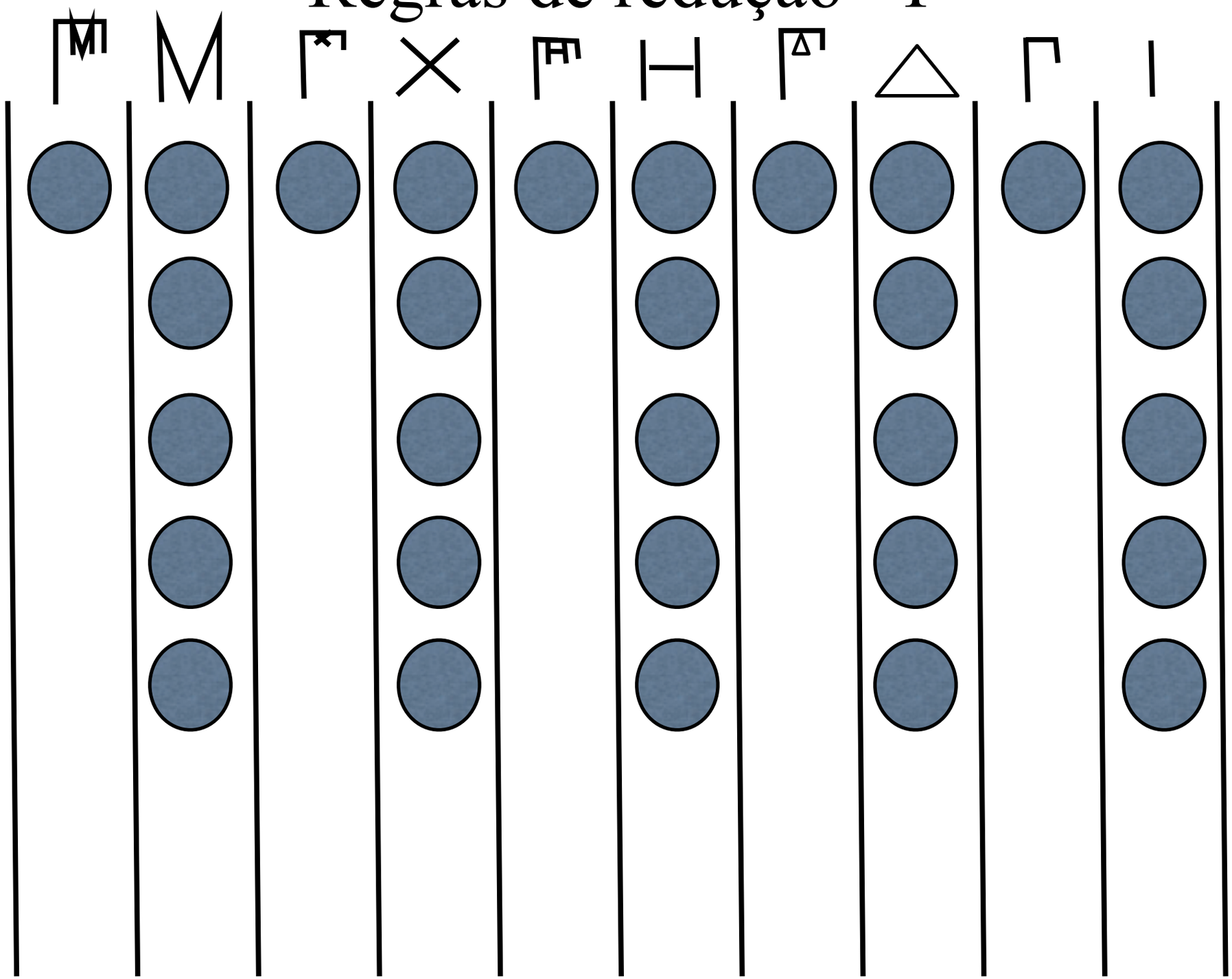
Ι	Κ	Λ	Μ	Ν	Ξ	Ο	Π	Ϟ
ι	κ	λ	μ	ν	ξ	ο	π	ϟ
10	20	30	40	50	60	70	80	90

Ρ	Σ	Τ	Υ	Φ	Χ	Ψ	Ω	Ϡ
ρ	σ	τ	υ	ϕ	χ	ψ	ω	ϡ
100	200	300	400	500	600	700	800	900

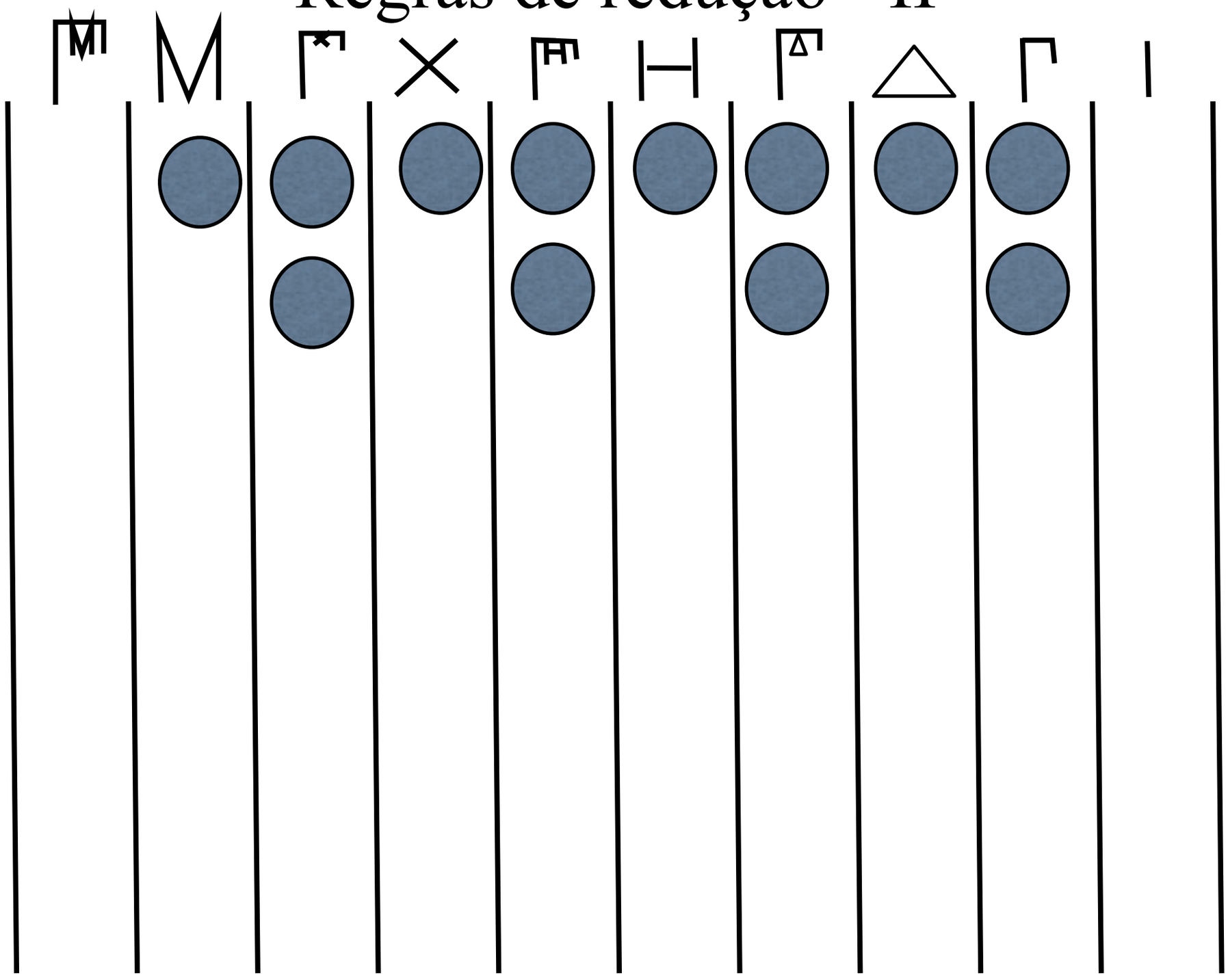
Grécia



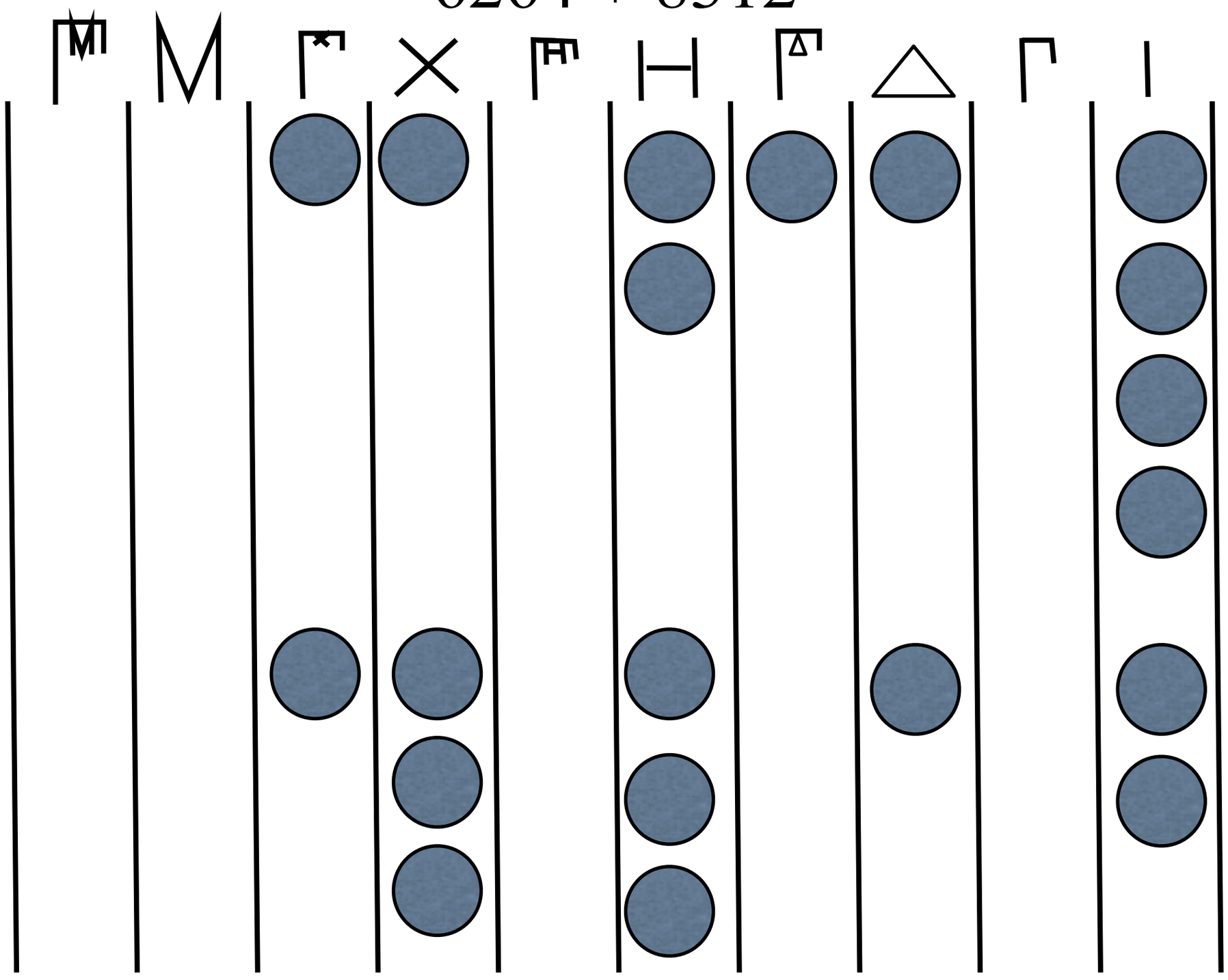
Regras de redução - I



Regras de redução - II



6264 + 8312



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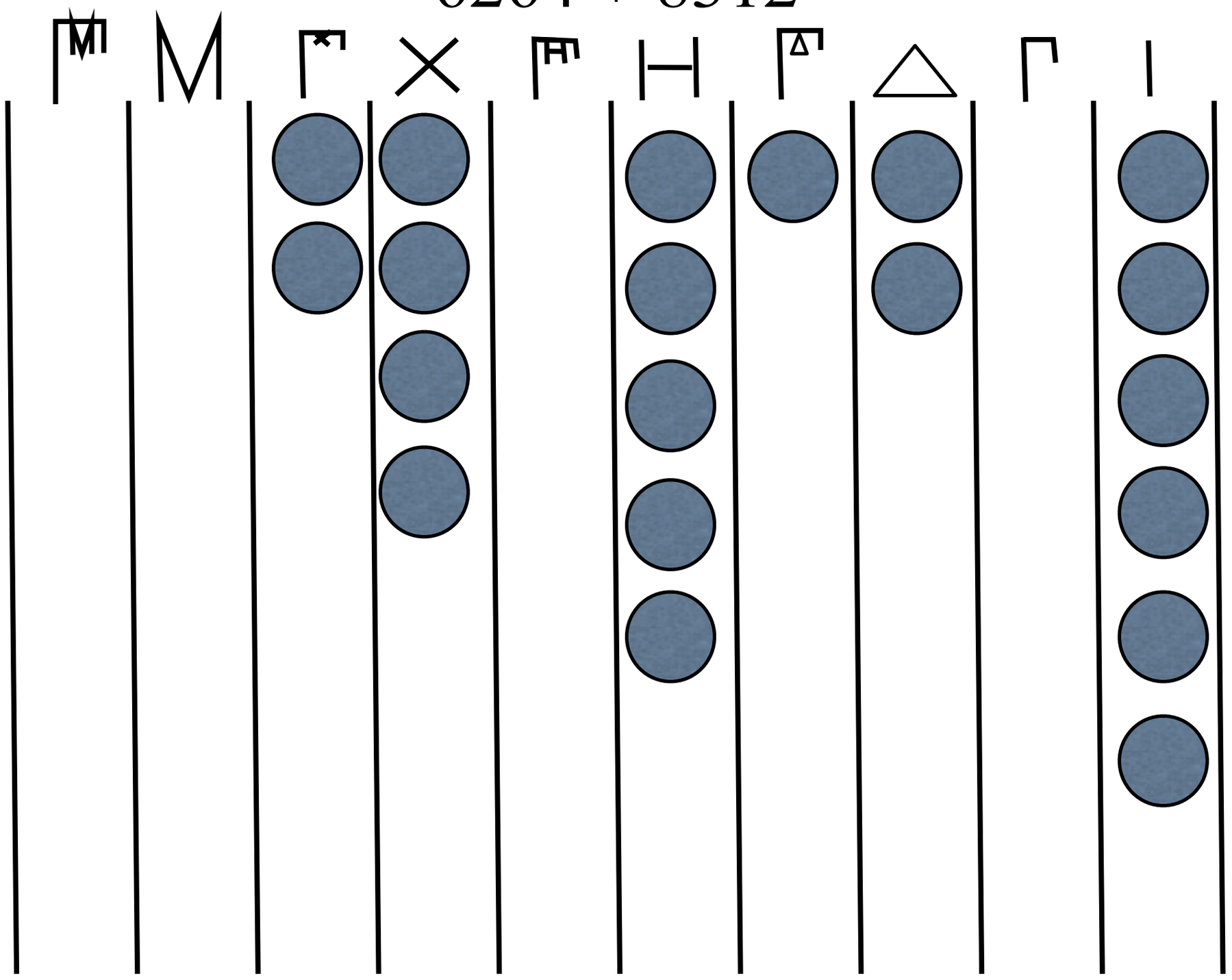

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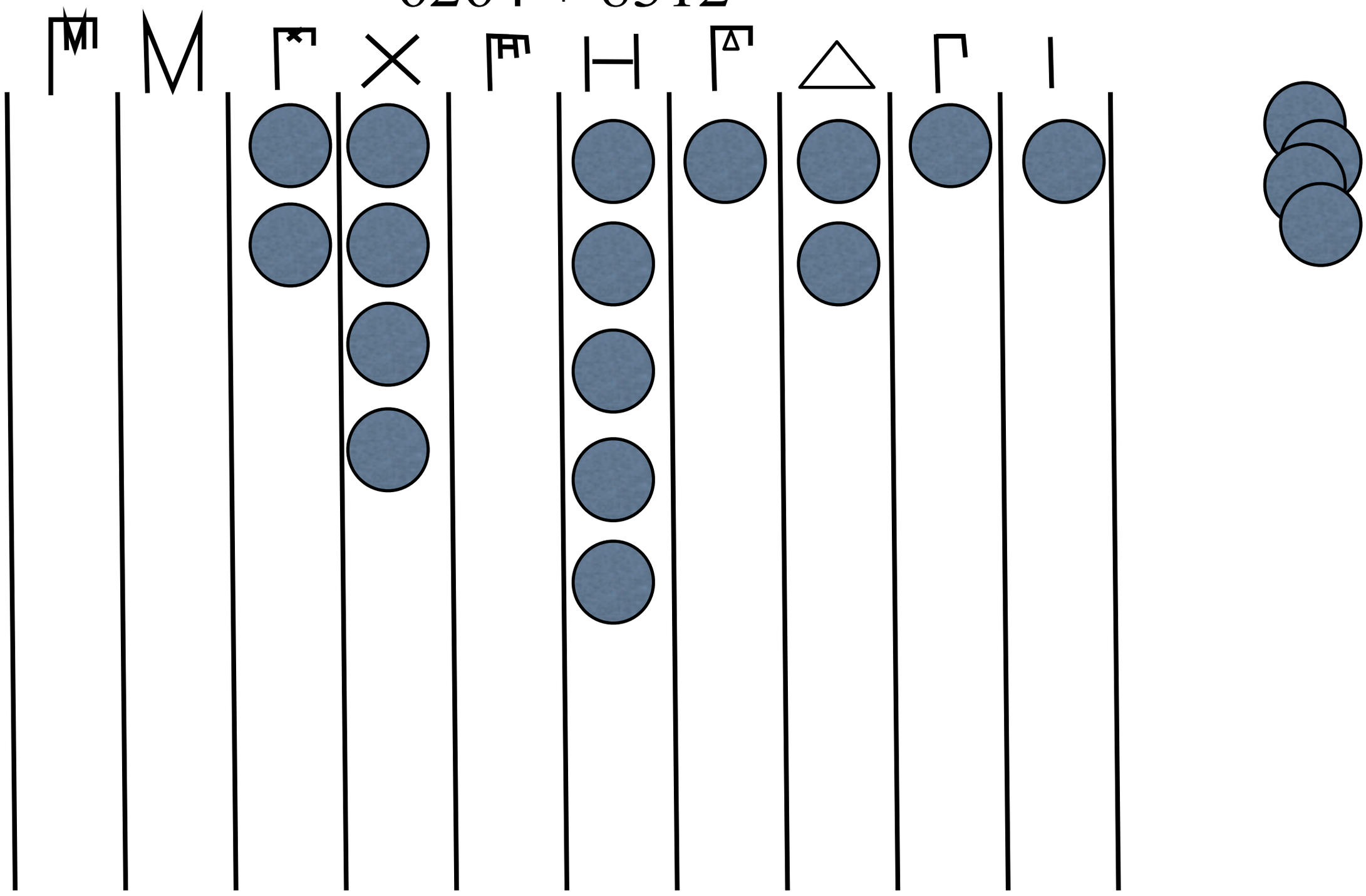

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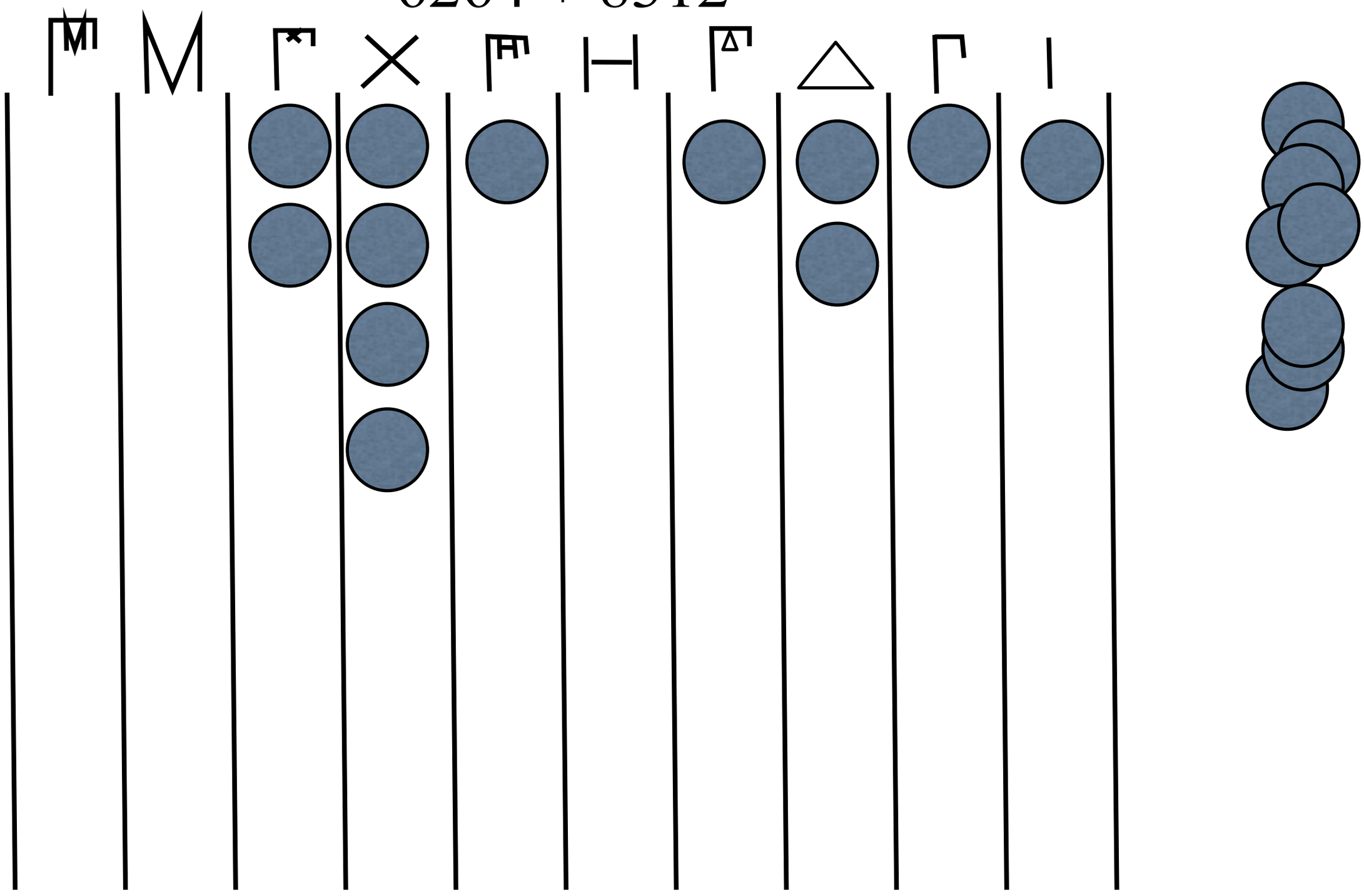
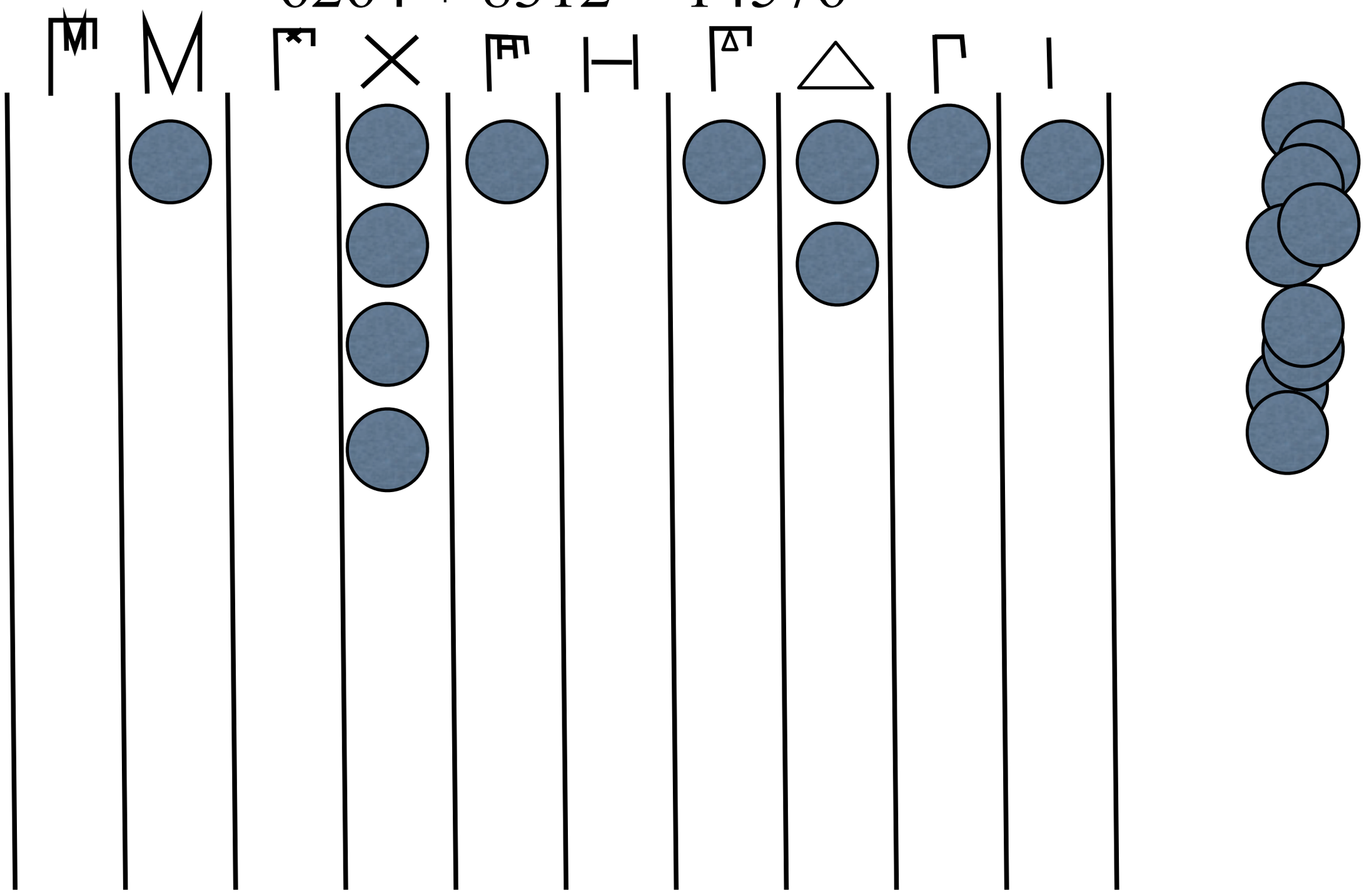
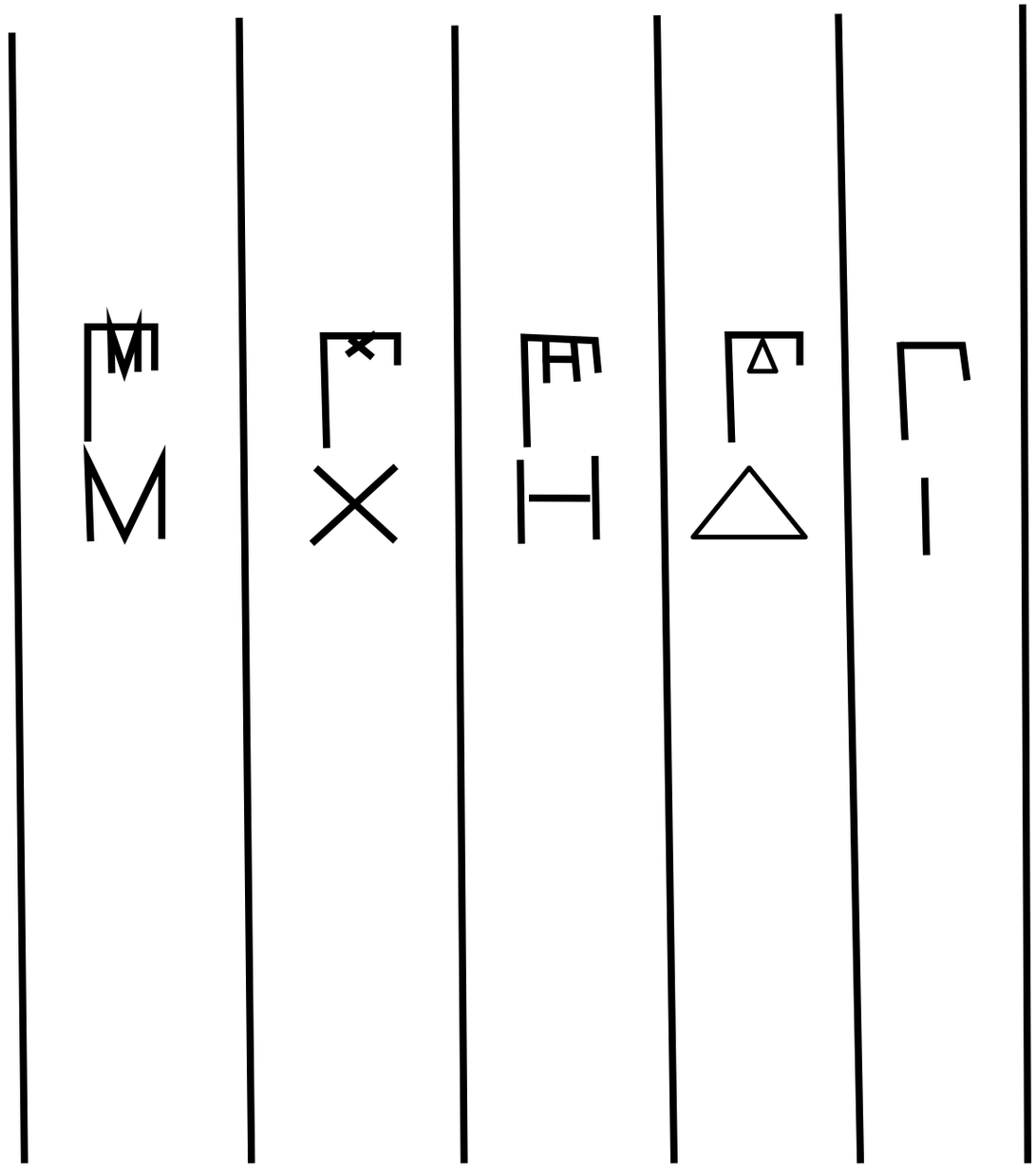


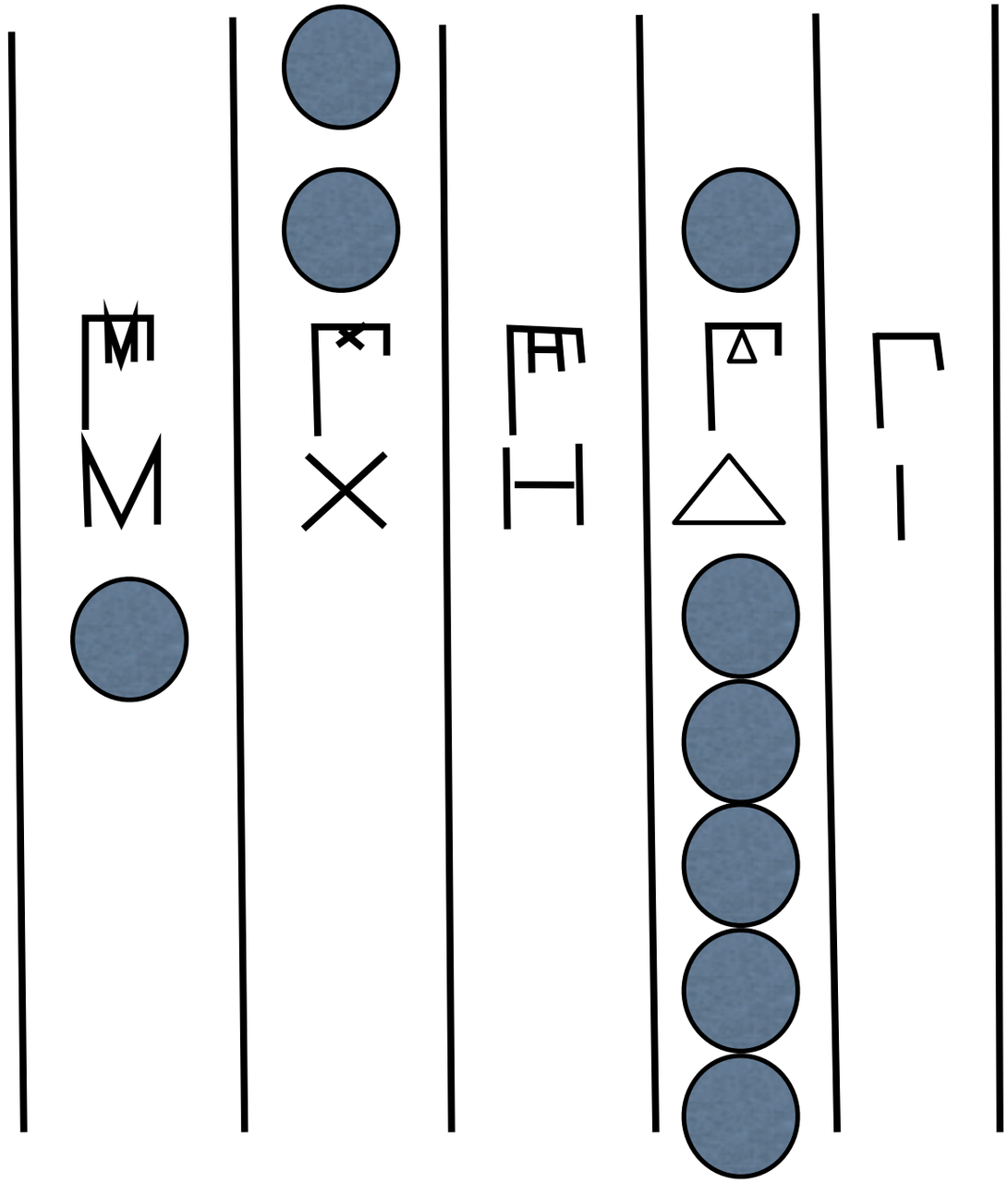
$$6264 + 8312 = 14576$$



Colunas decimais



Regras de redução



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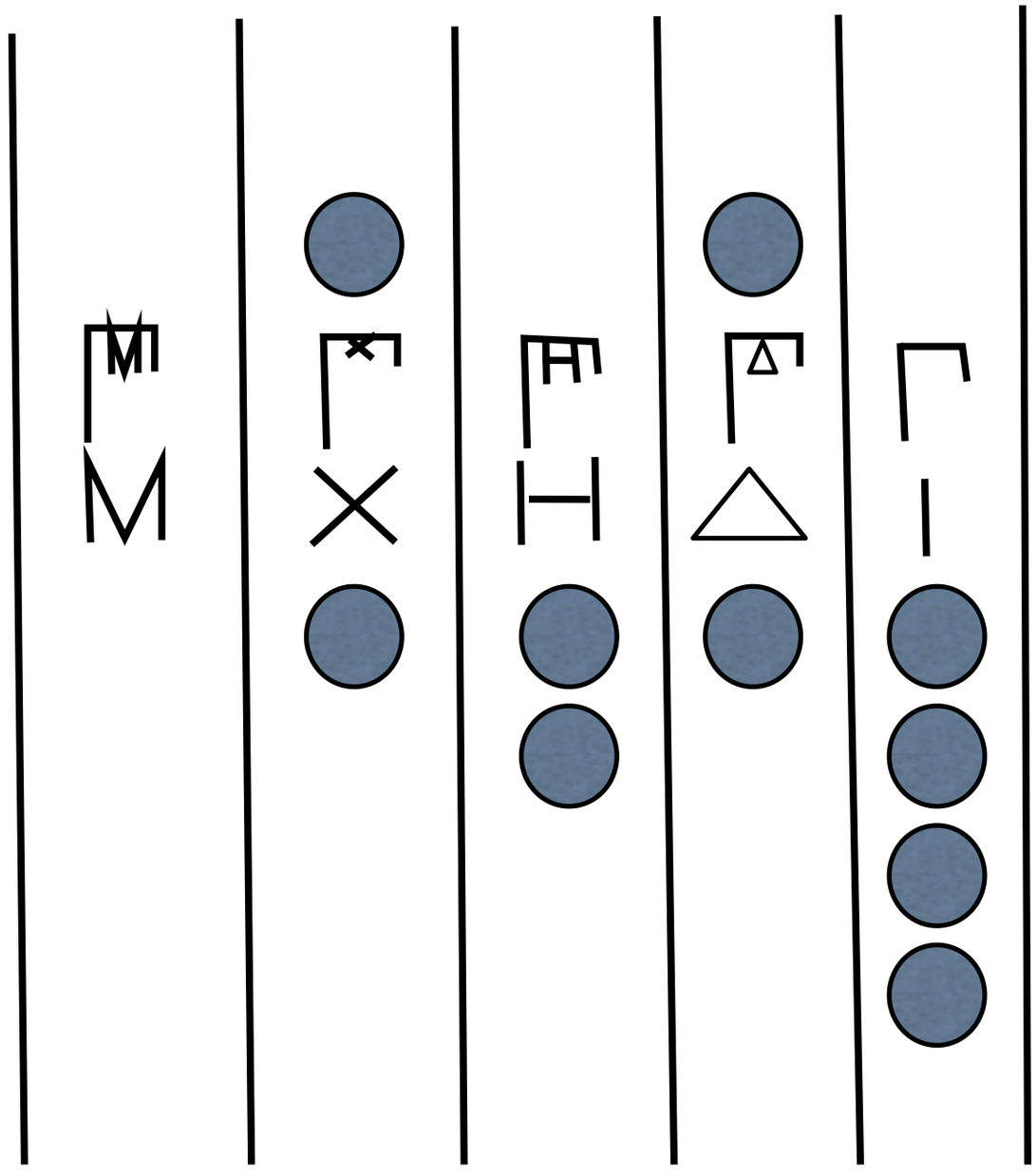

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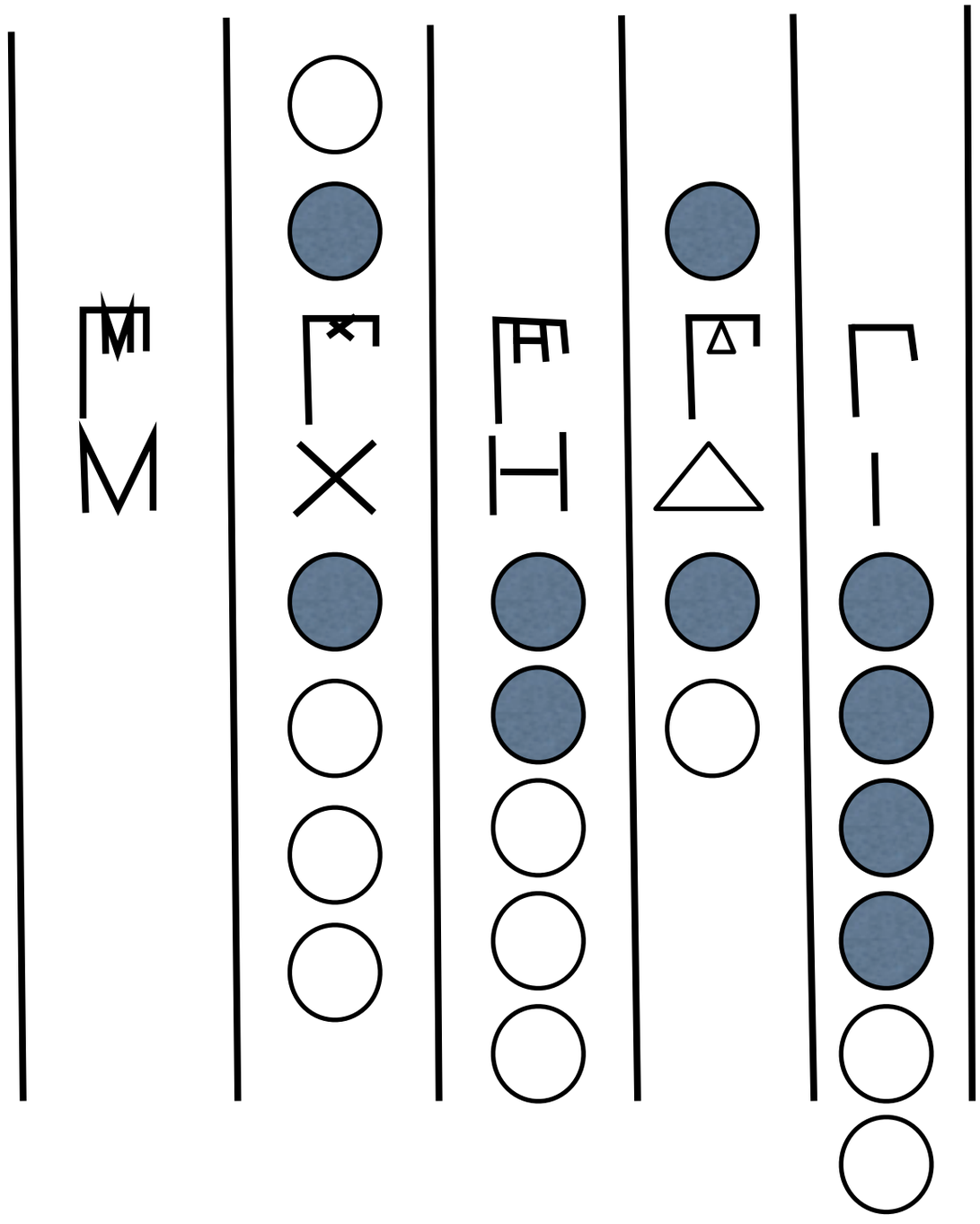

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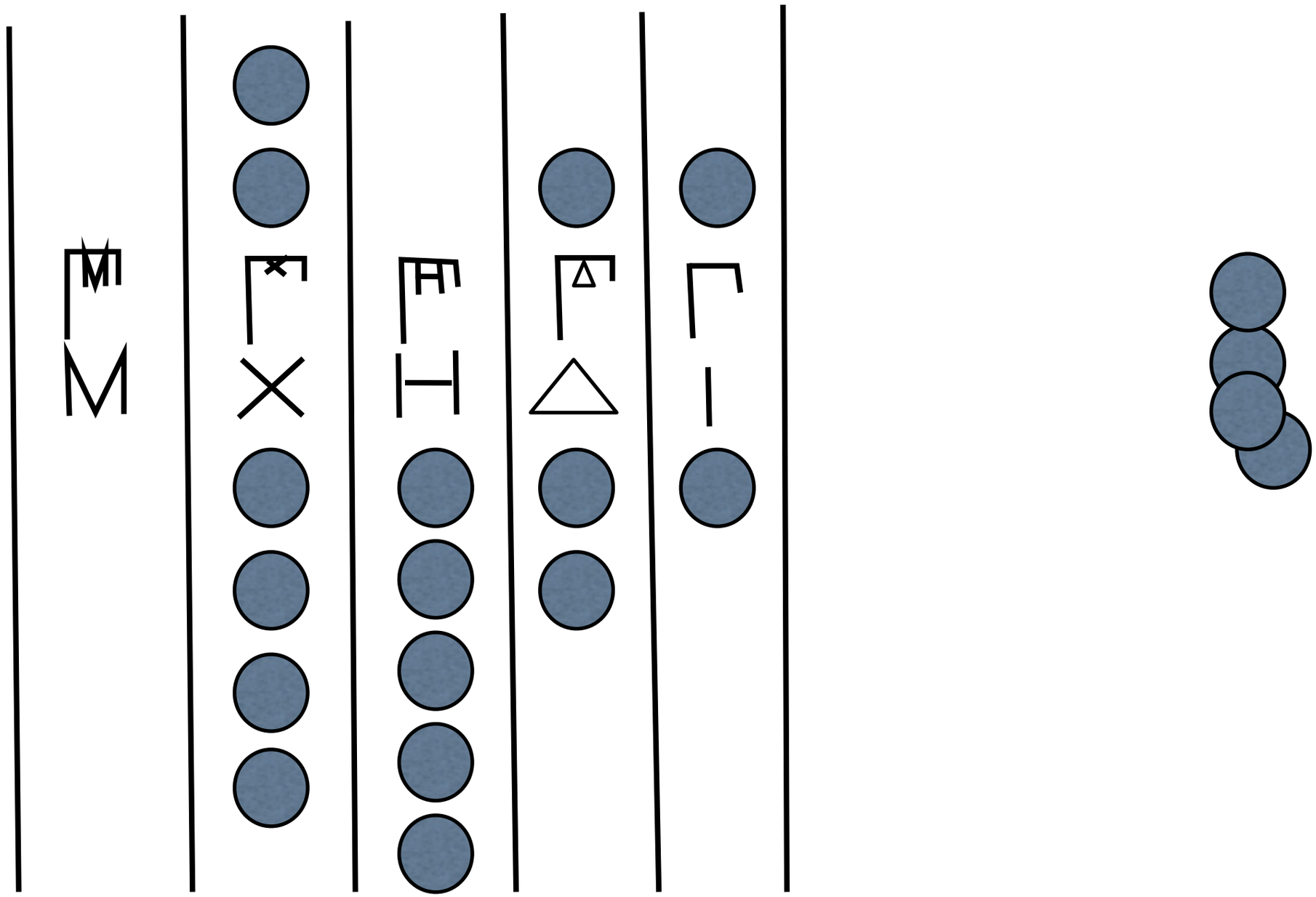

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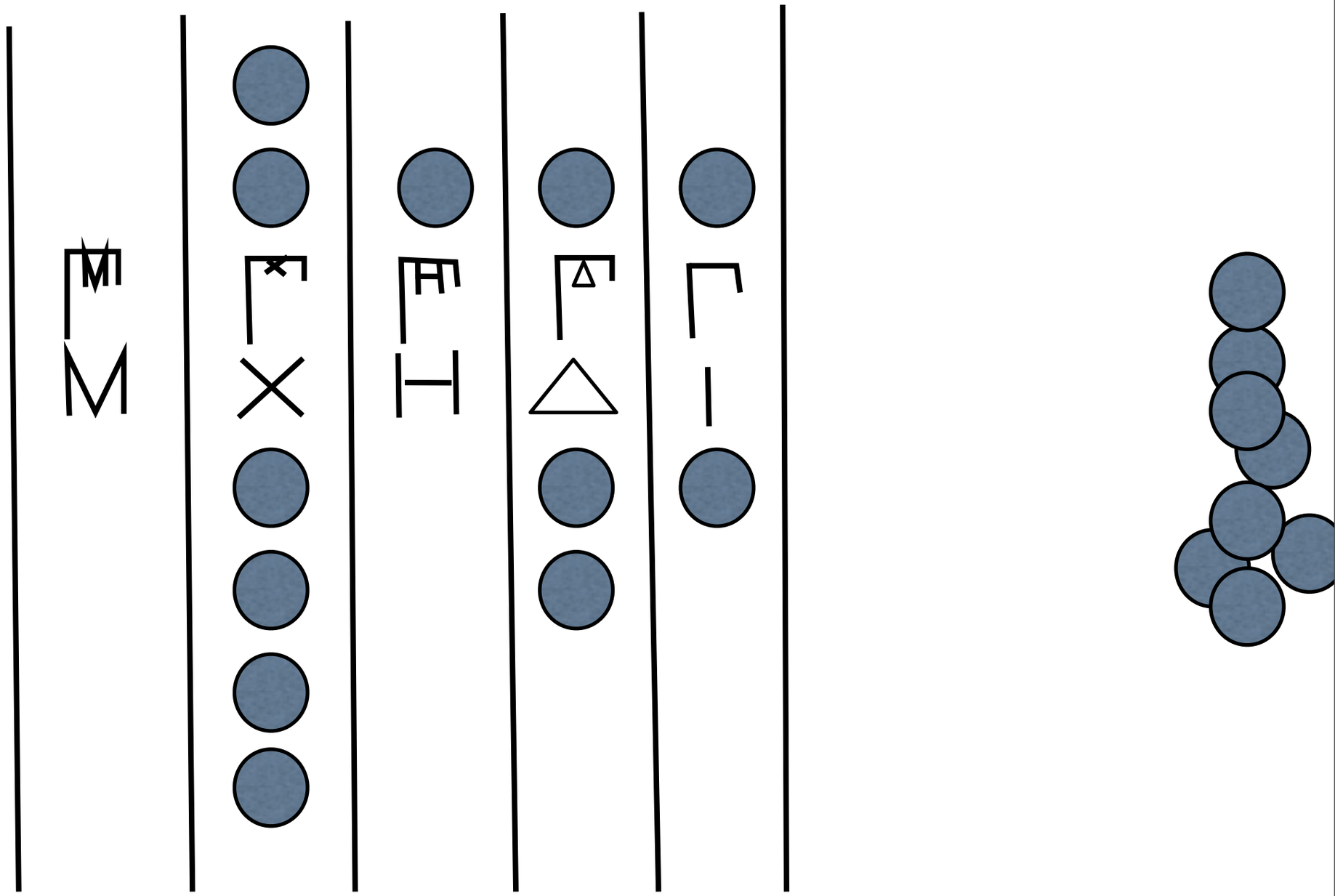
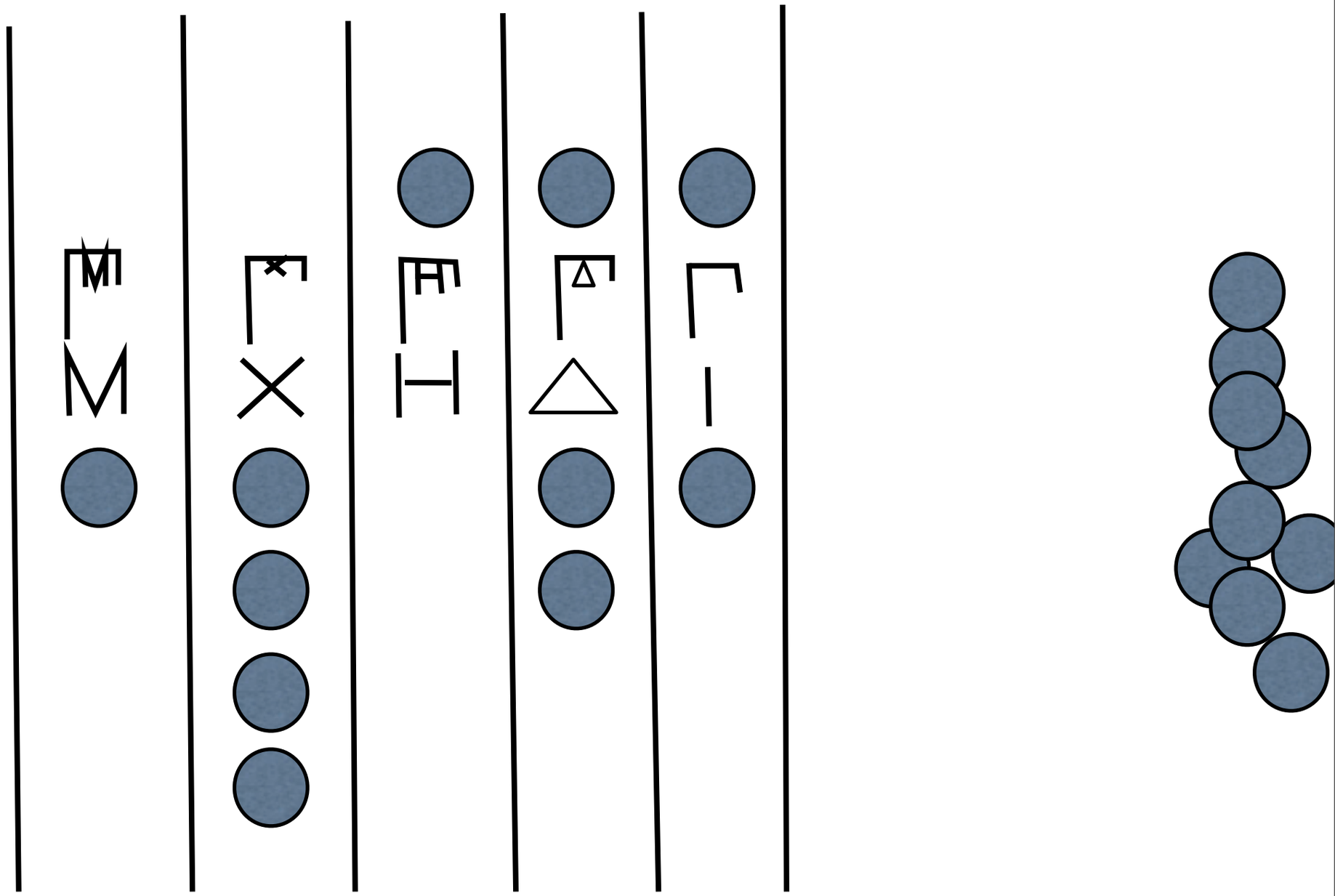


$$6264 + 8312 = 14576$$



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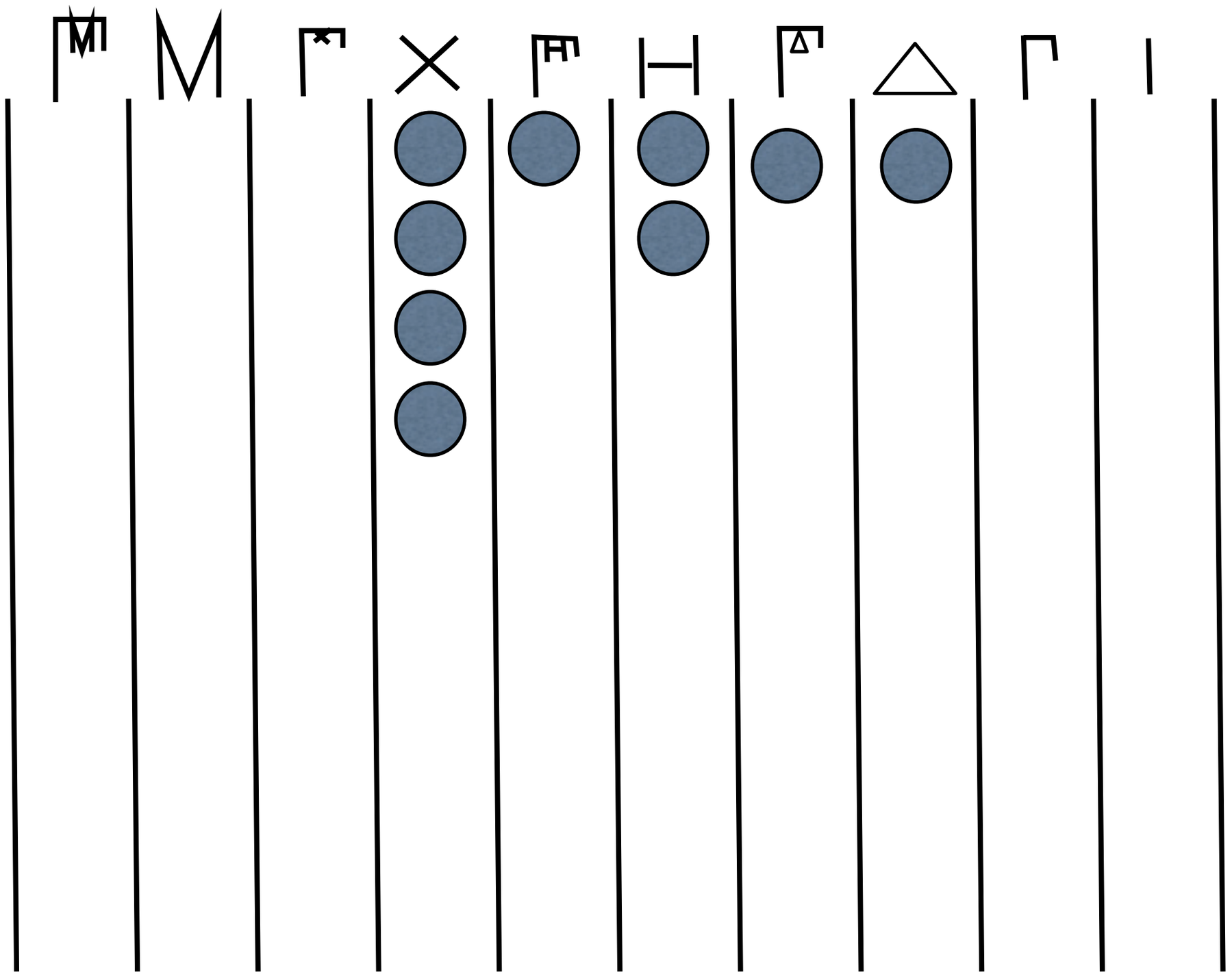

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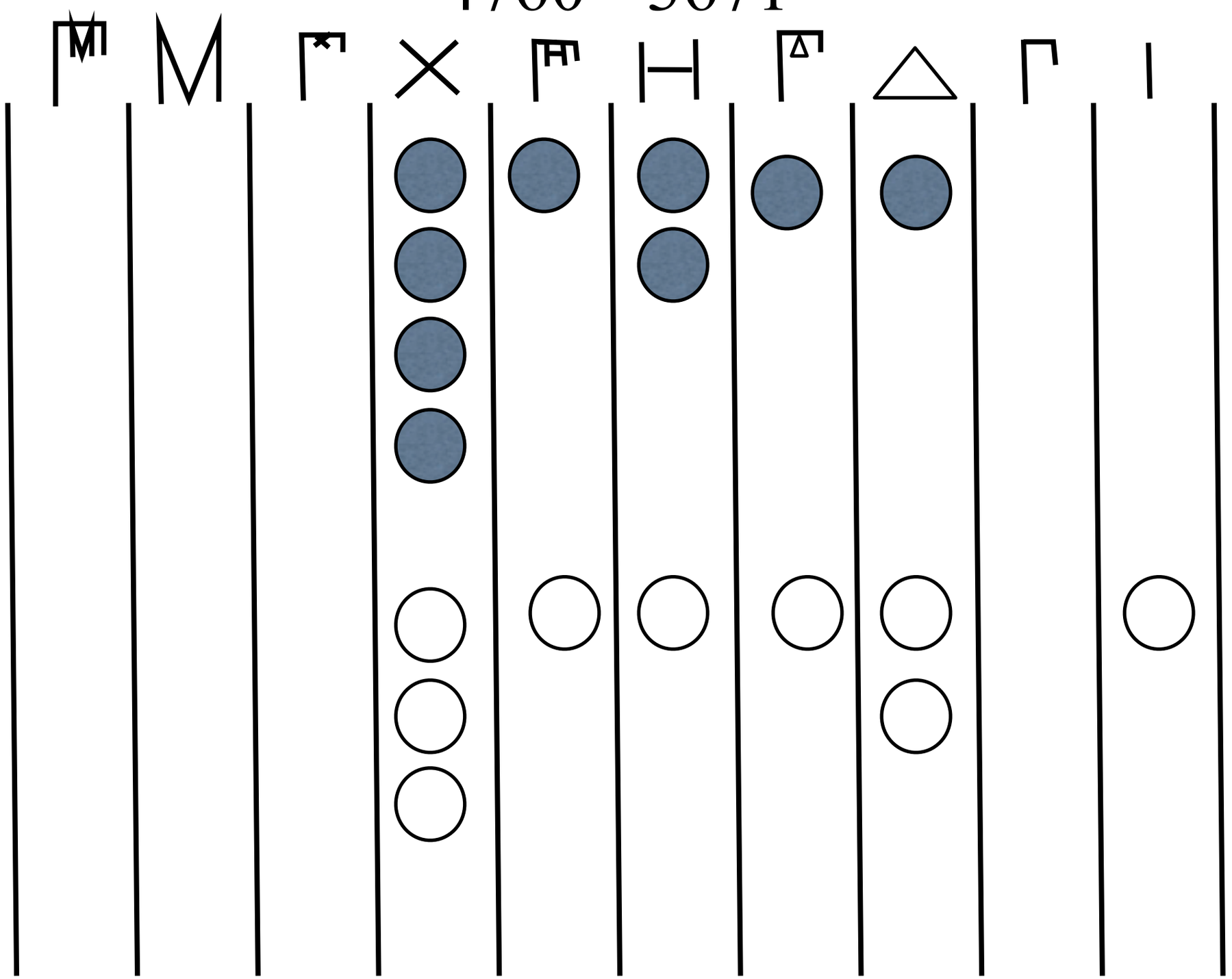

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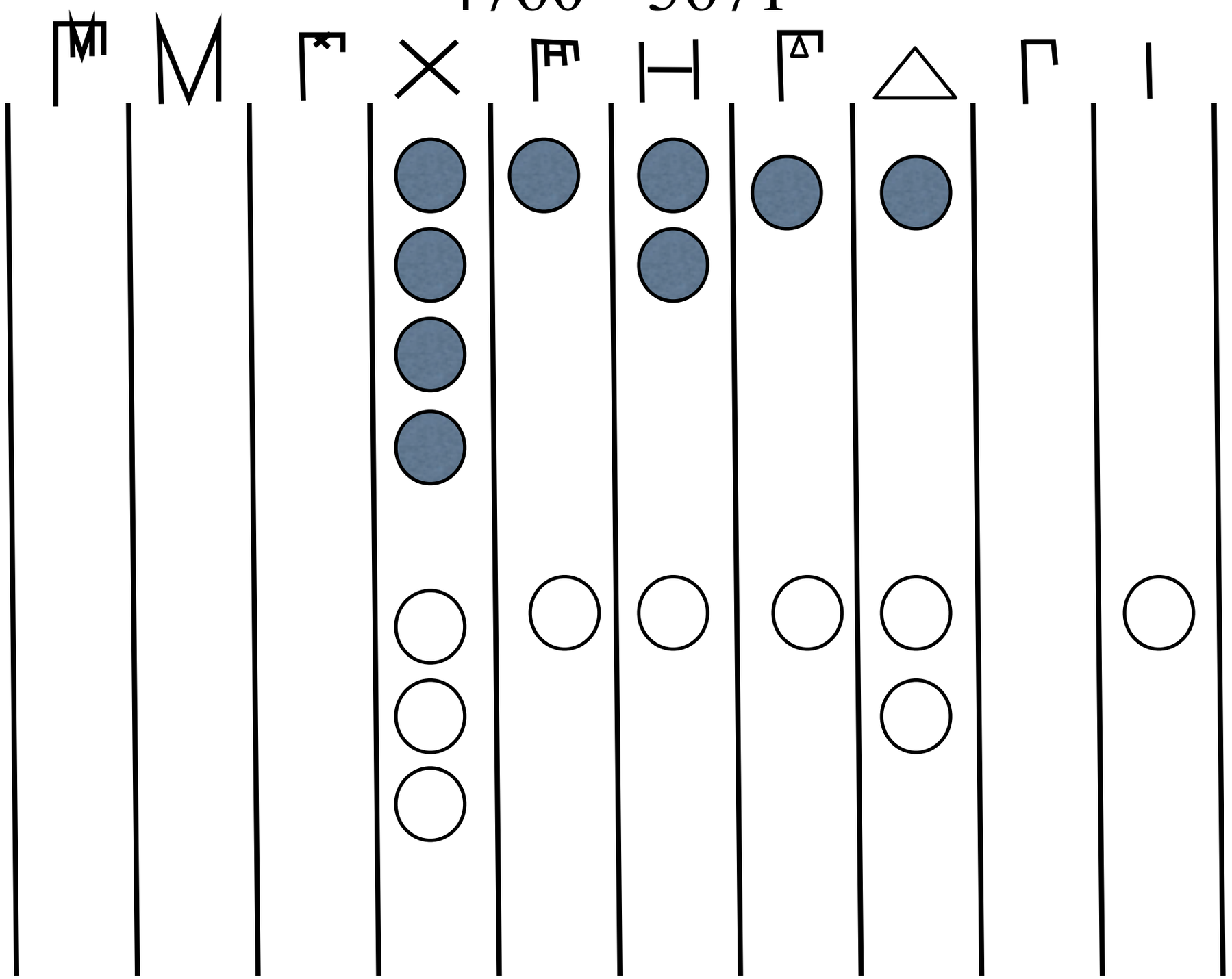

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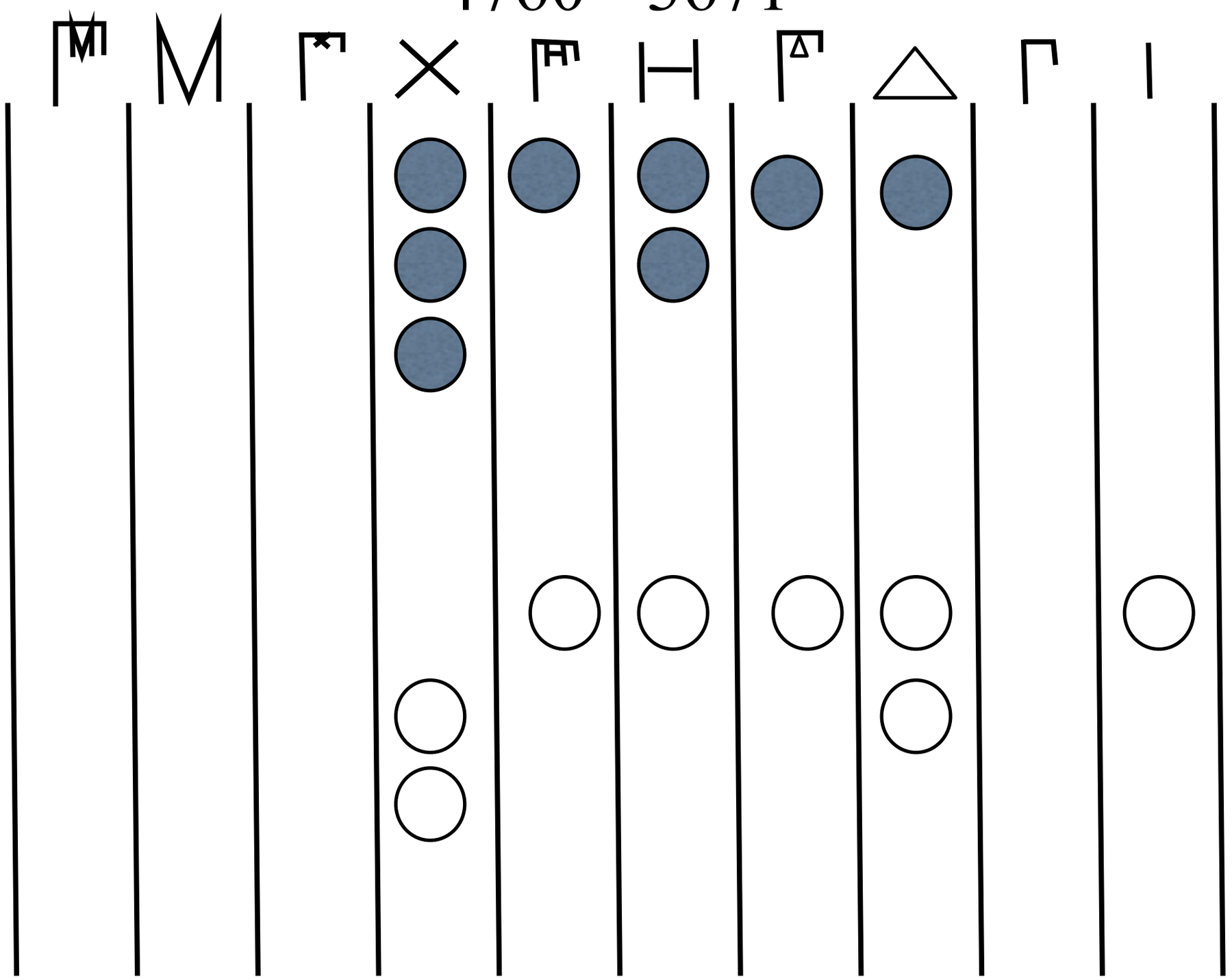

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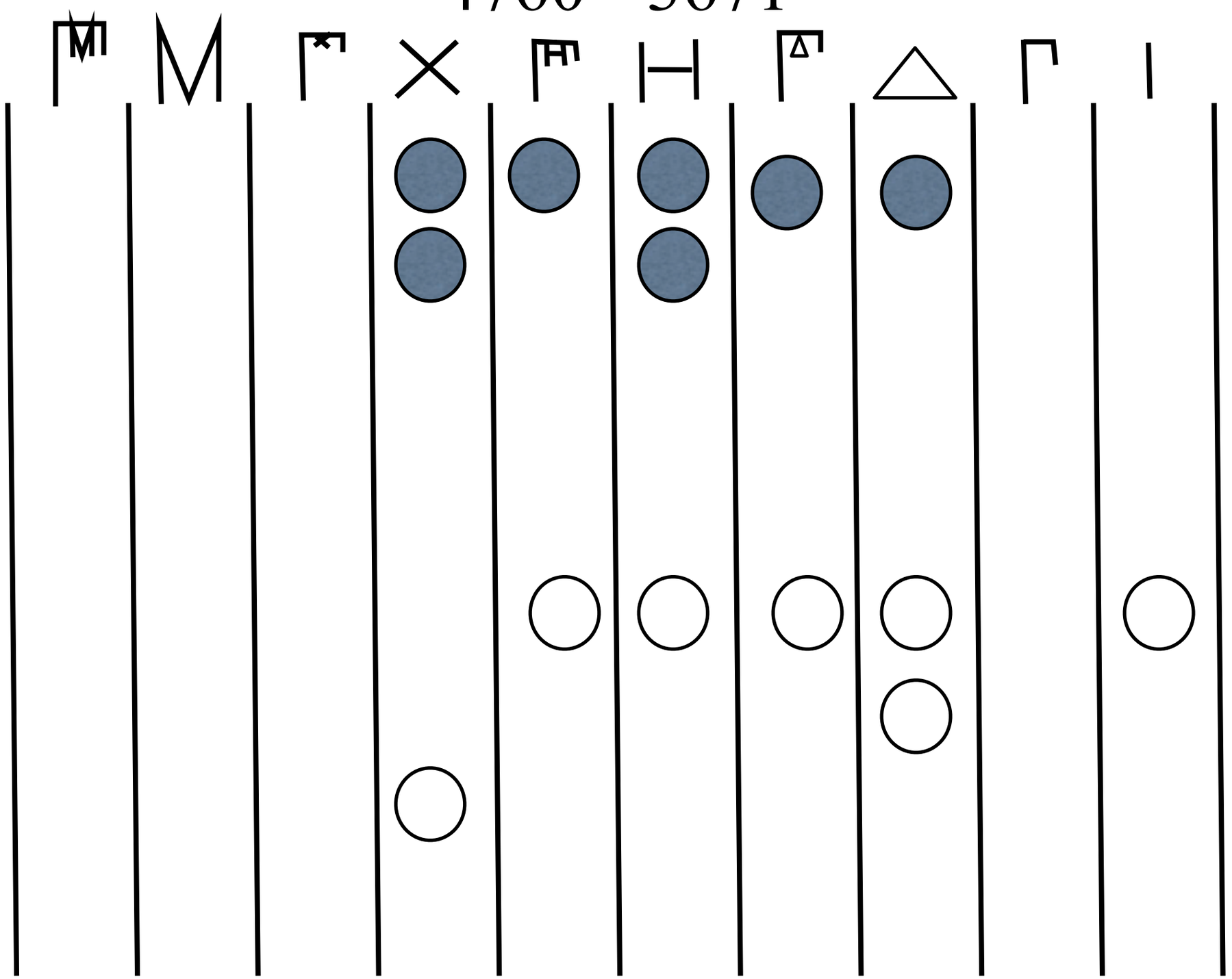

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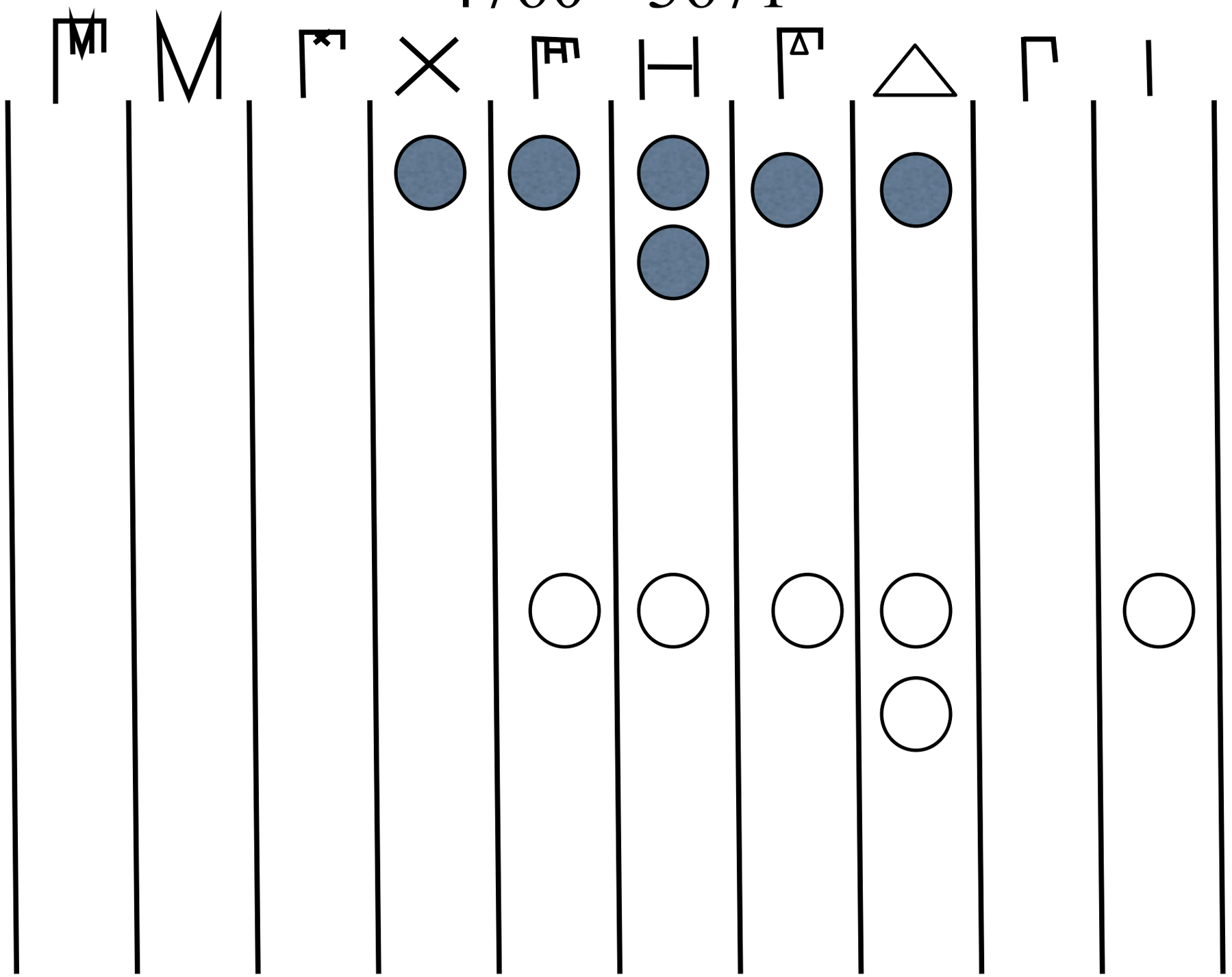

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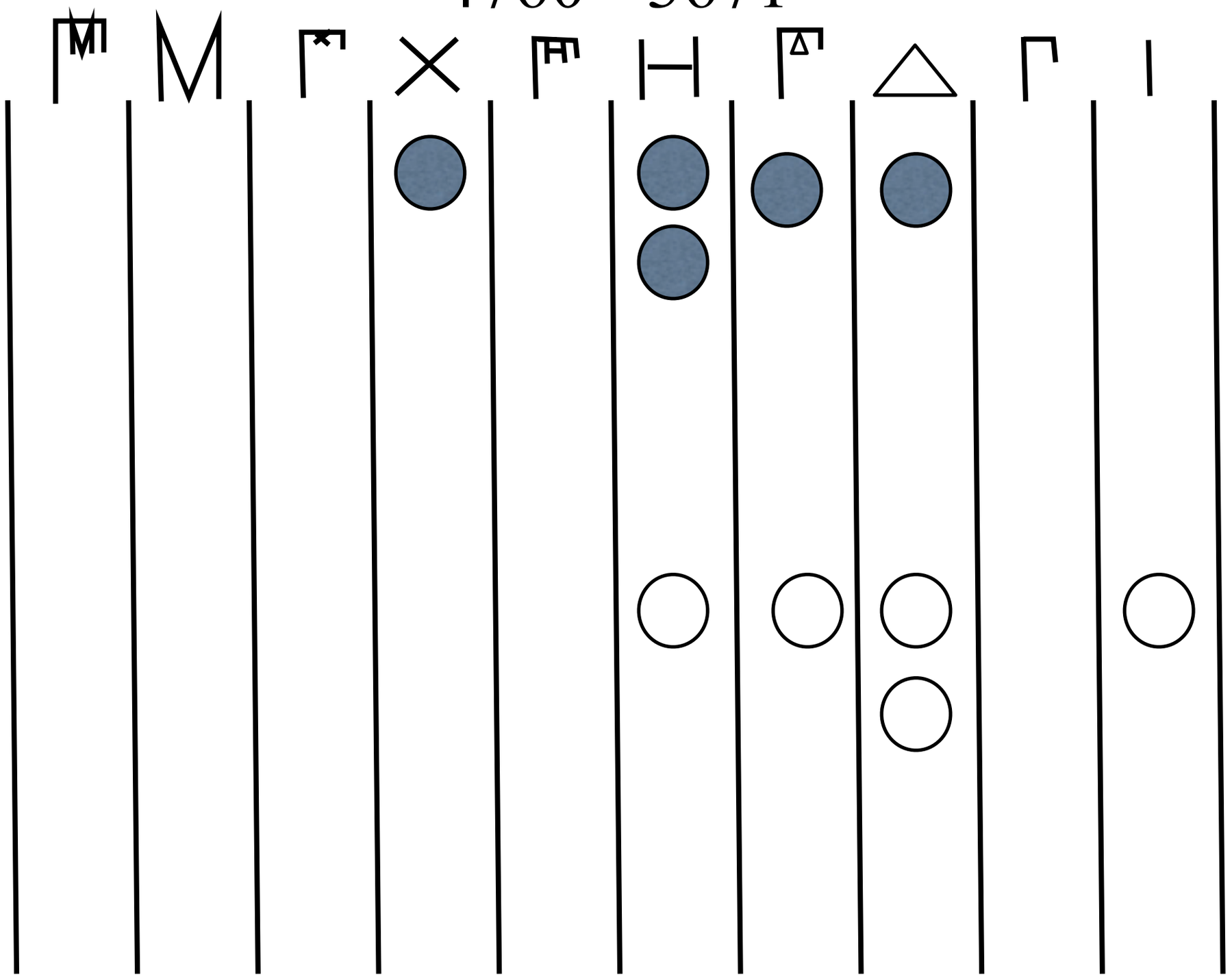

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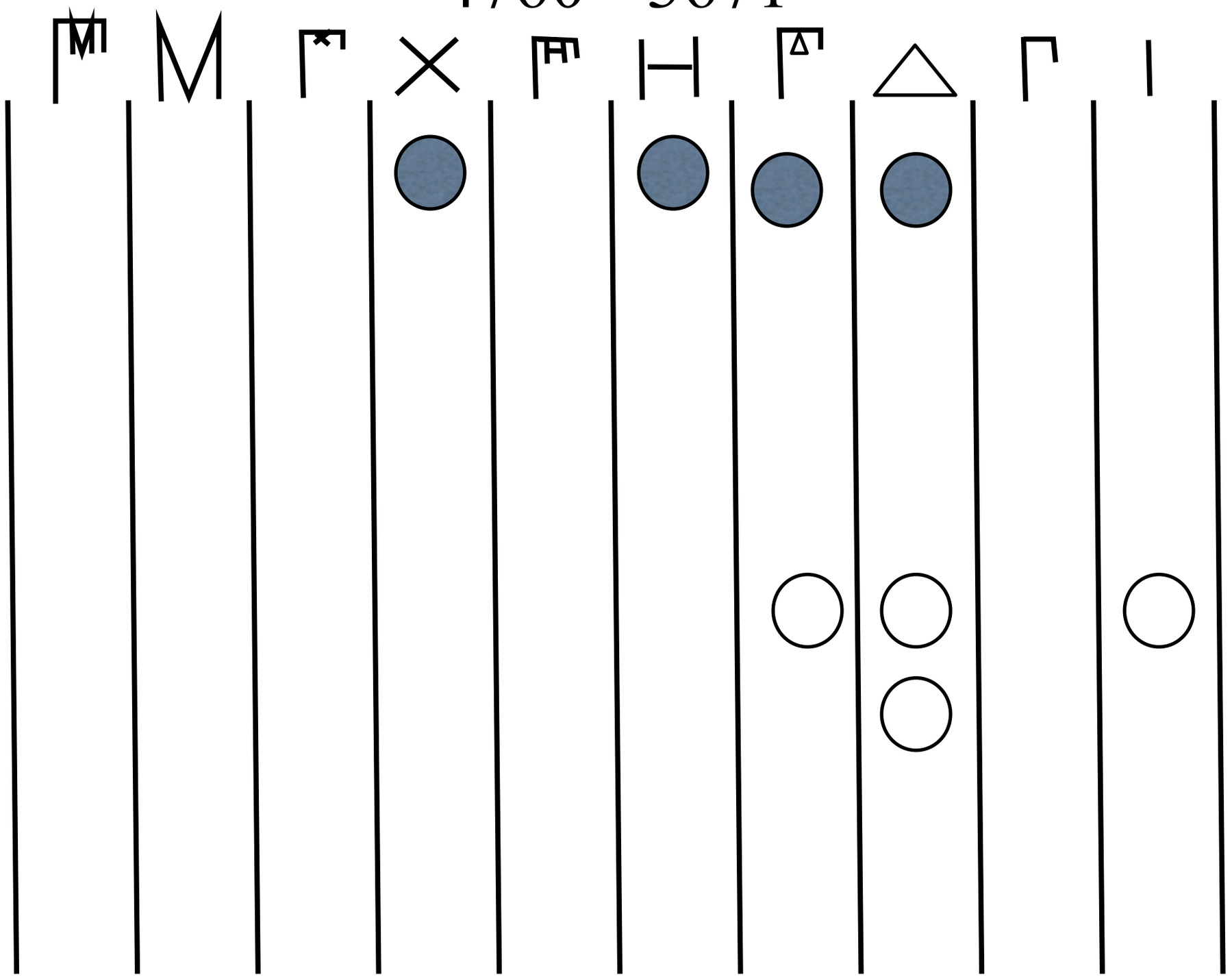

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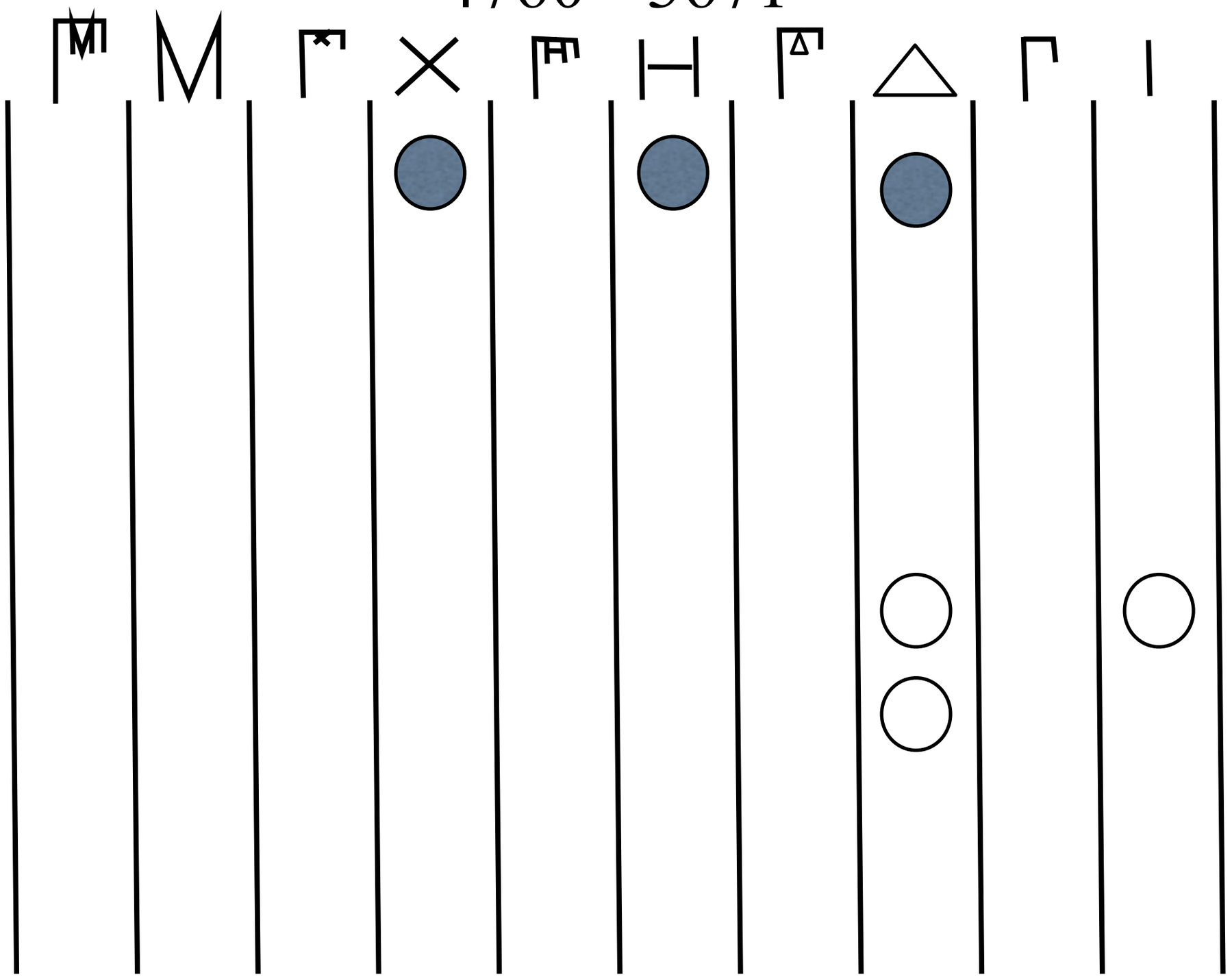

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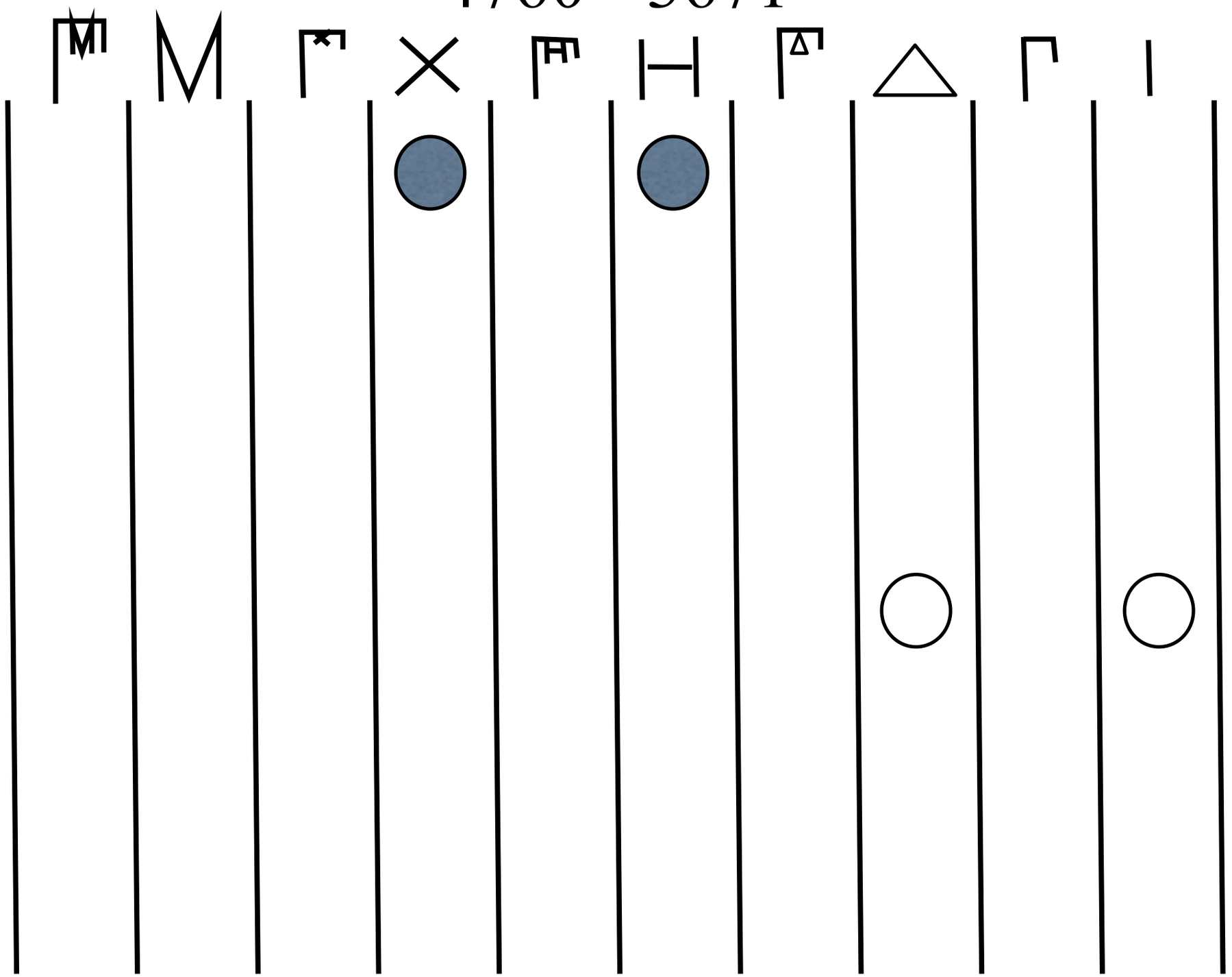

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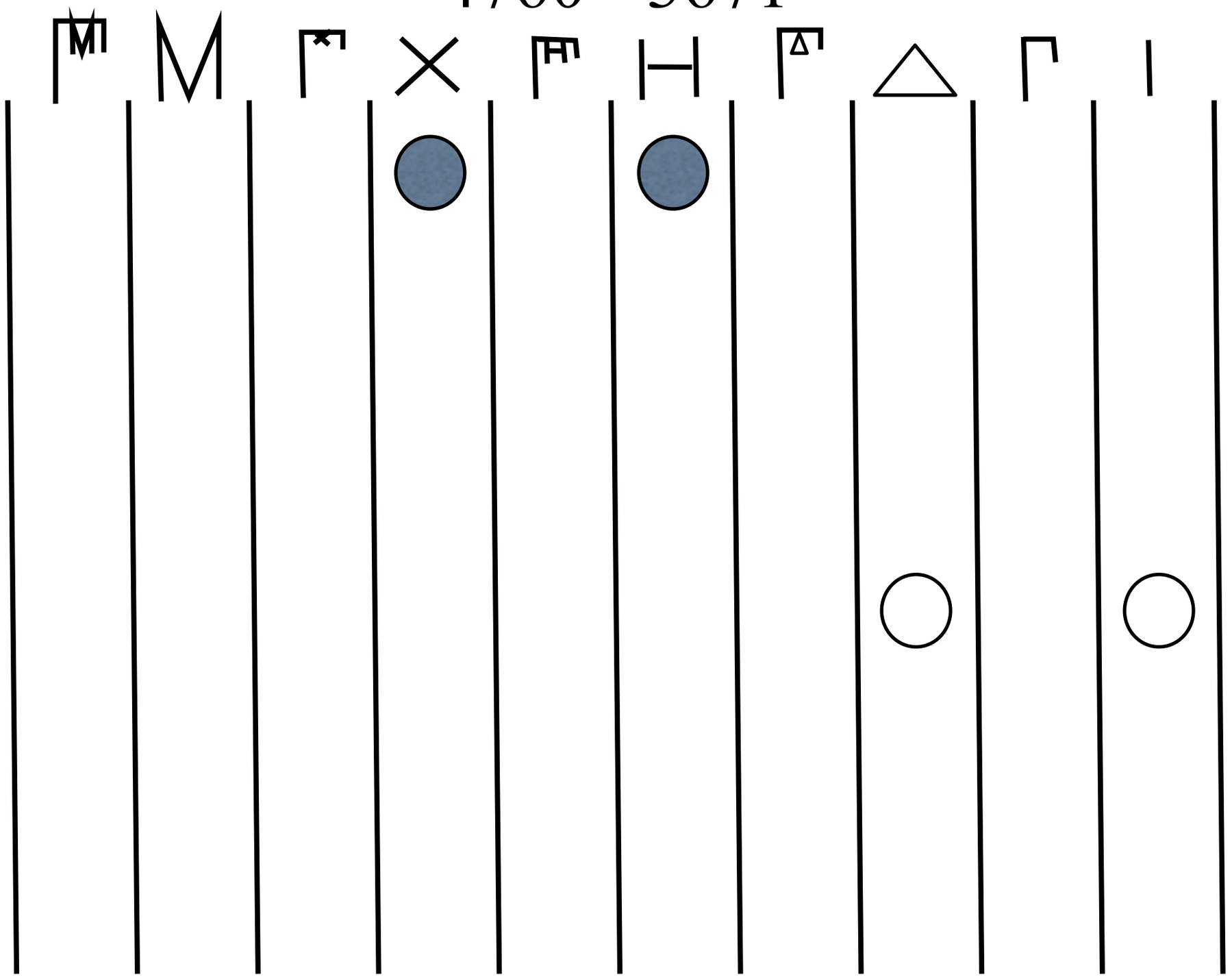

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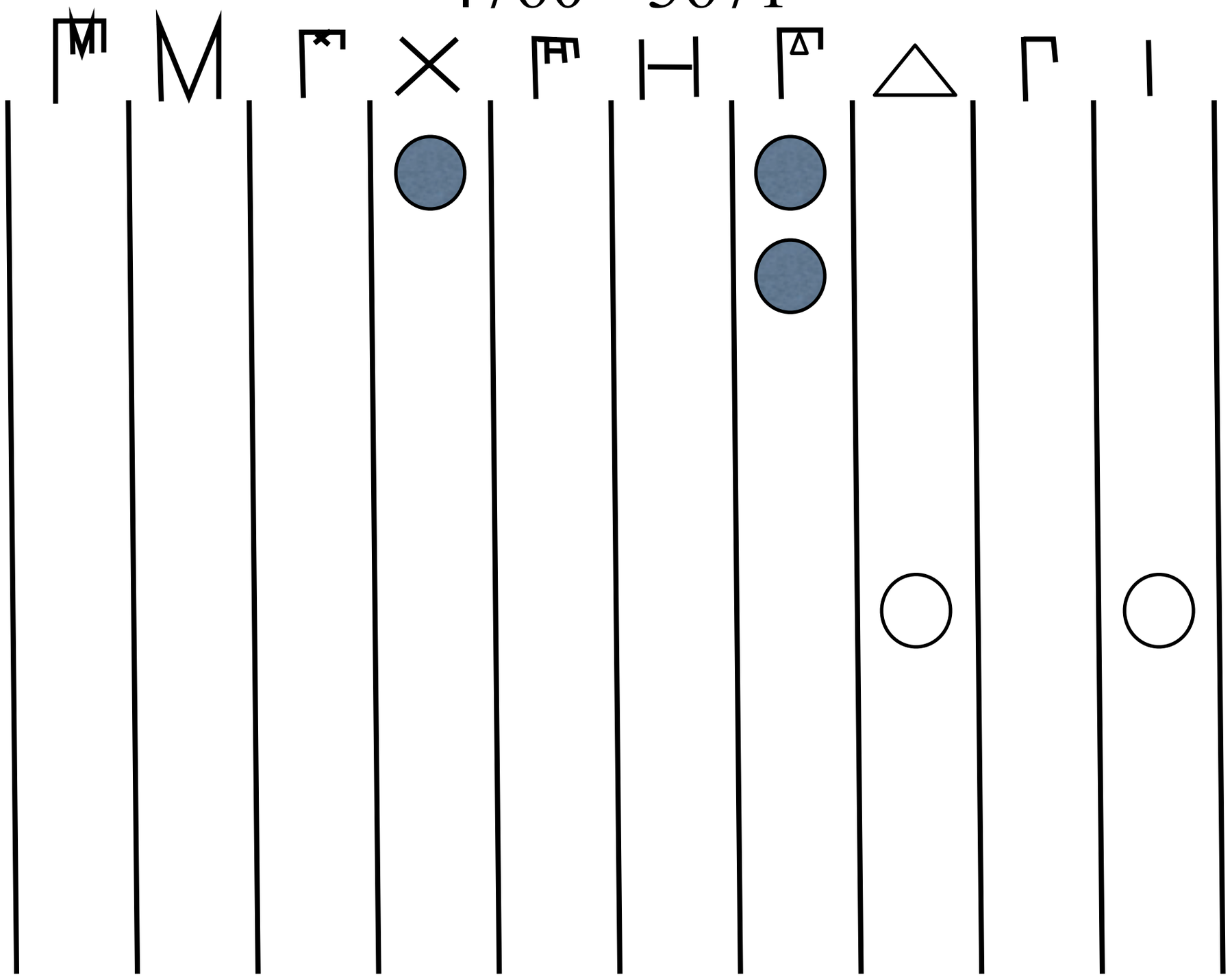

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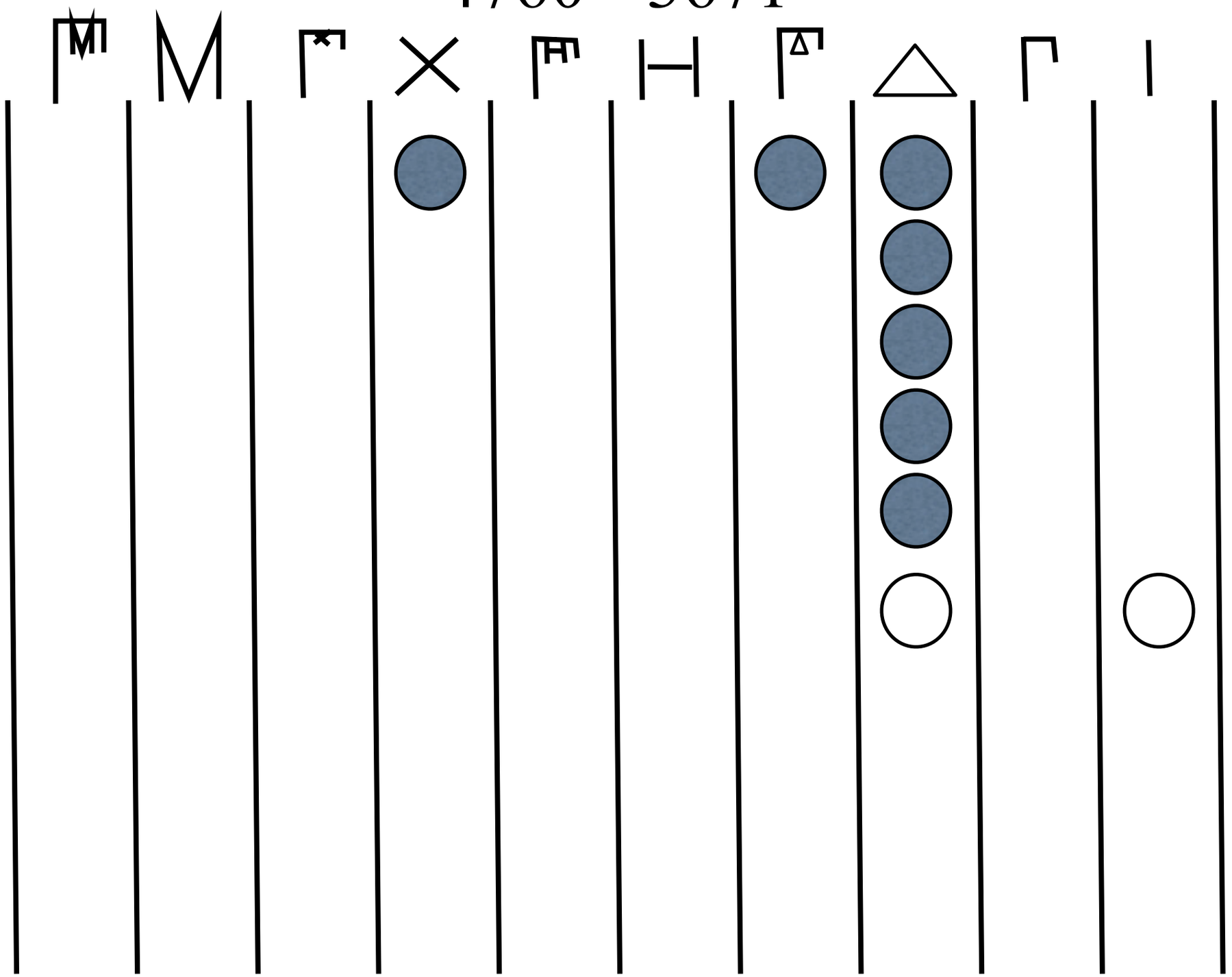

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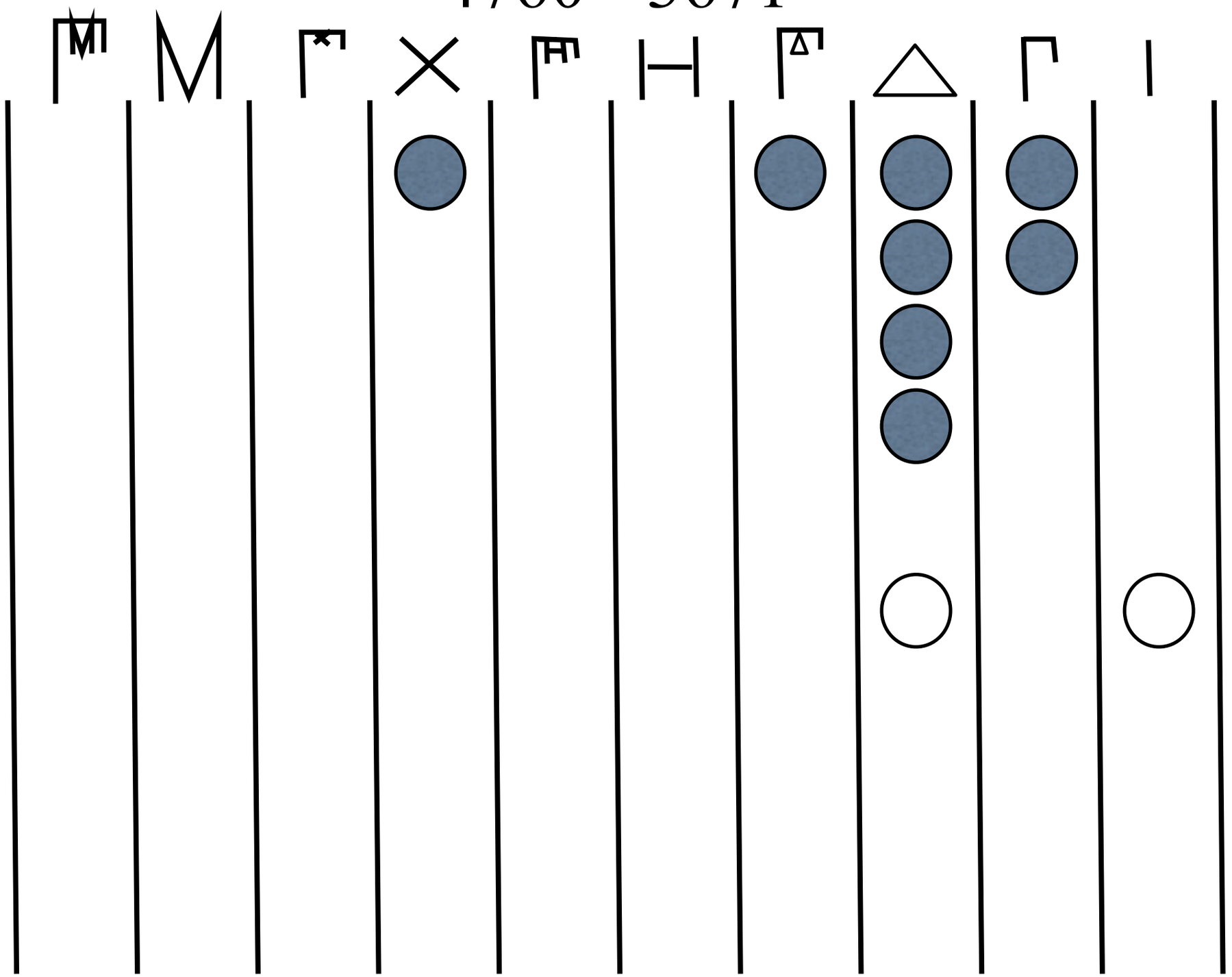

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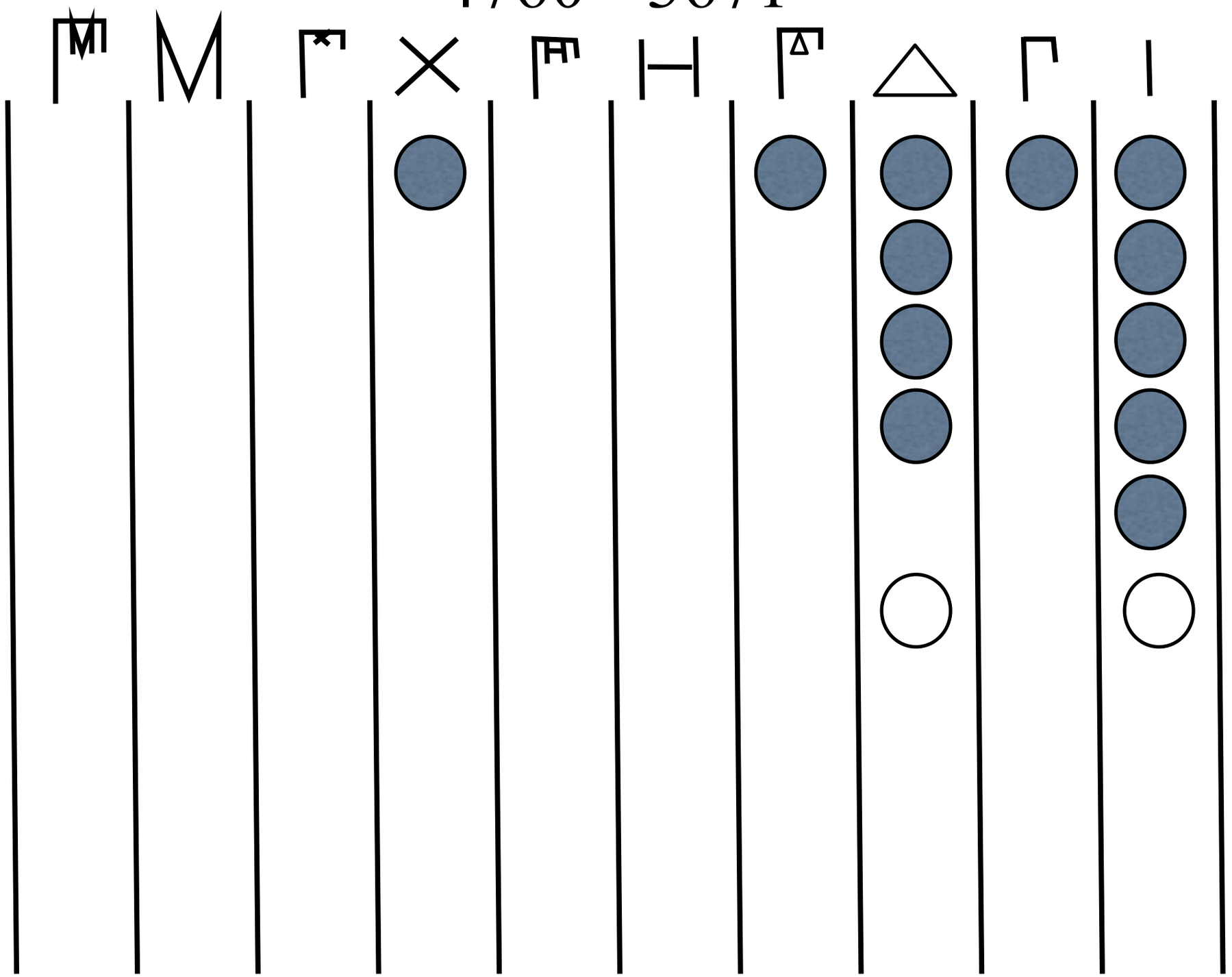

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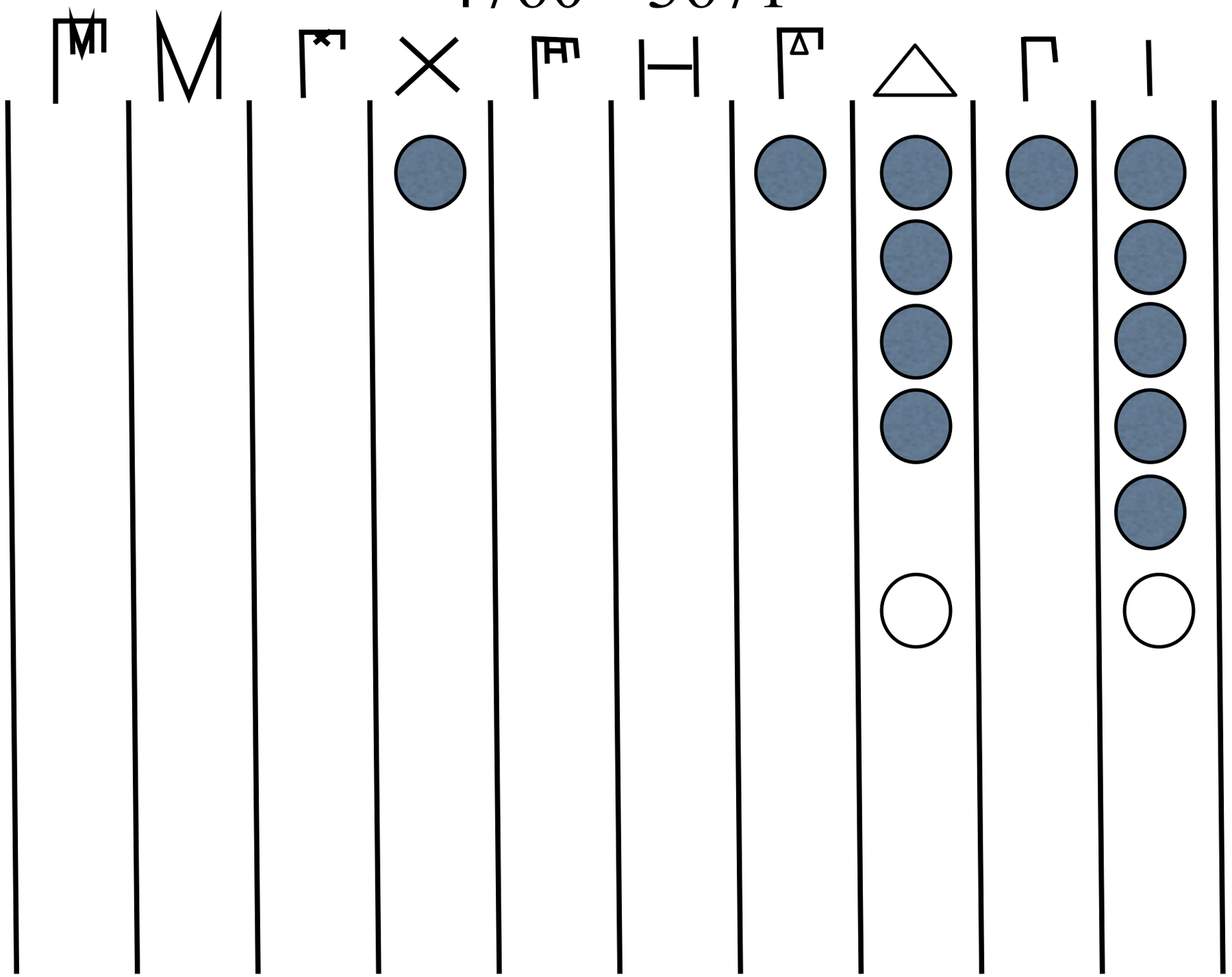

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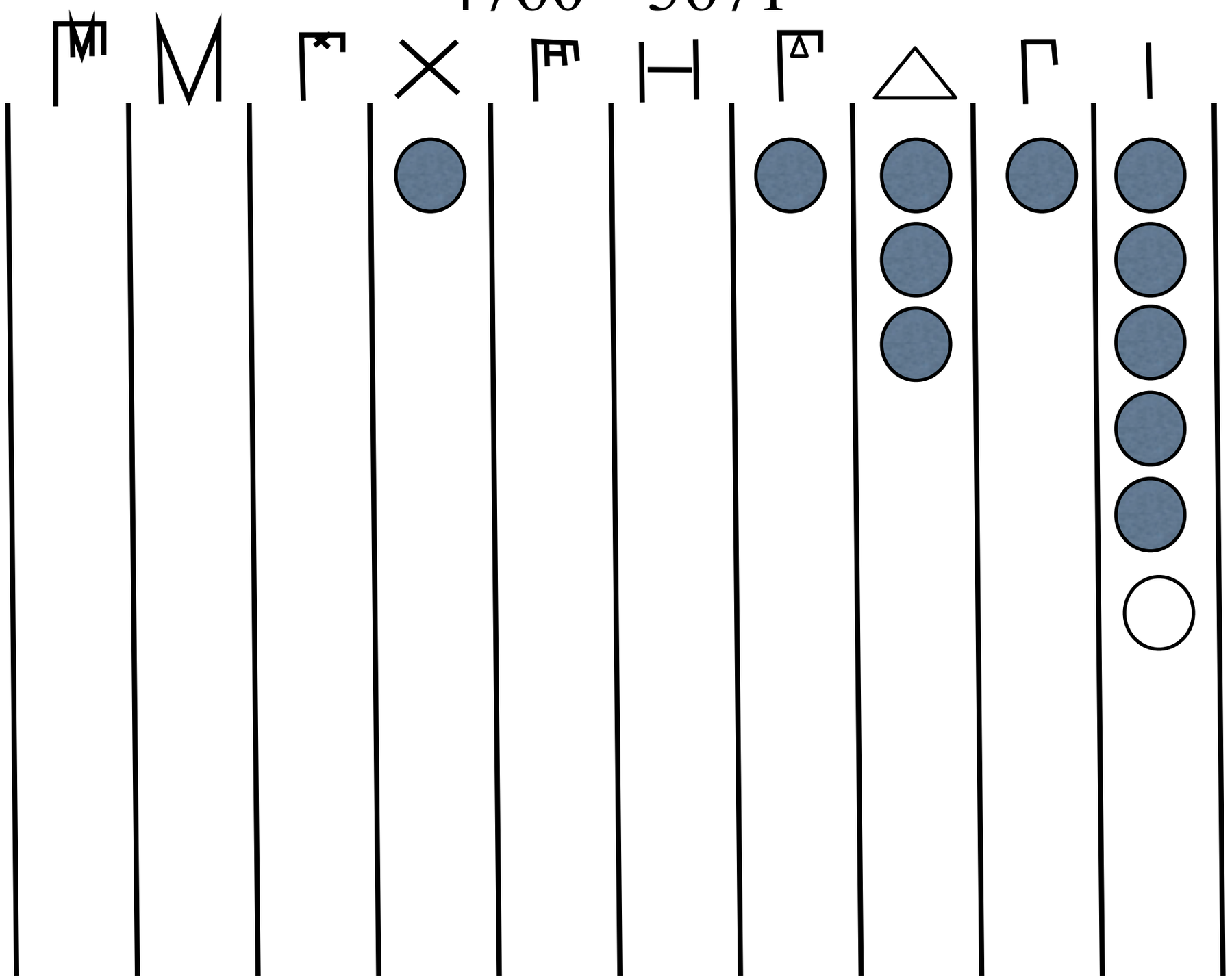

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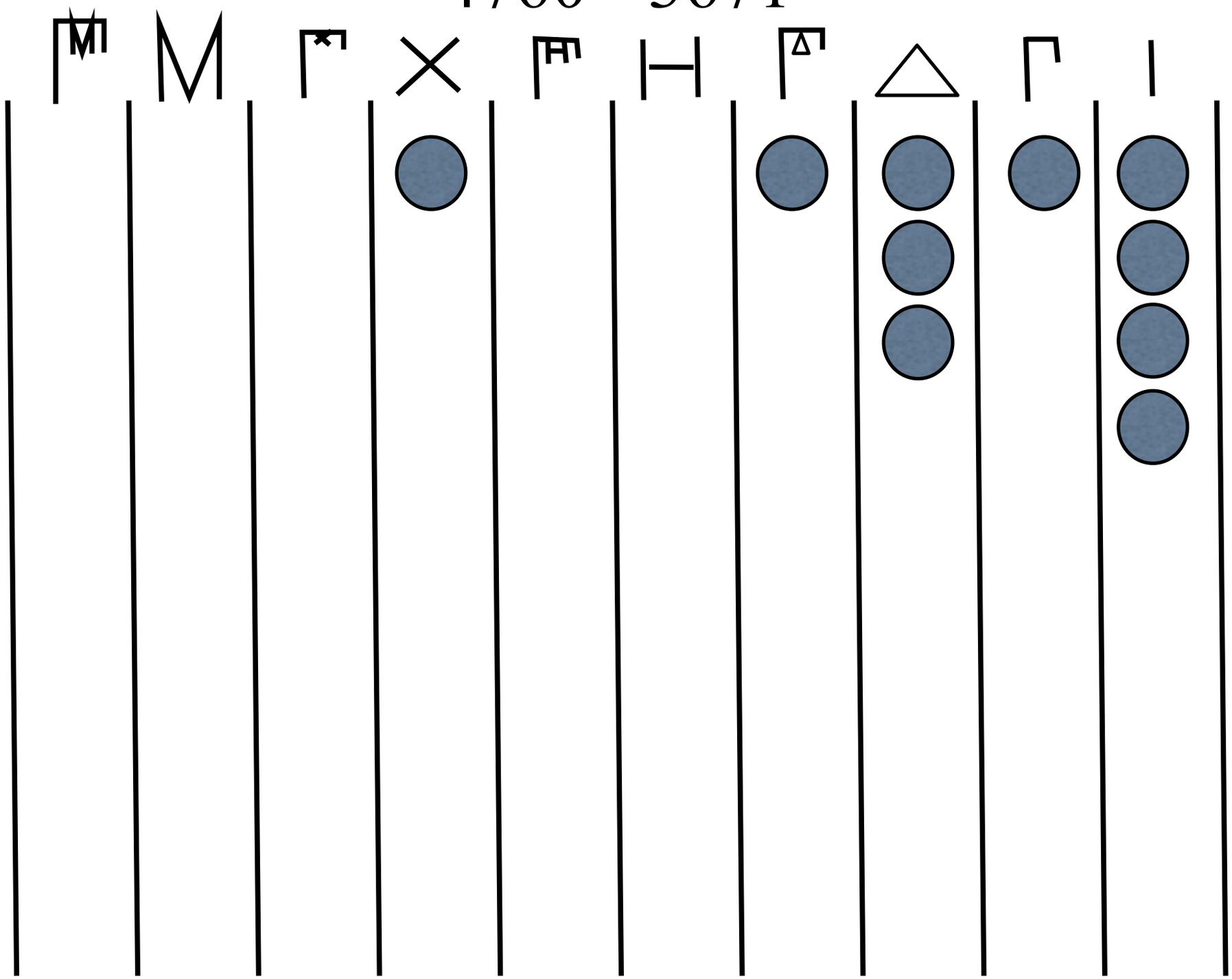

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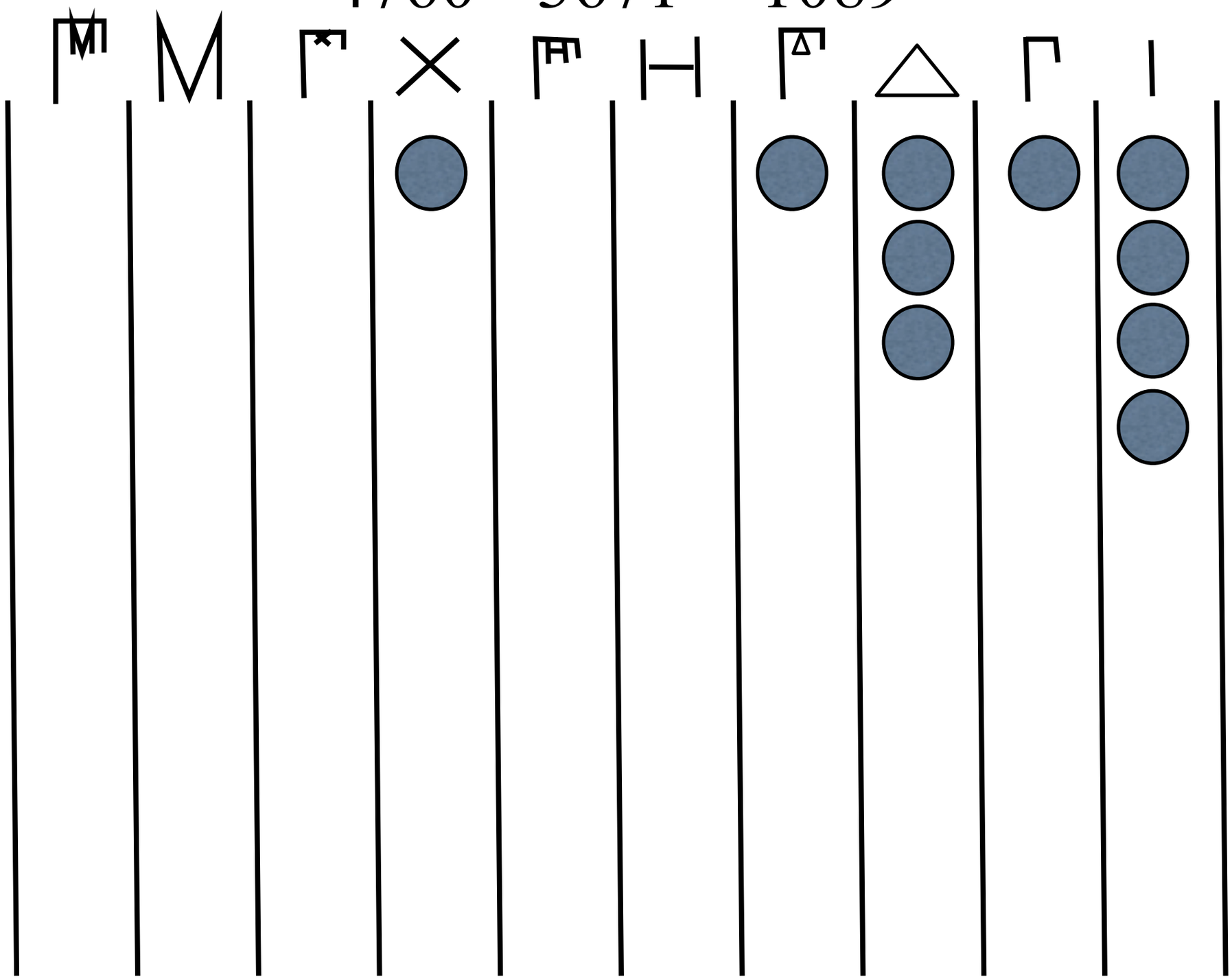

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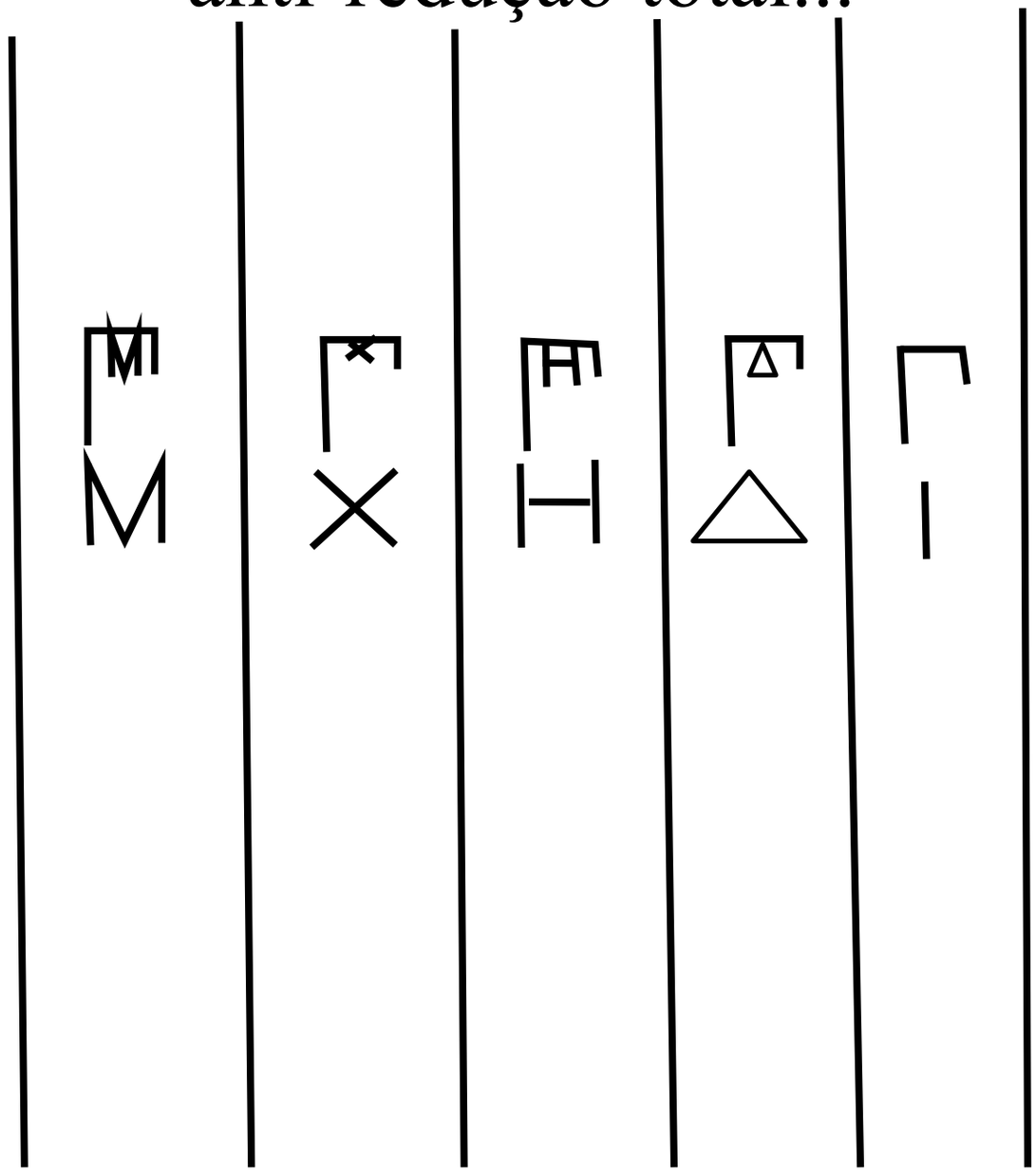
em colunas decimais temos de usar anti-redução total...



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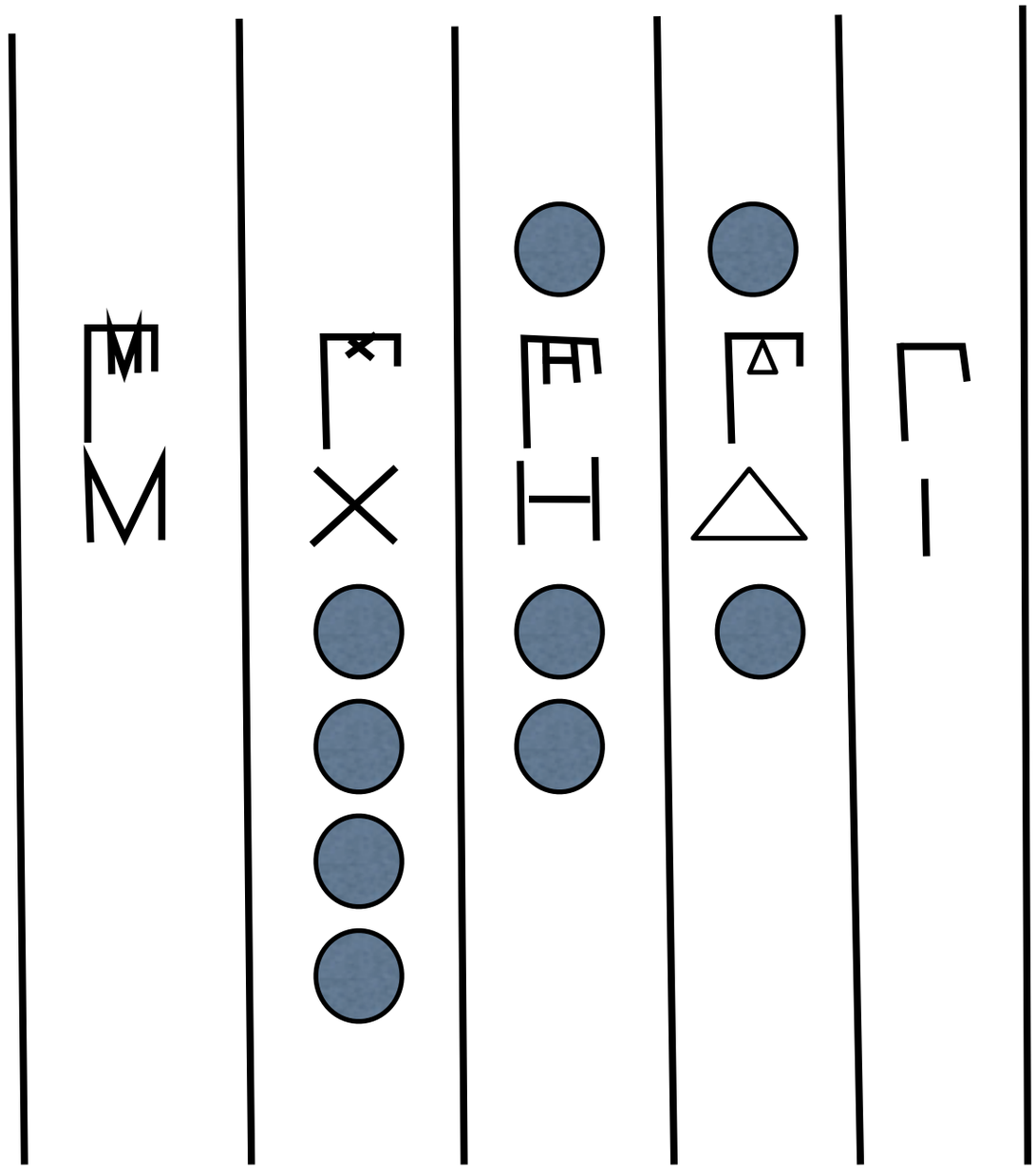

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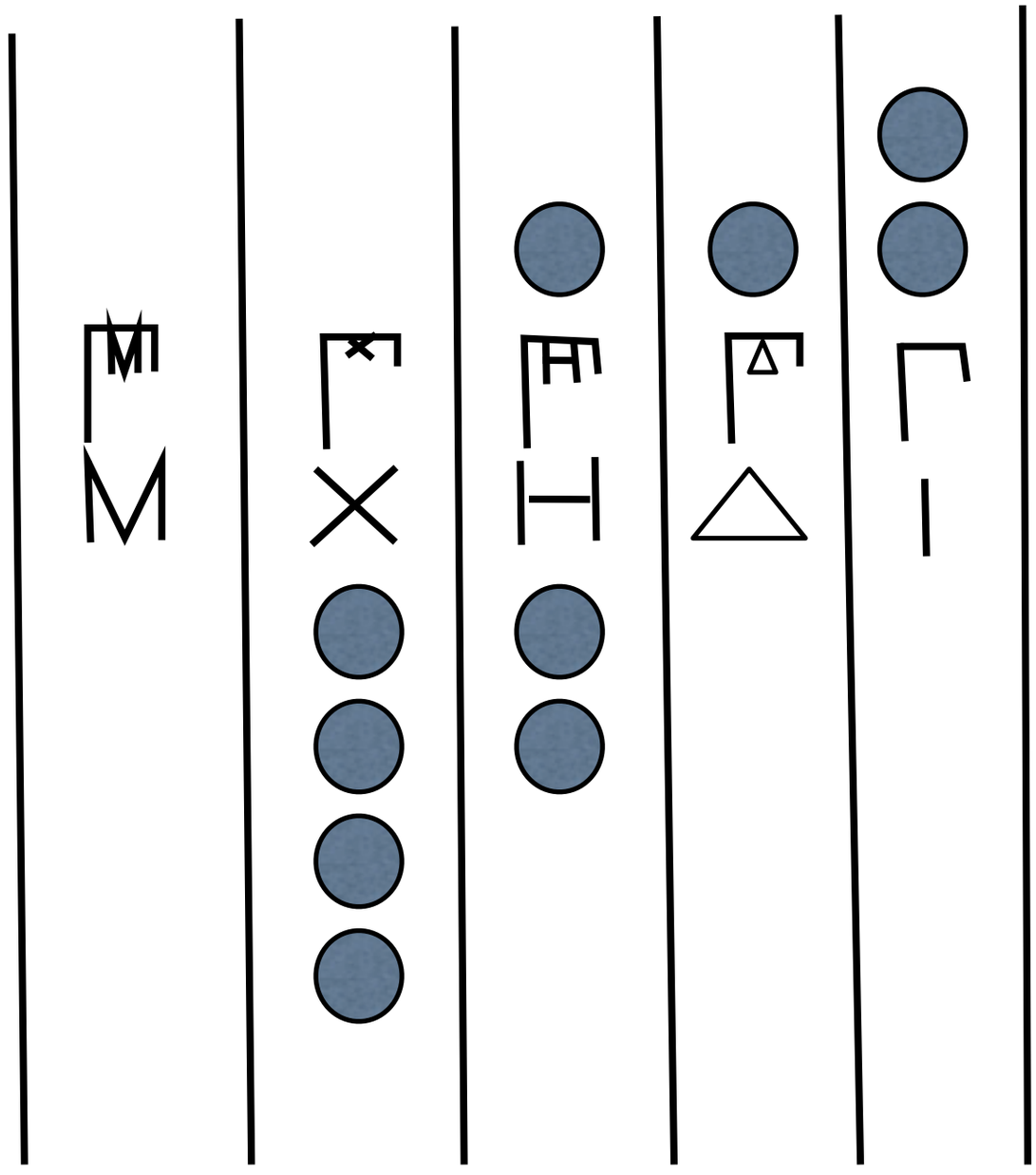

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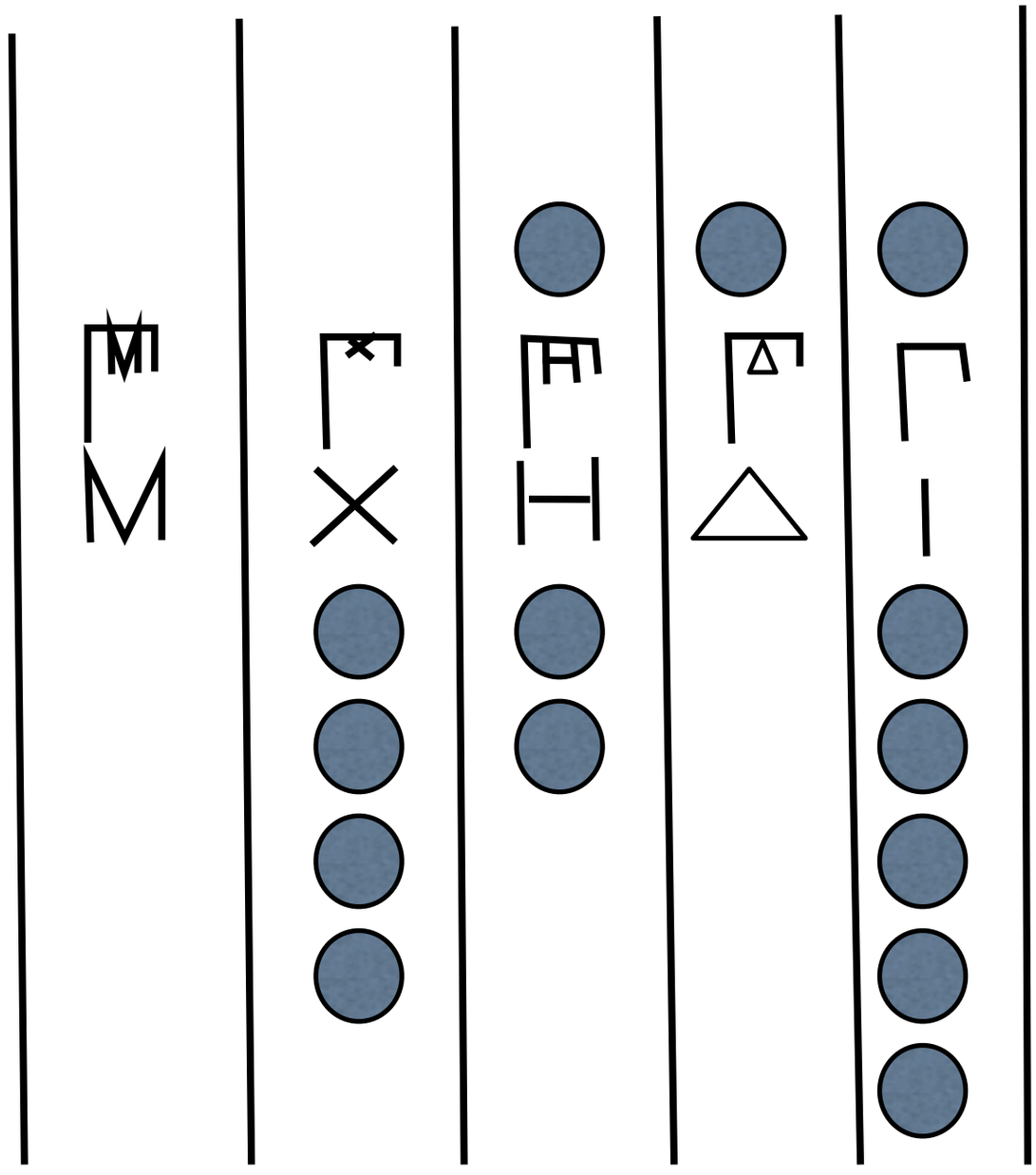

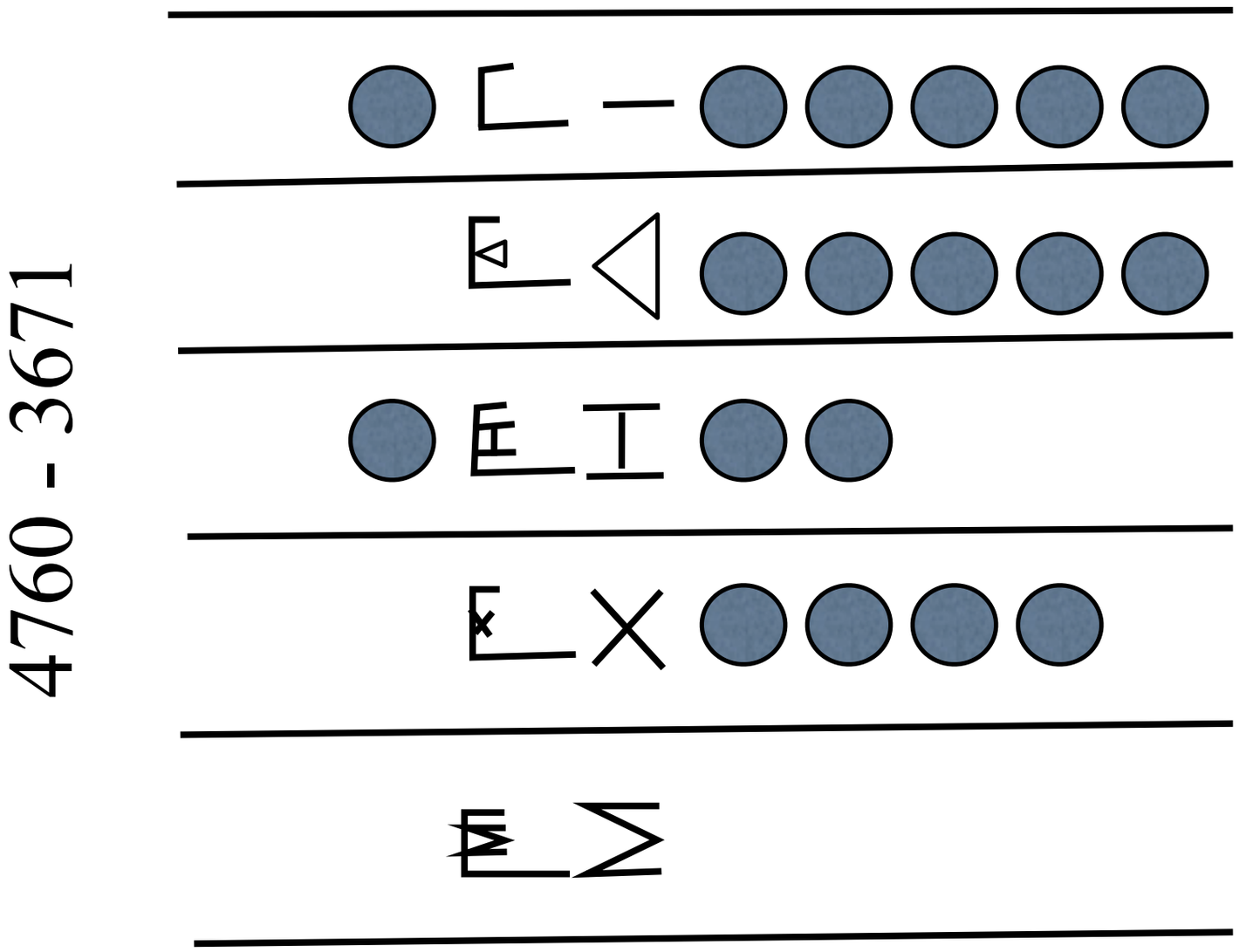

- 3671

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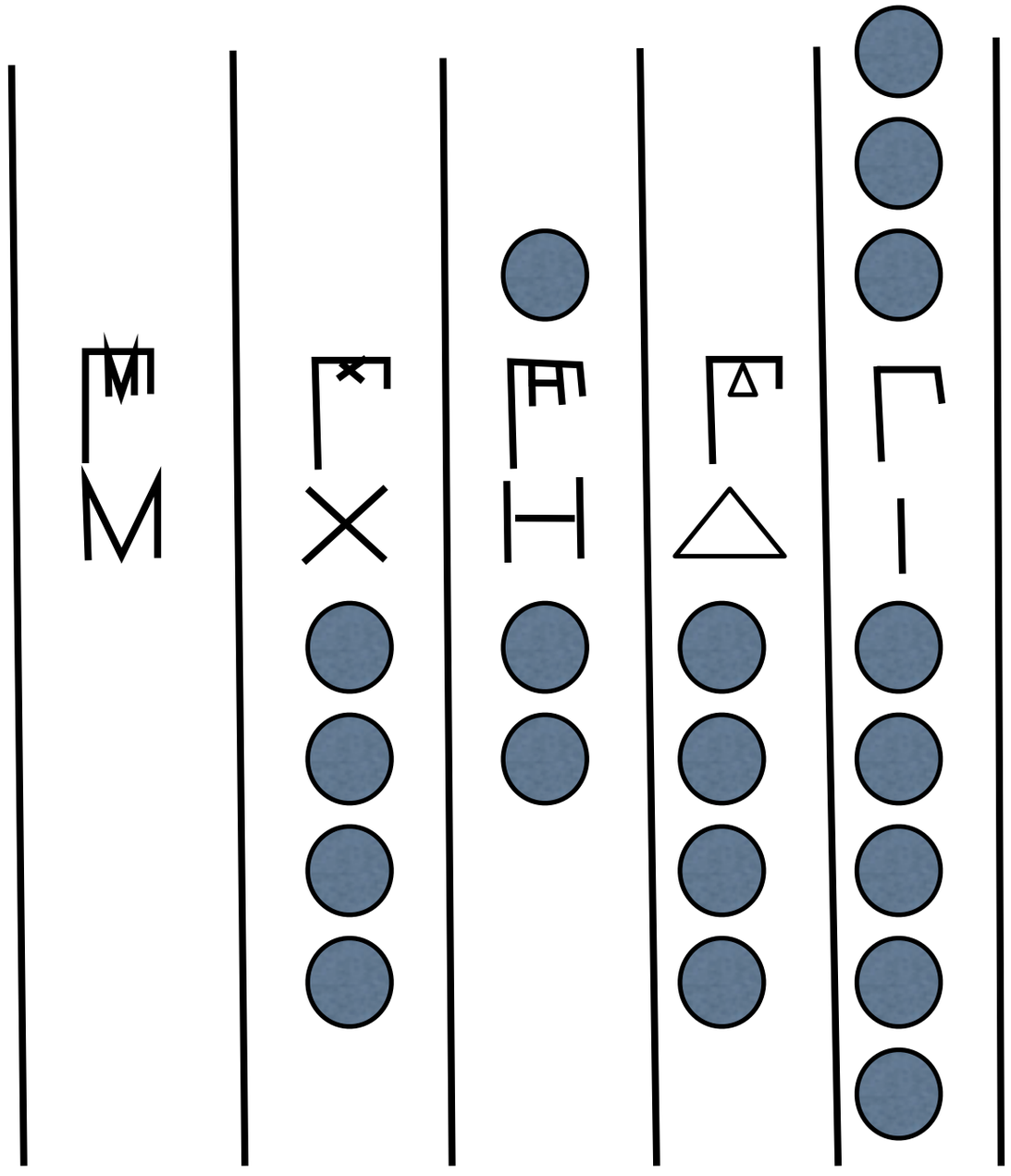
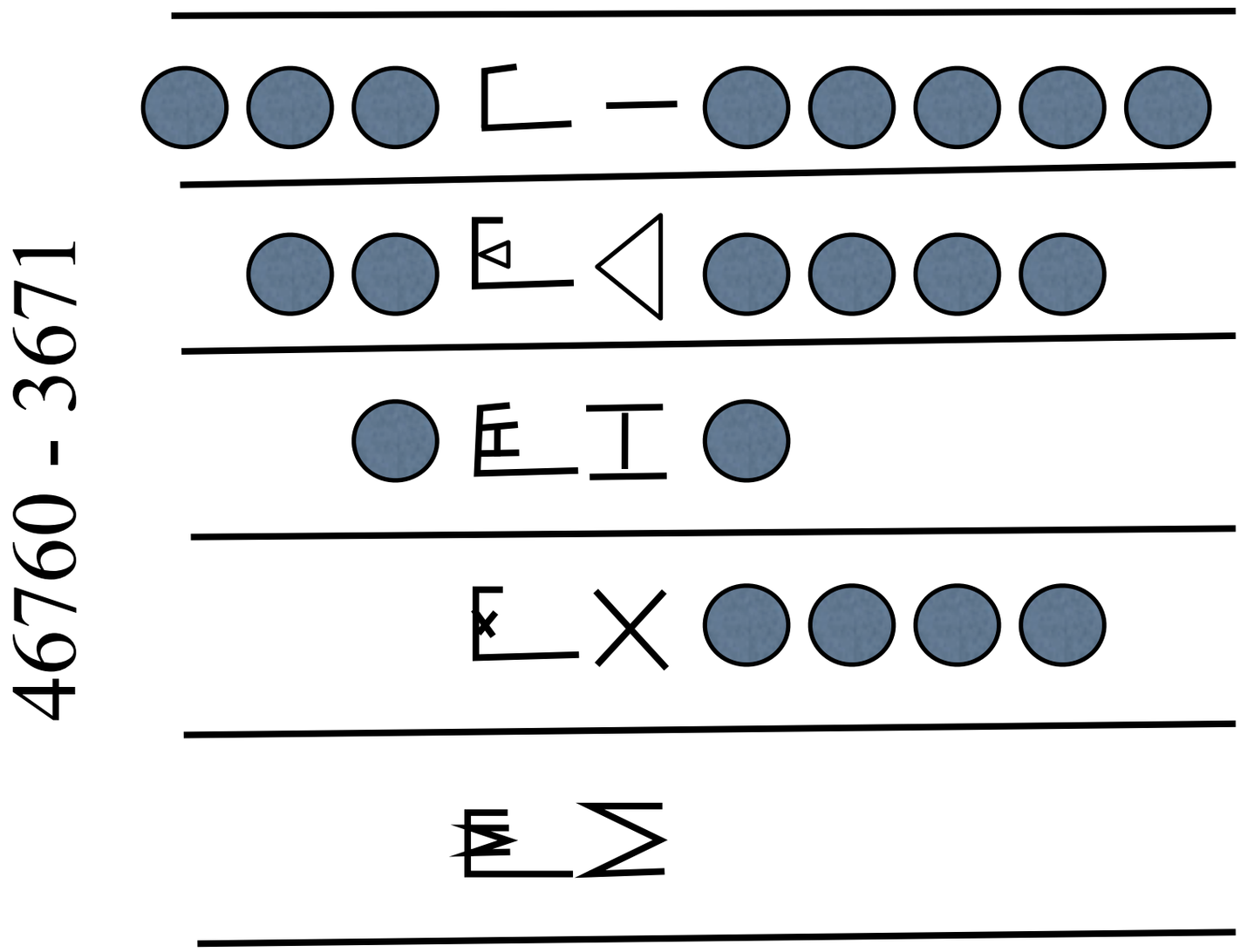
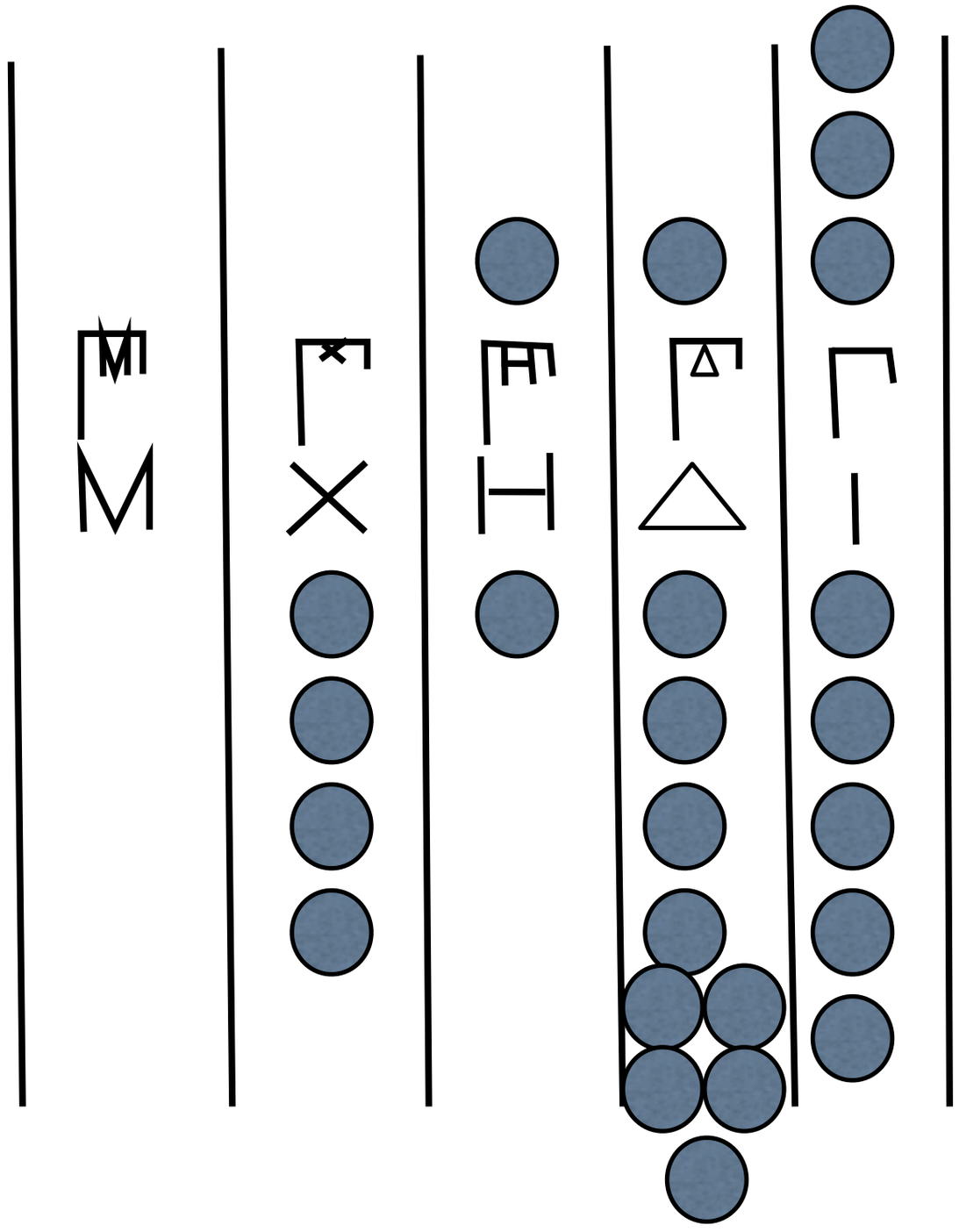
4760 - 3671





46760 - 3671

4760 - 3671



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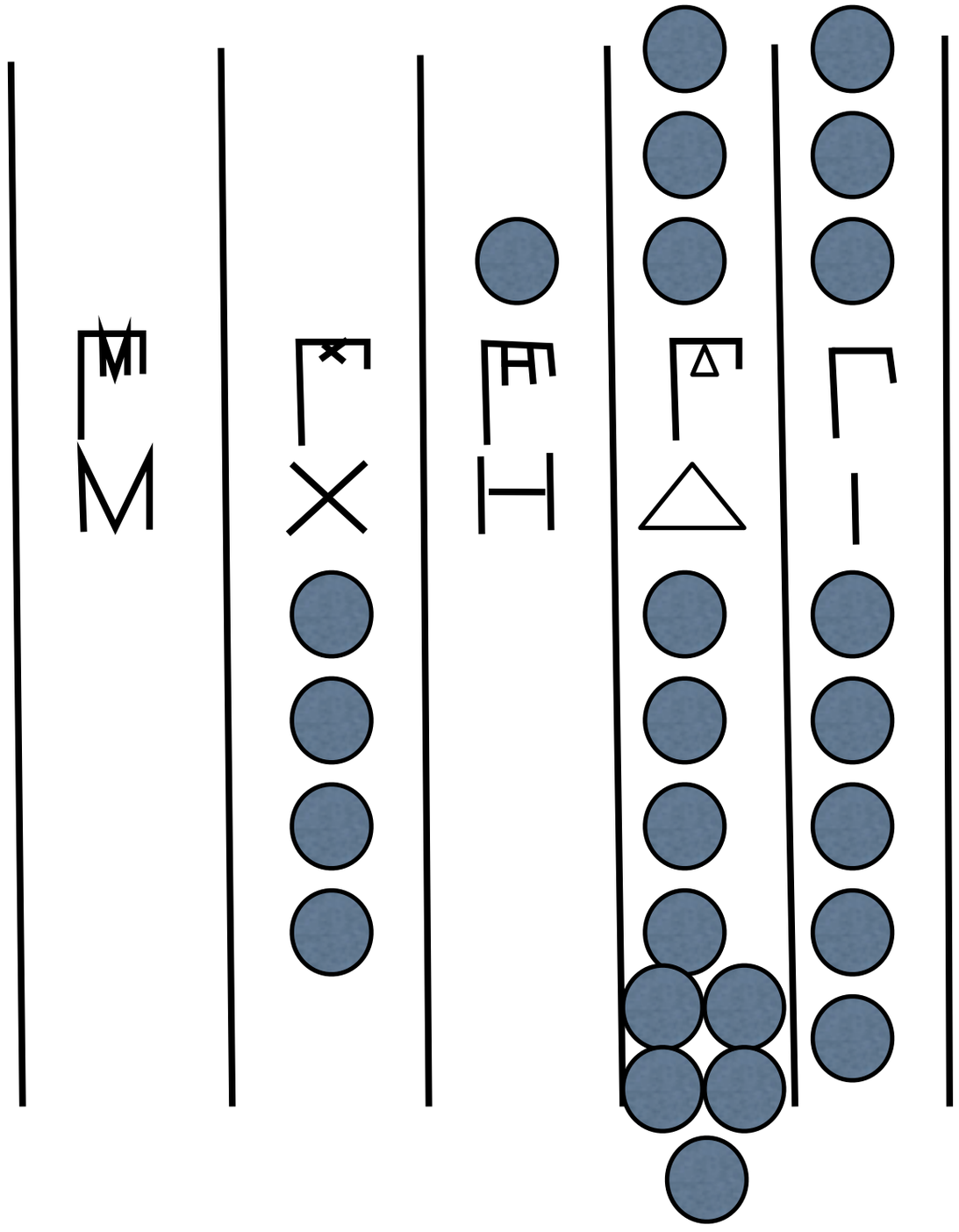

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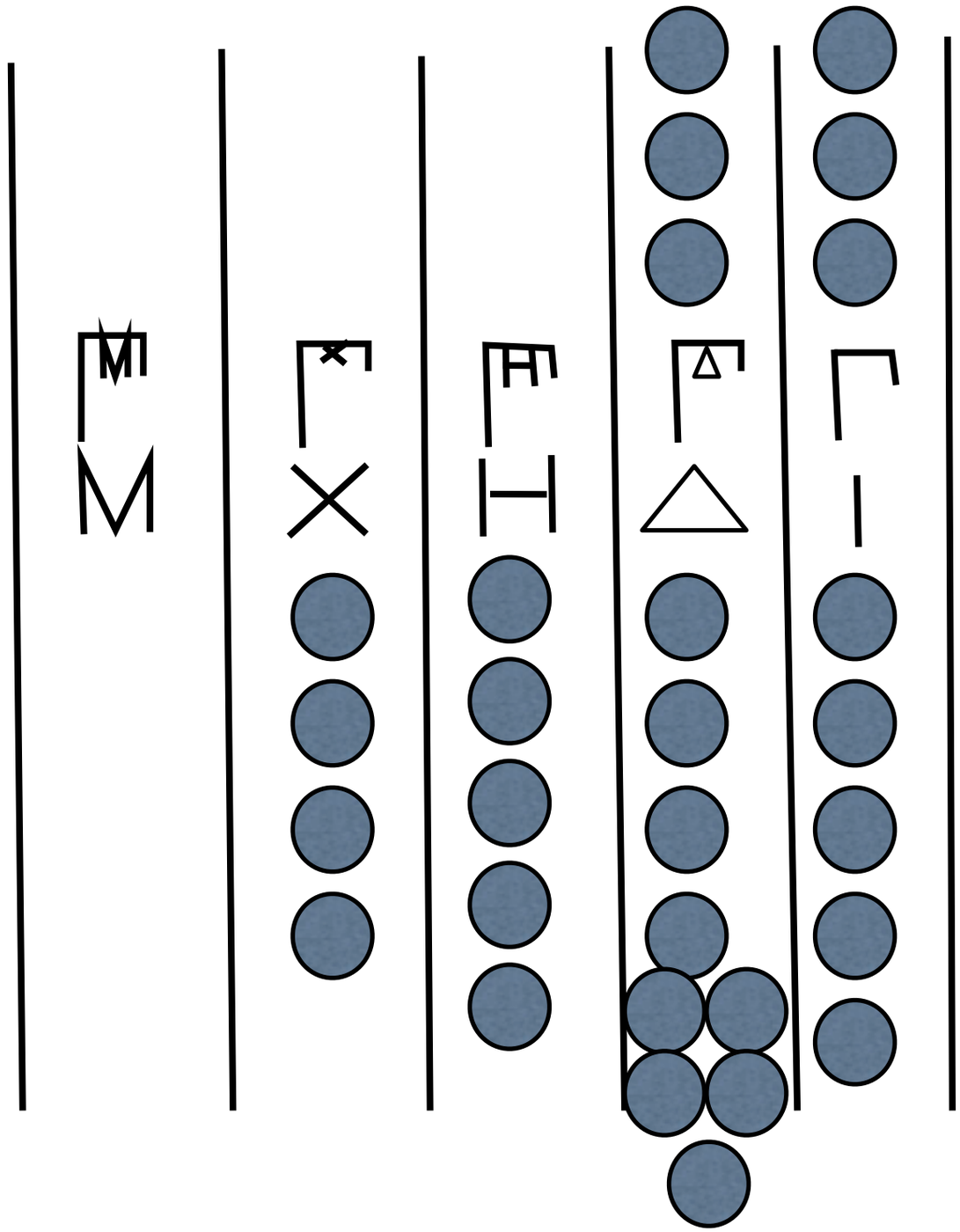

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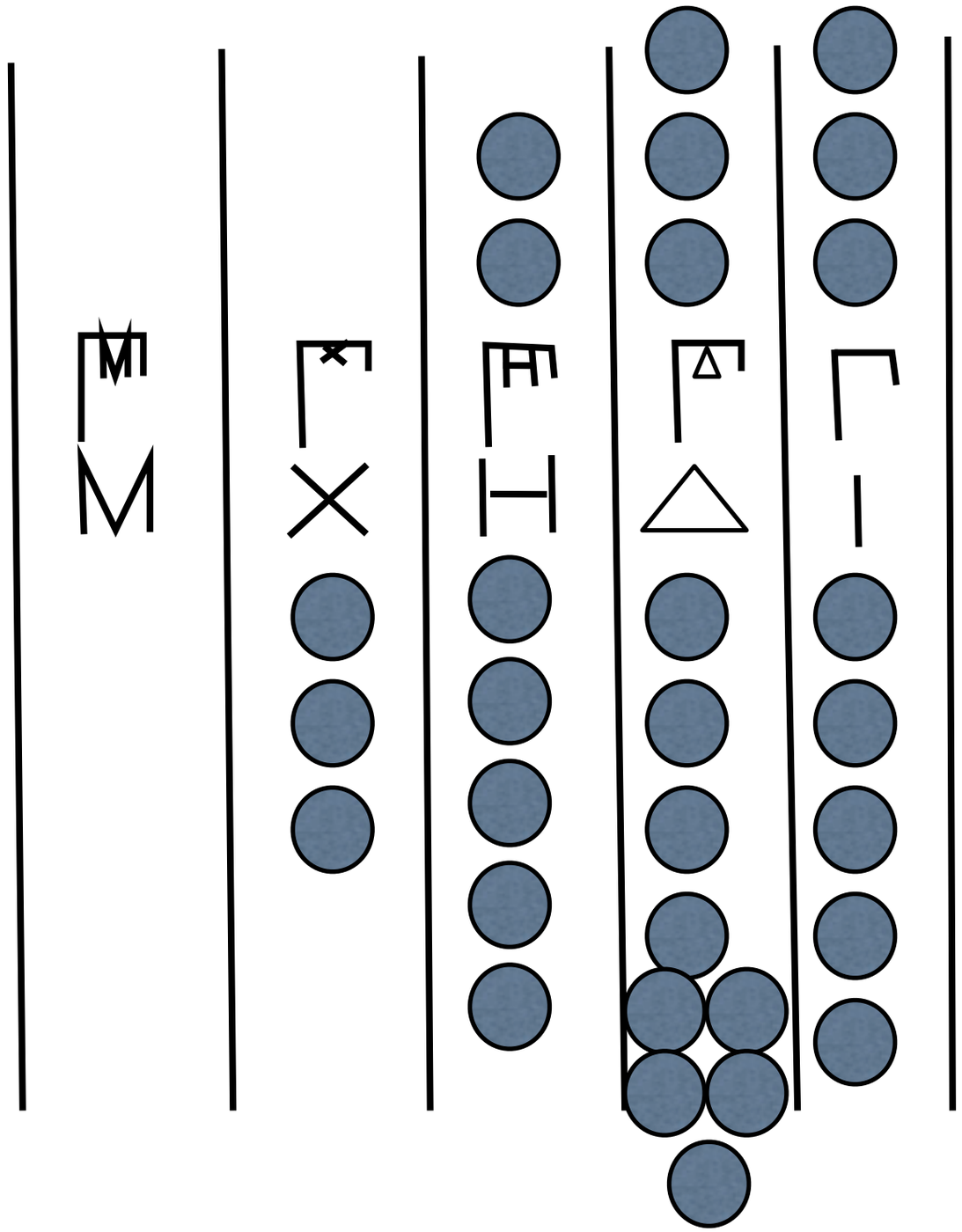
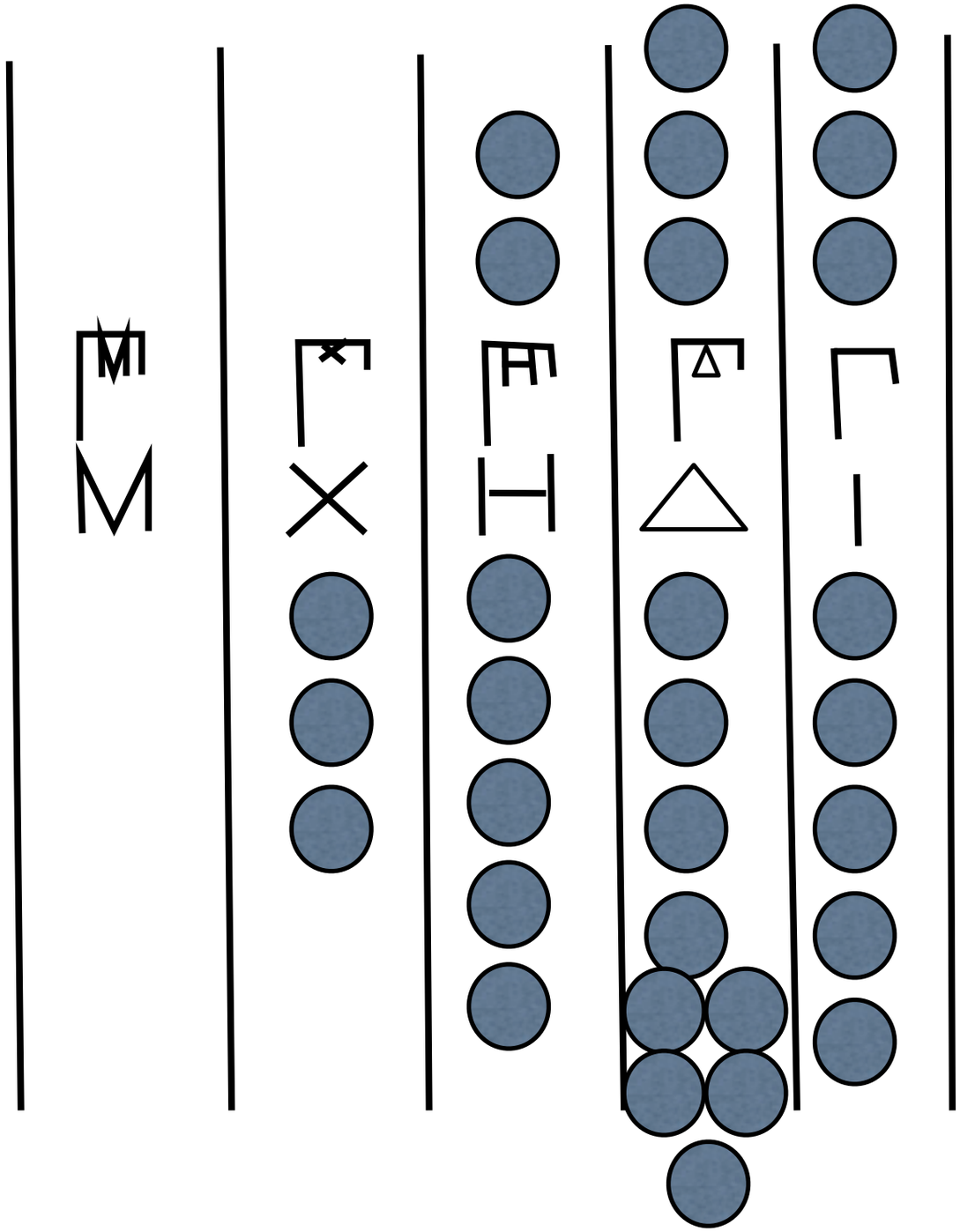


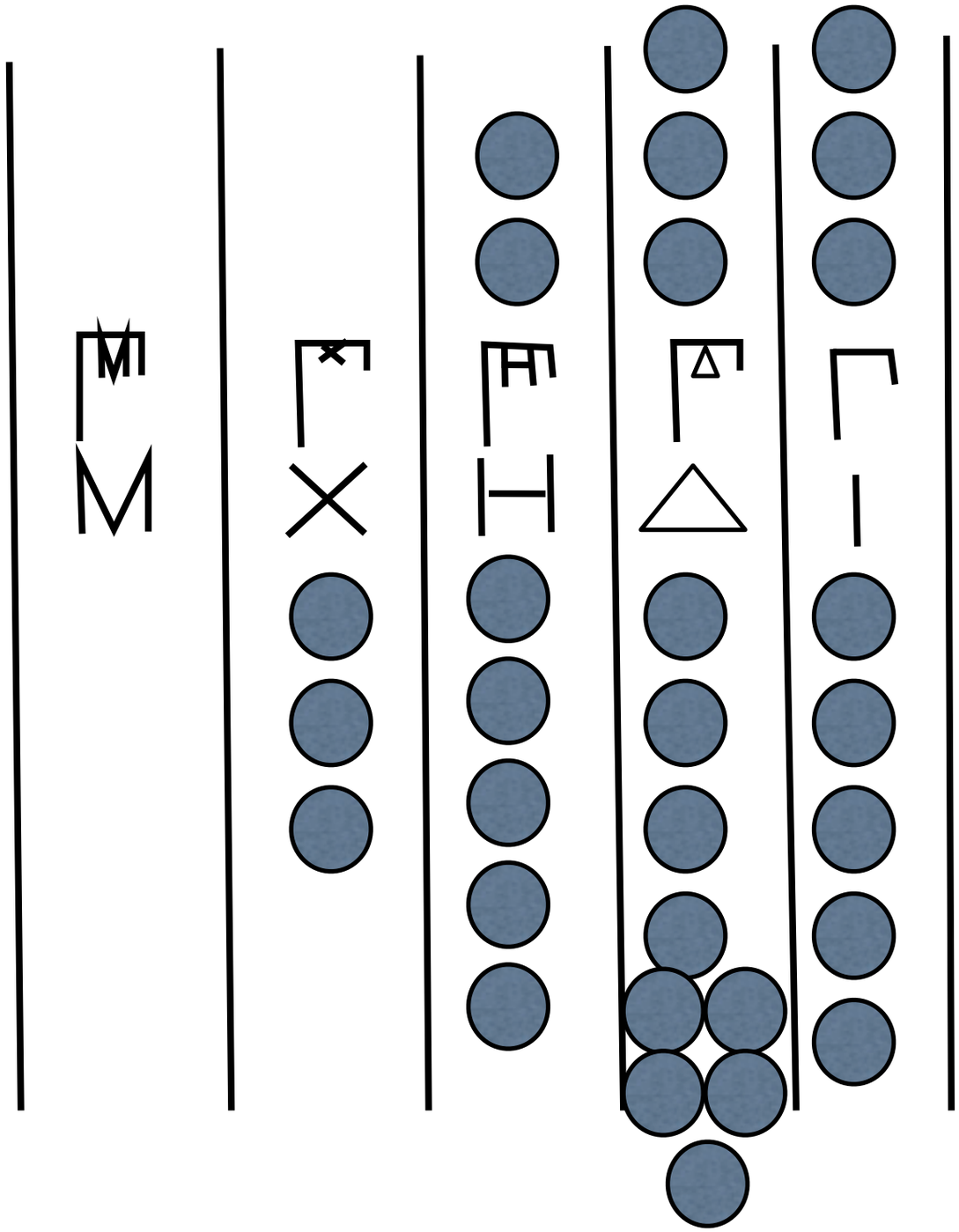
4760 - 3671

Agora retiramos 3671 usando a ordem natural (E⟶D)



4760 - 3671

3671



4760 - 3671

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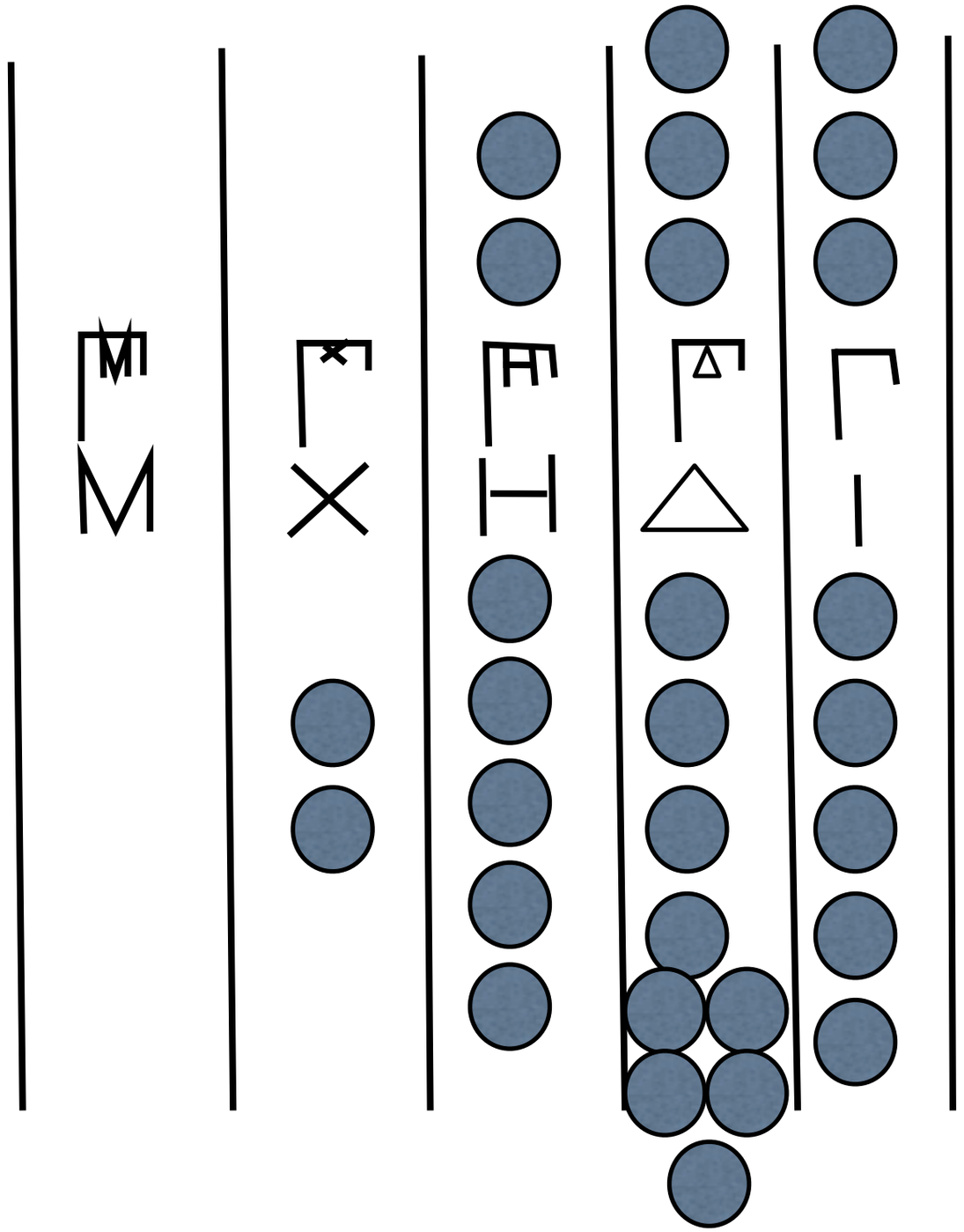

4760 - 3671

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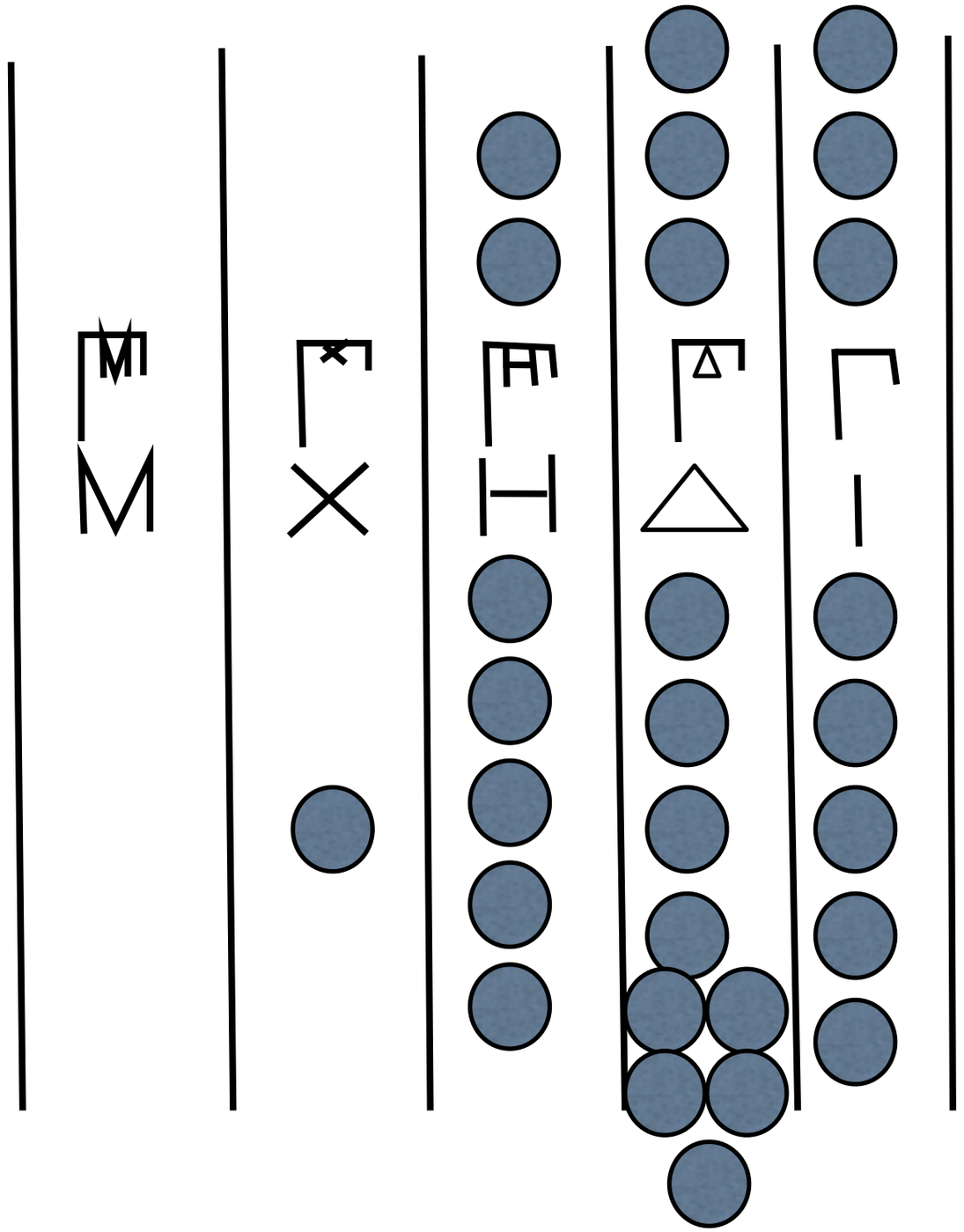

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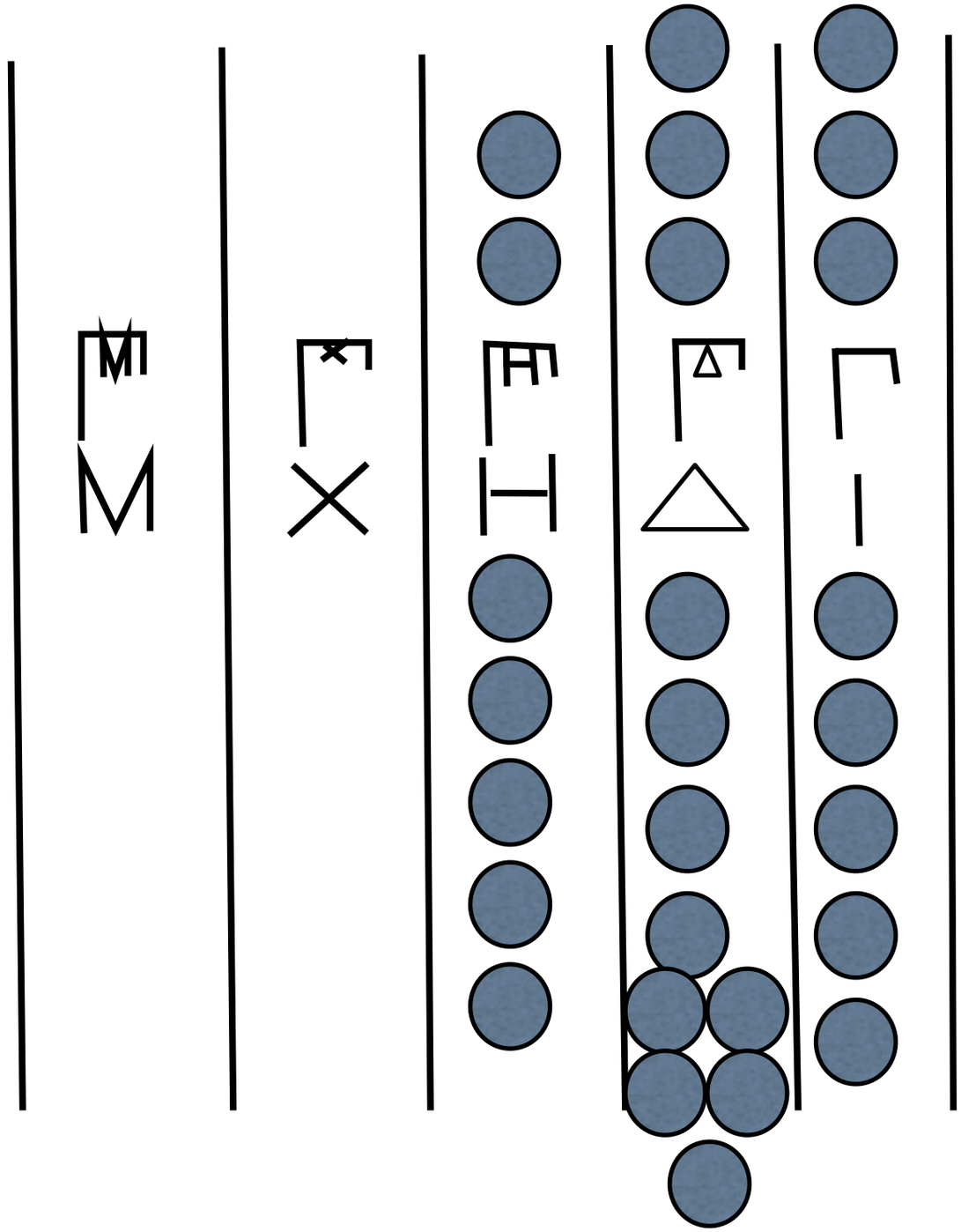

4760 - 3671

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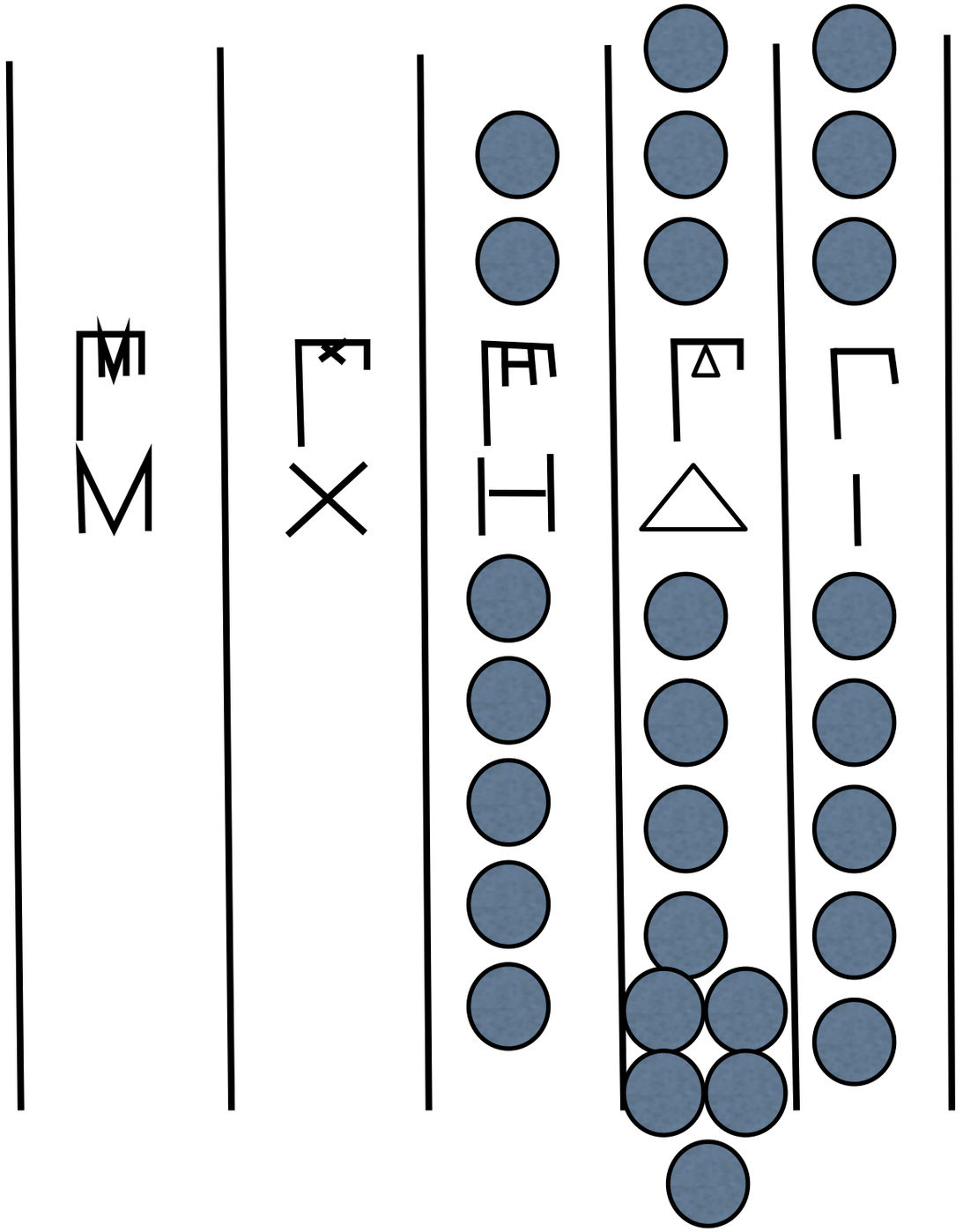

4760 - 3671

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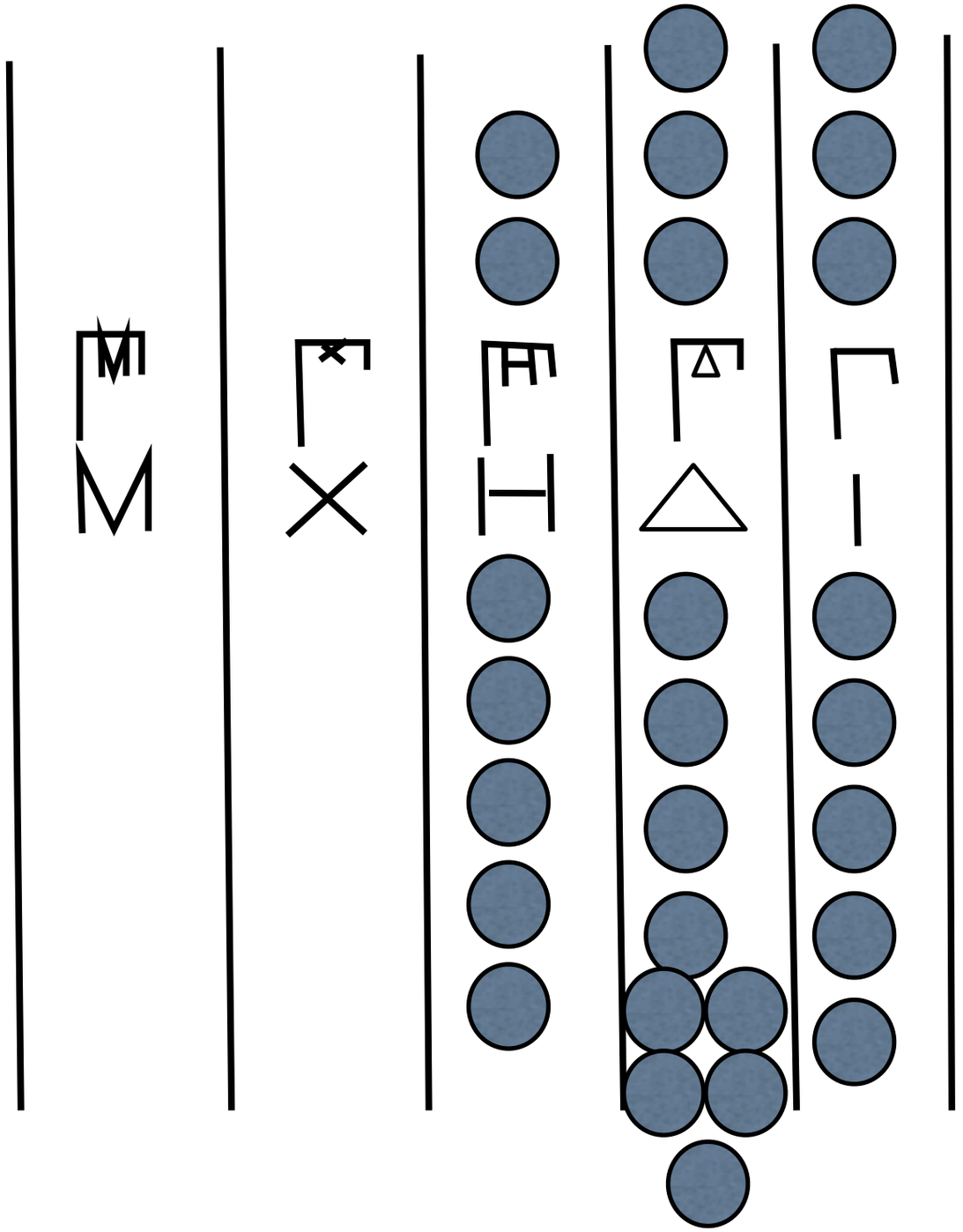

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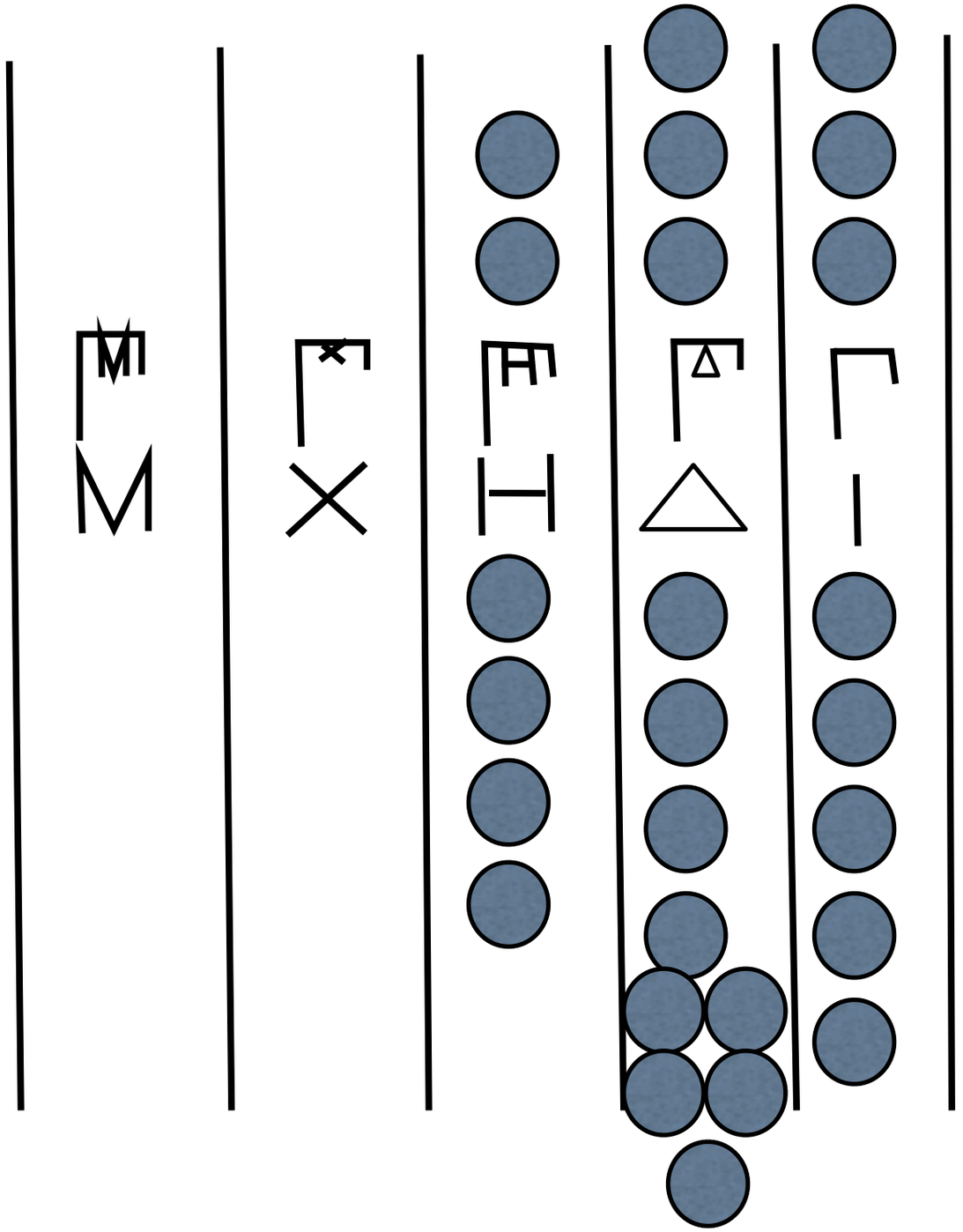

4760 - 3671

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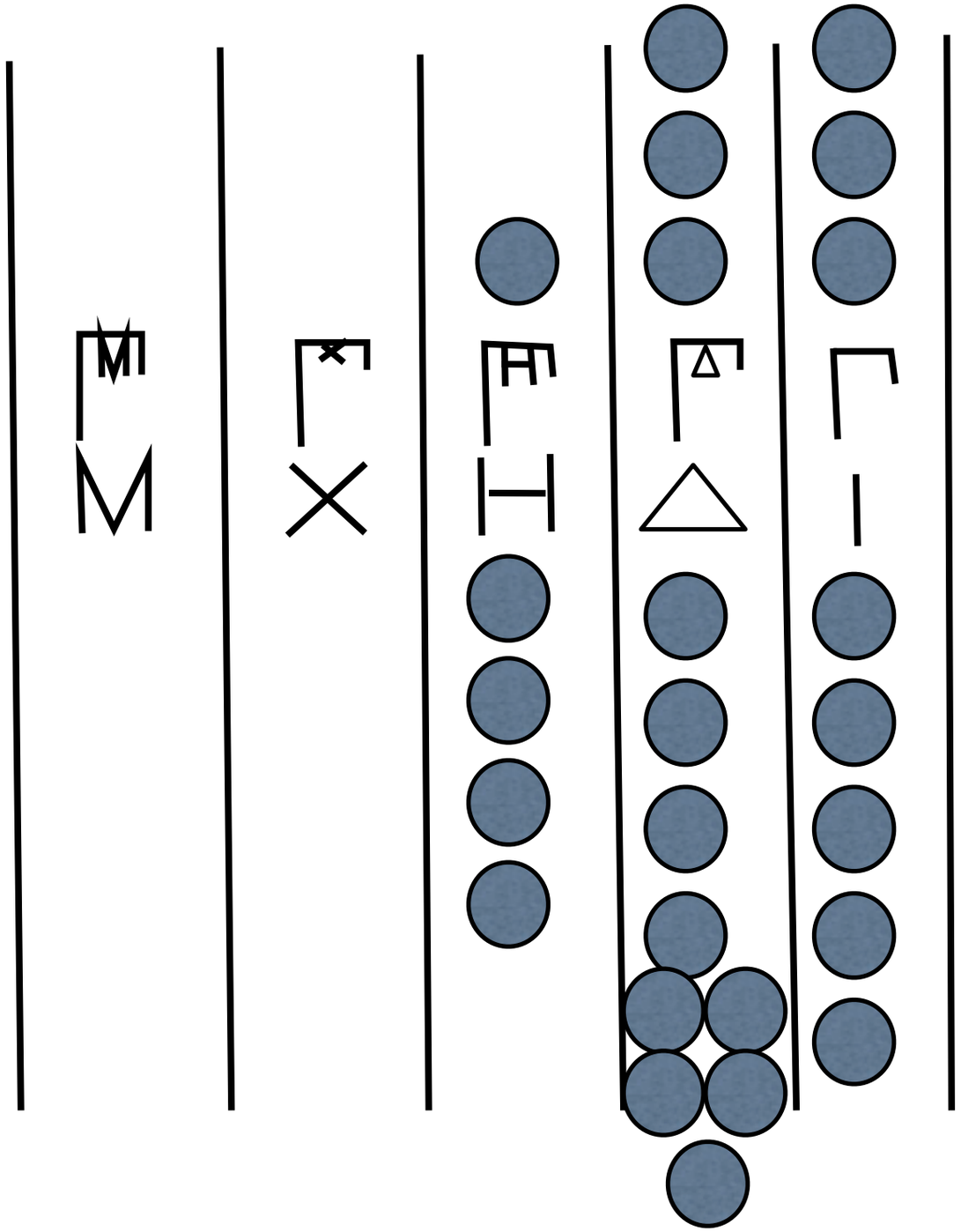

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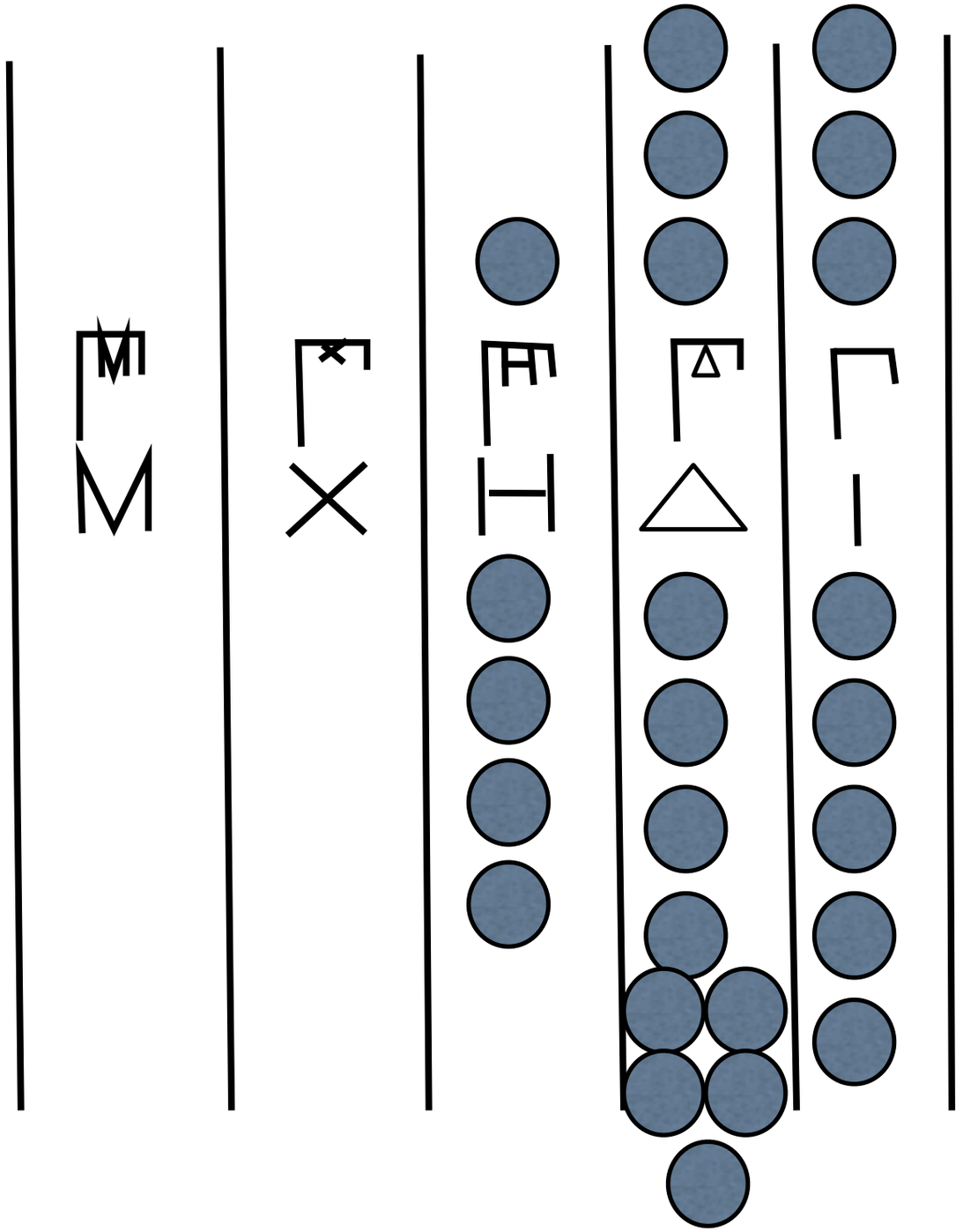

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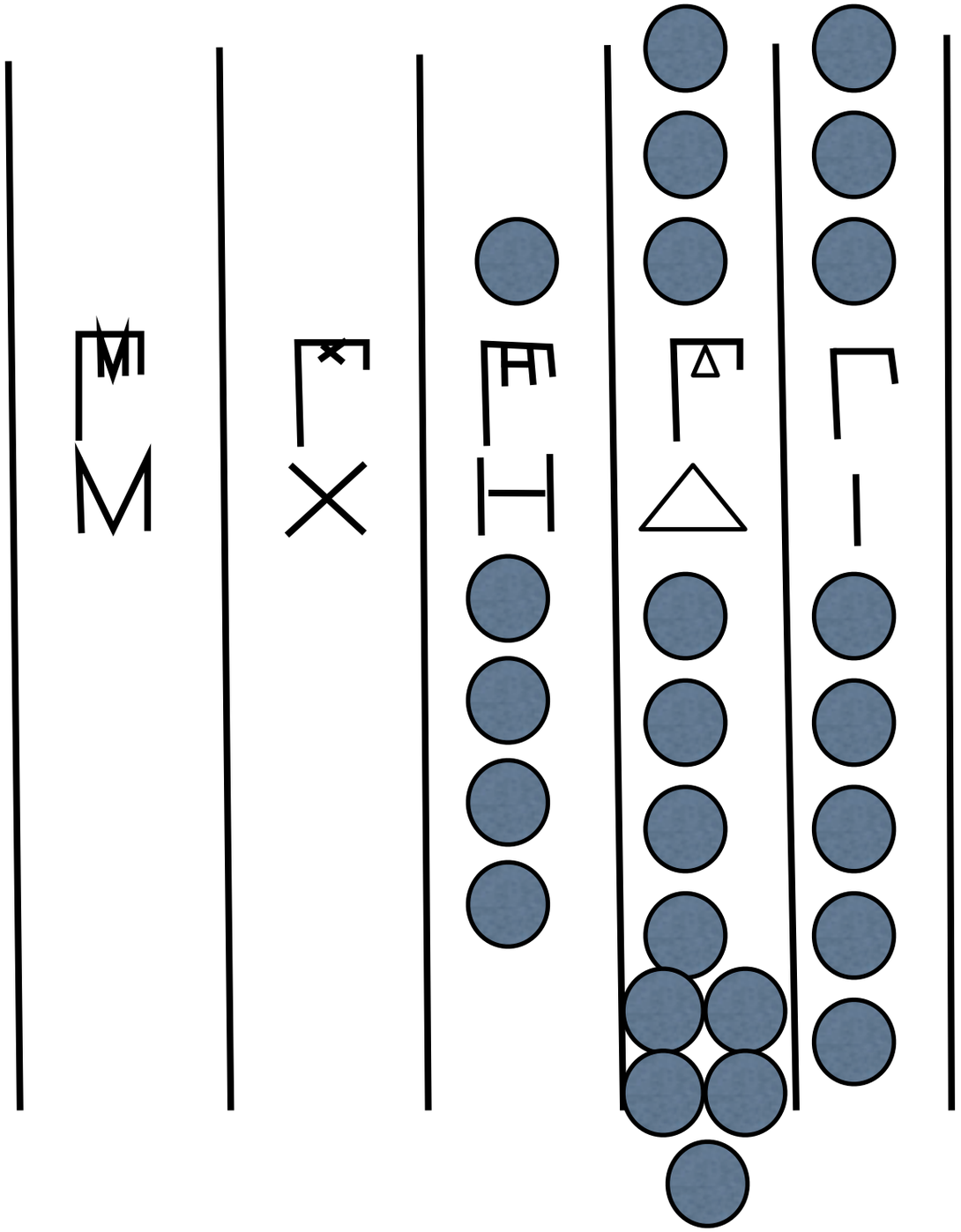
4760 - 3671



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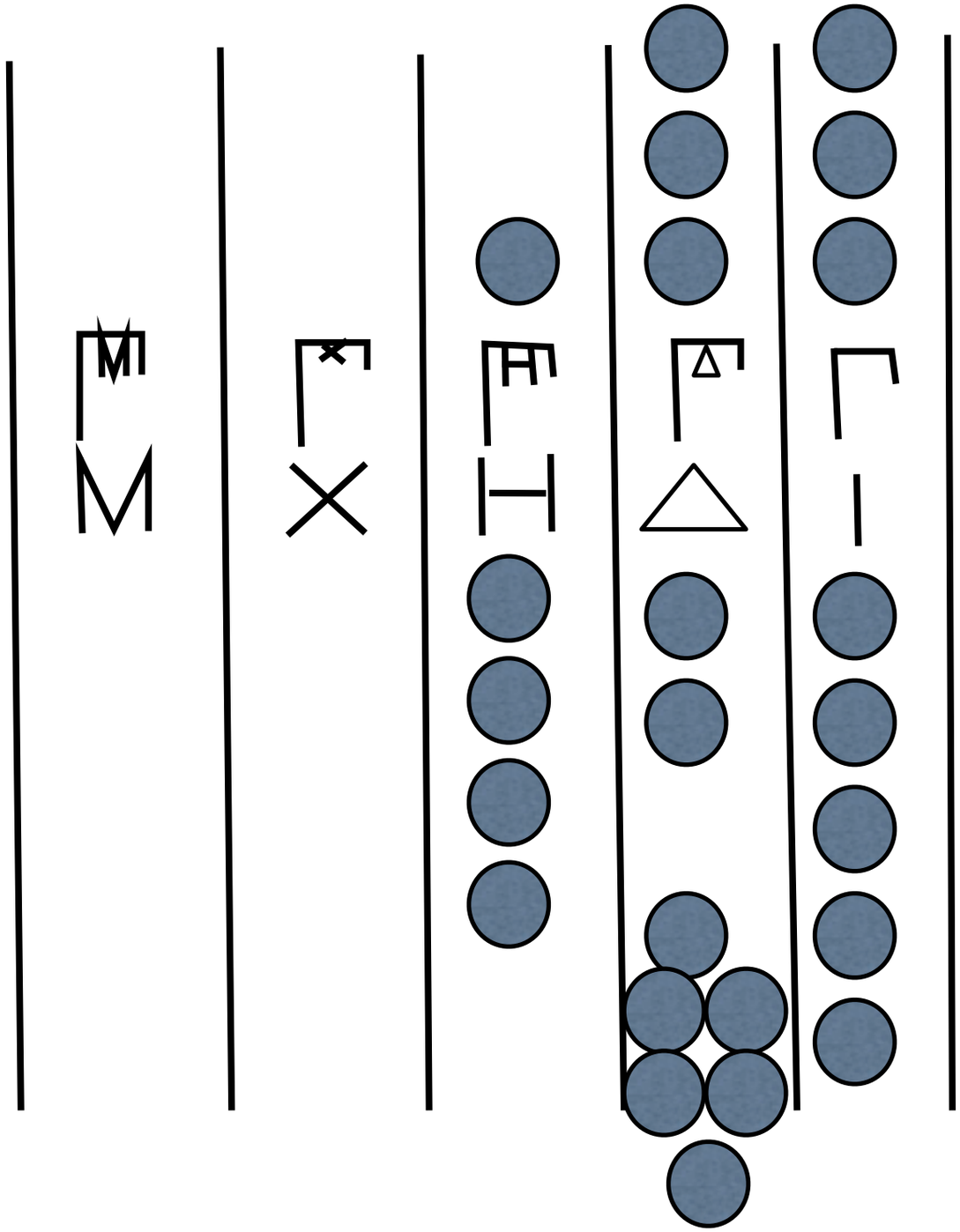
4760 - 3671

71



4760 - 3671

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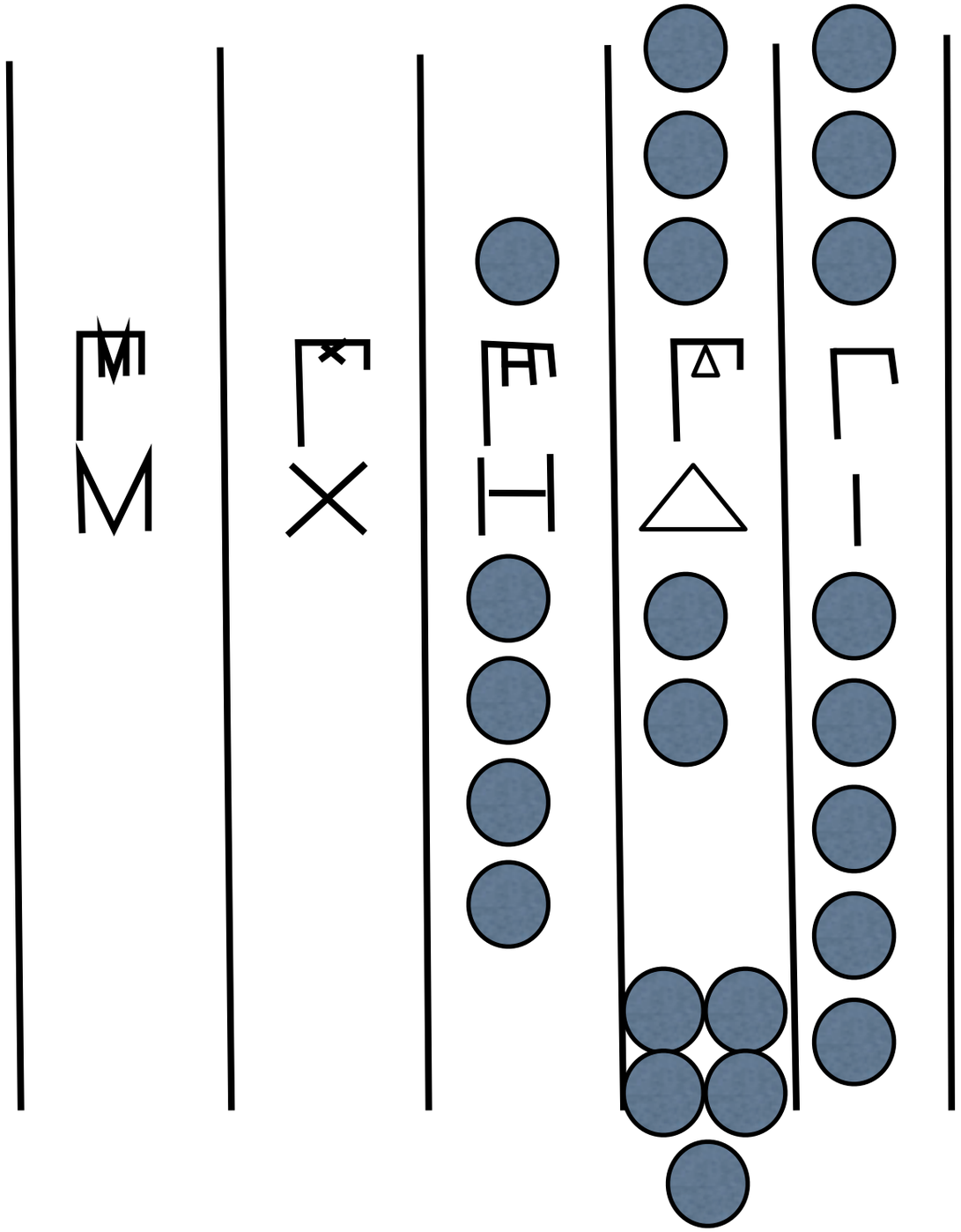

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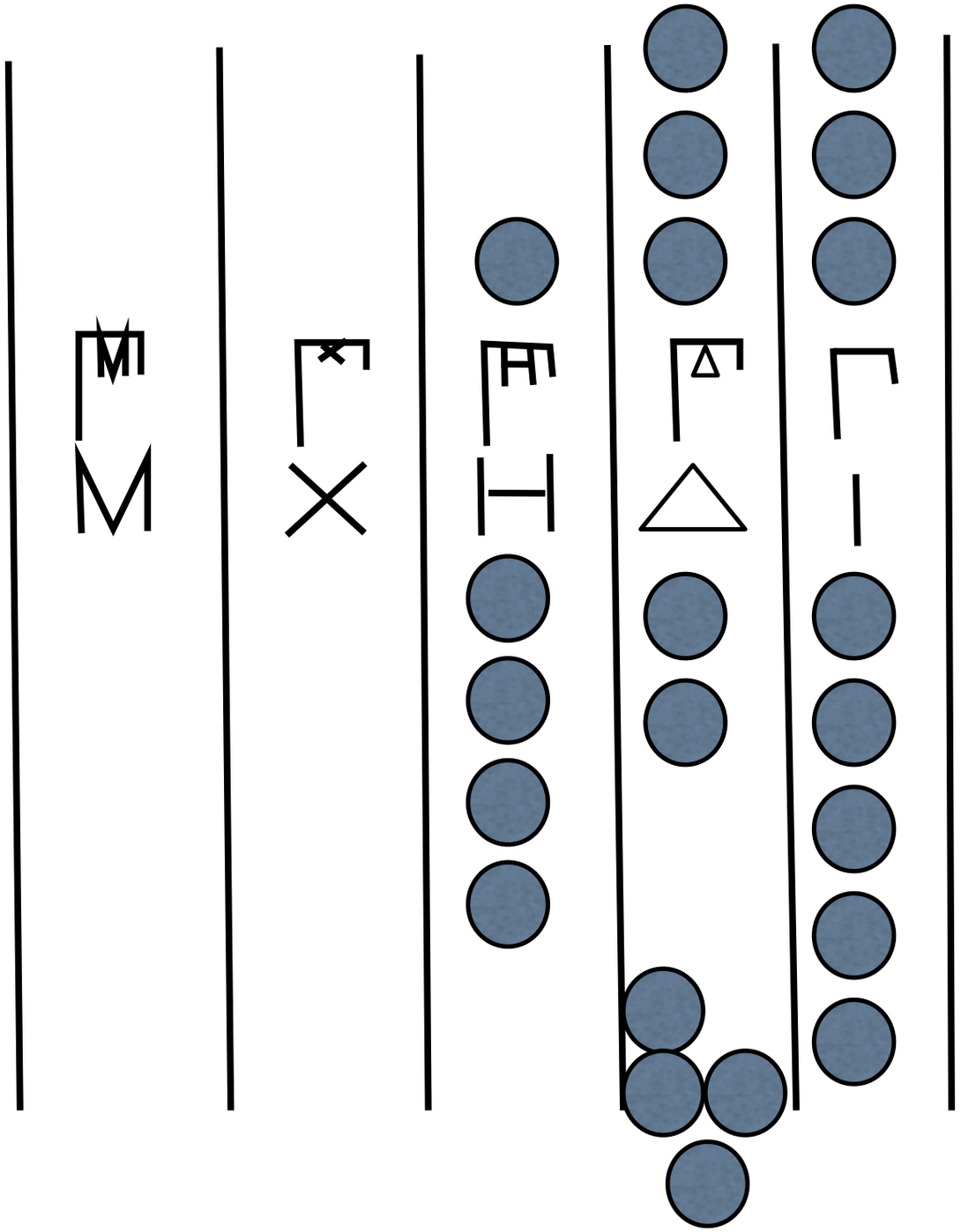

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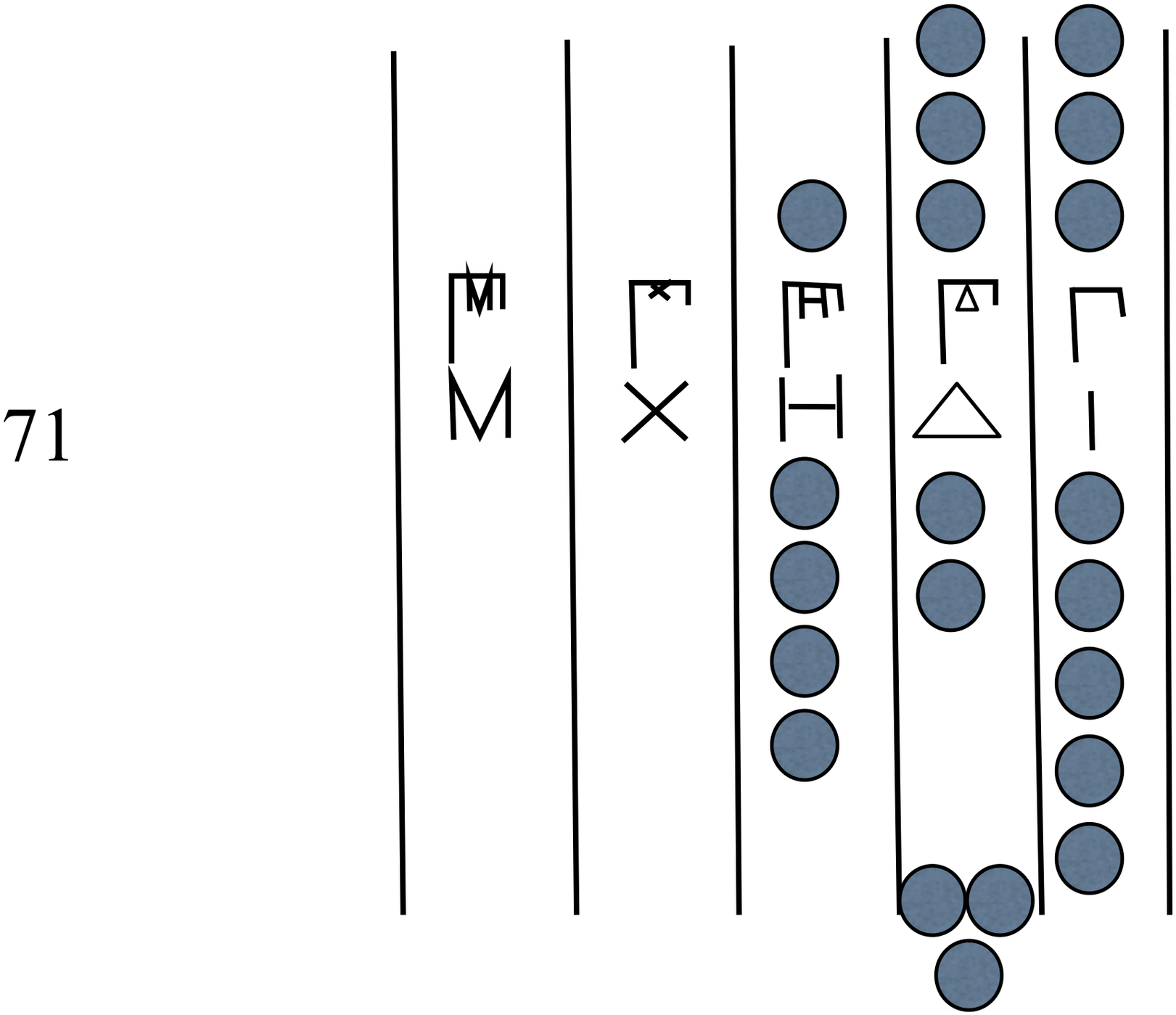
4760 - 3671



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4760 - 3671

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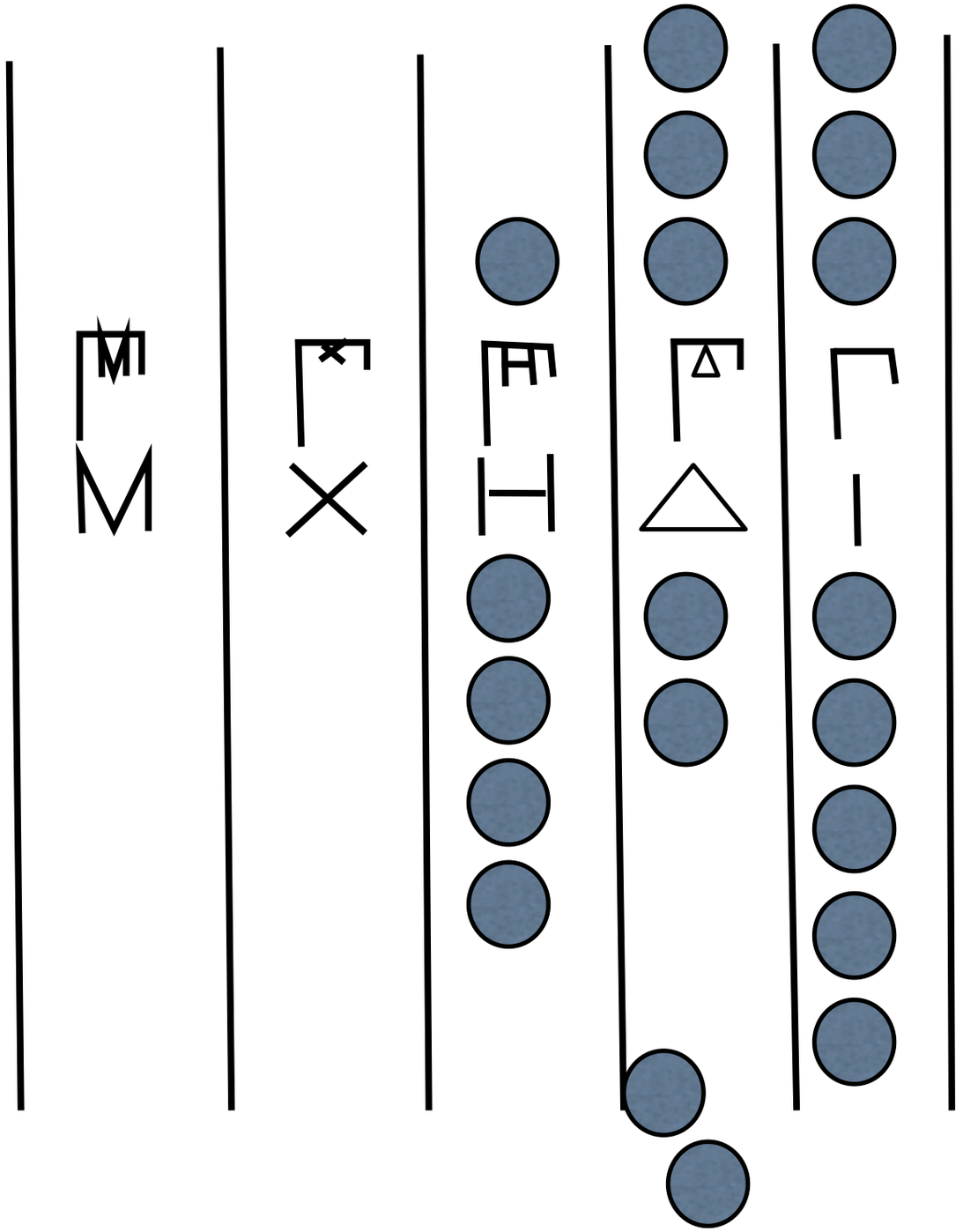

4760 - 3671

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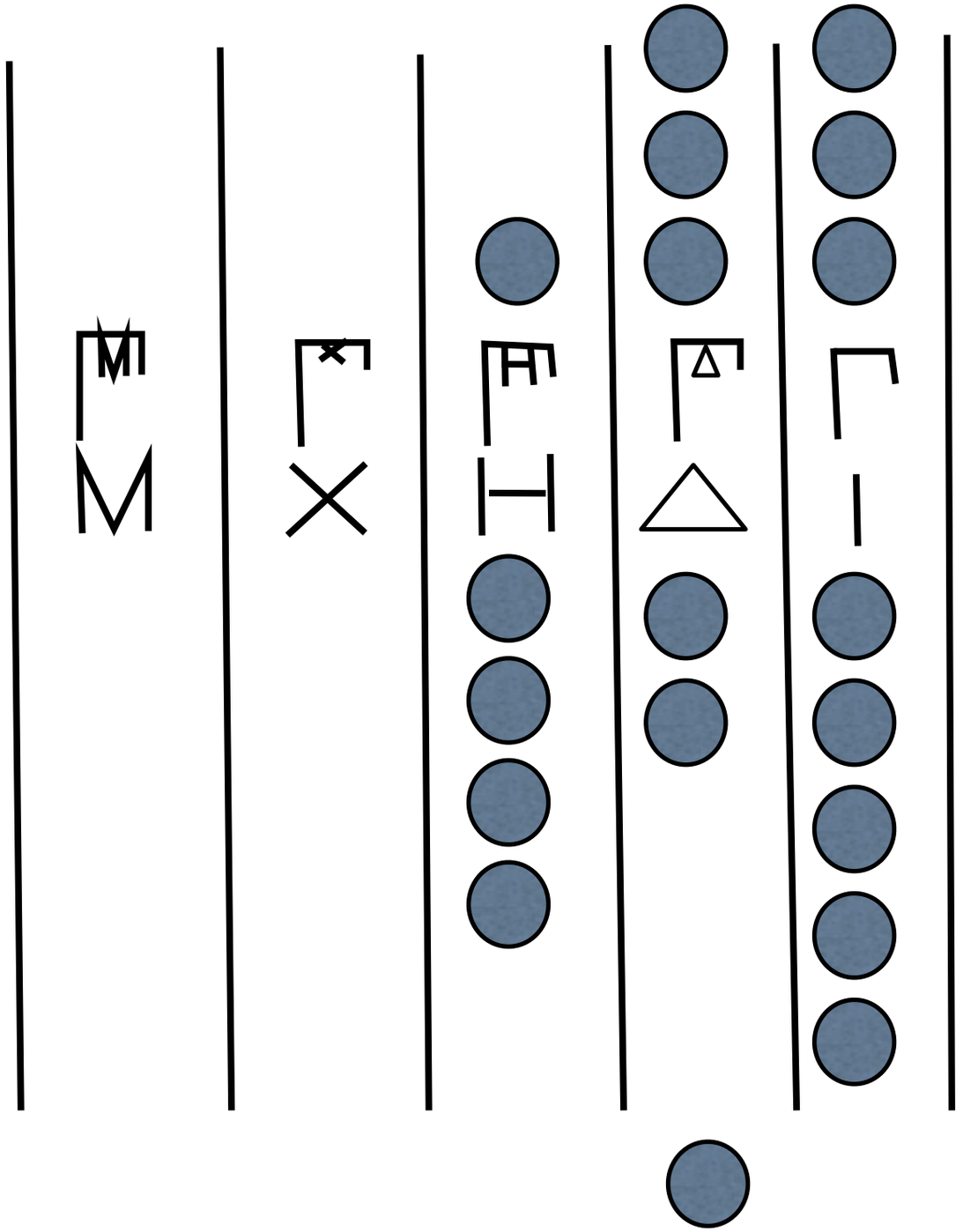

4760 - 3671

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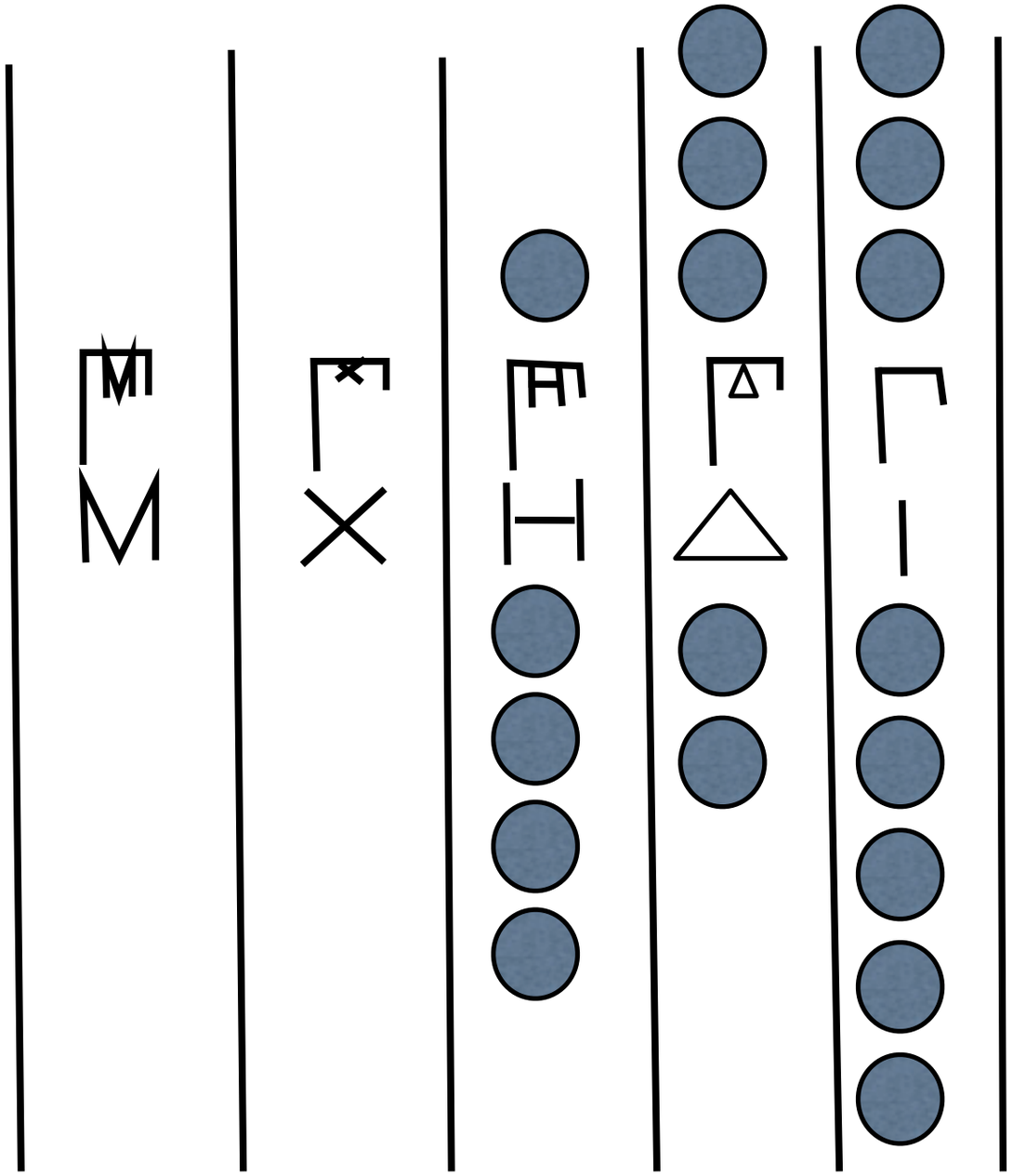

4760 - 3671

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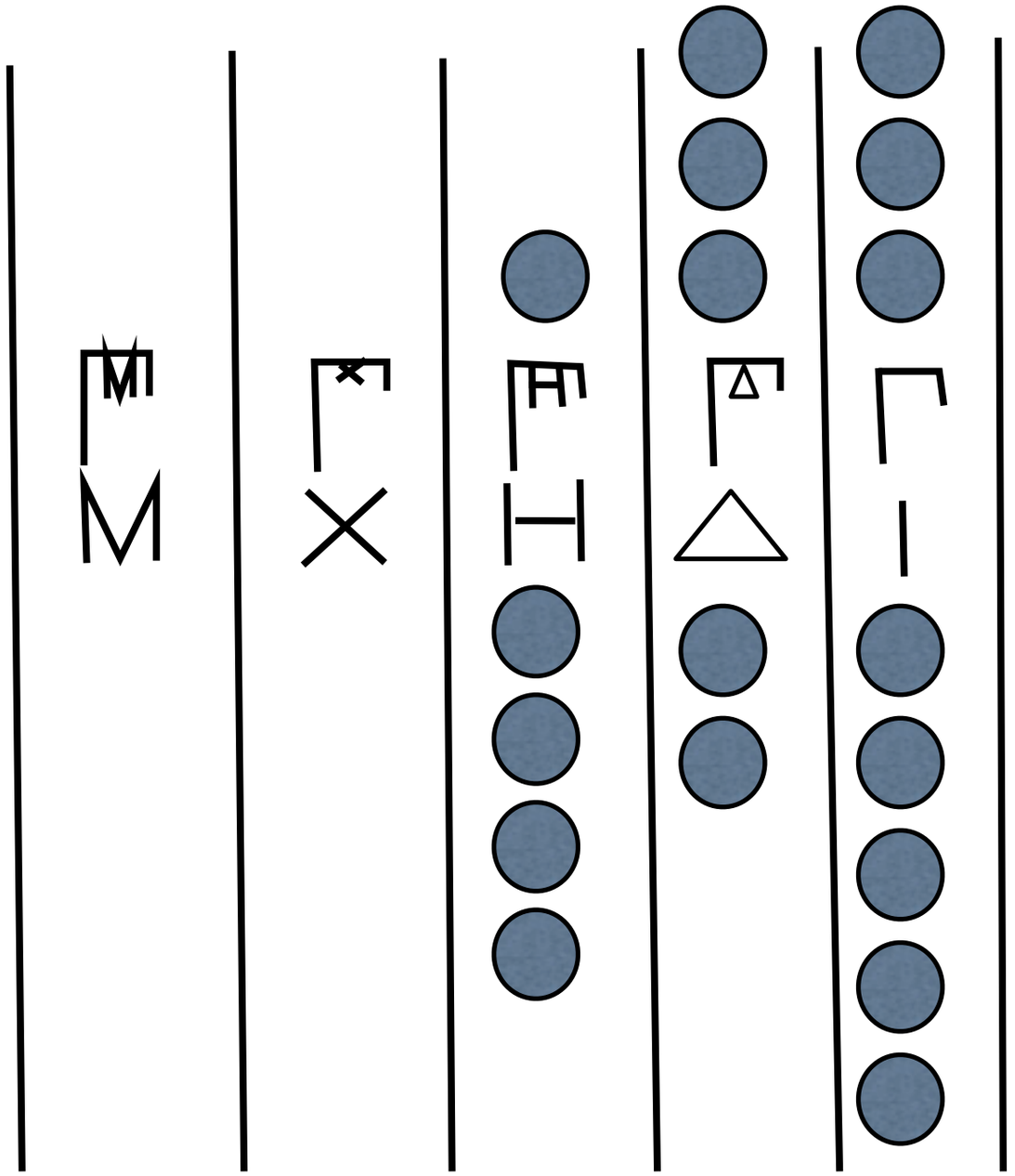

4760 - 3671

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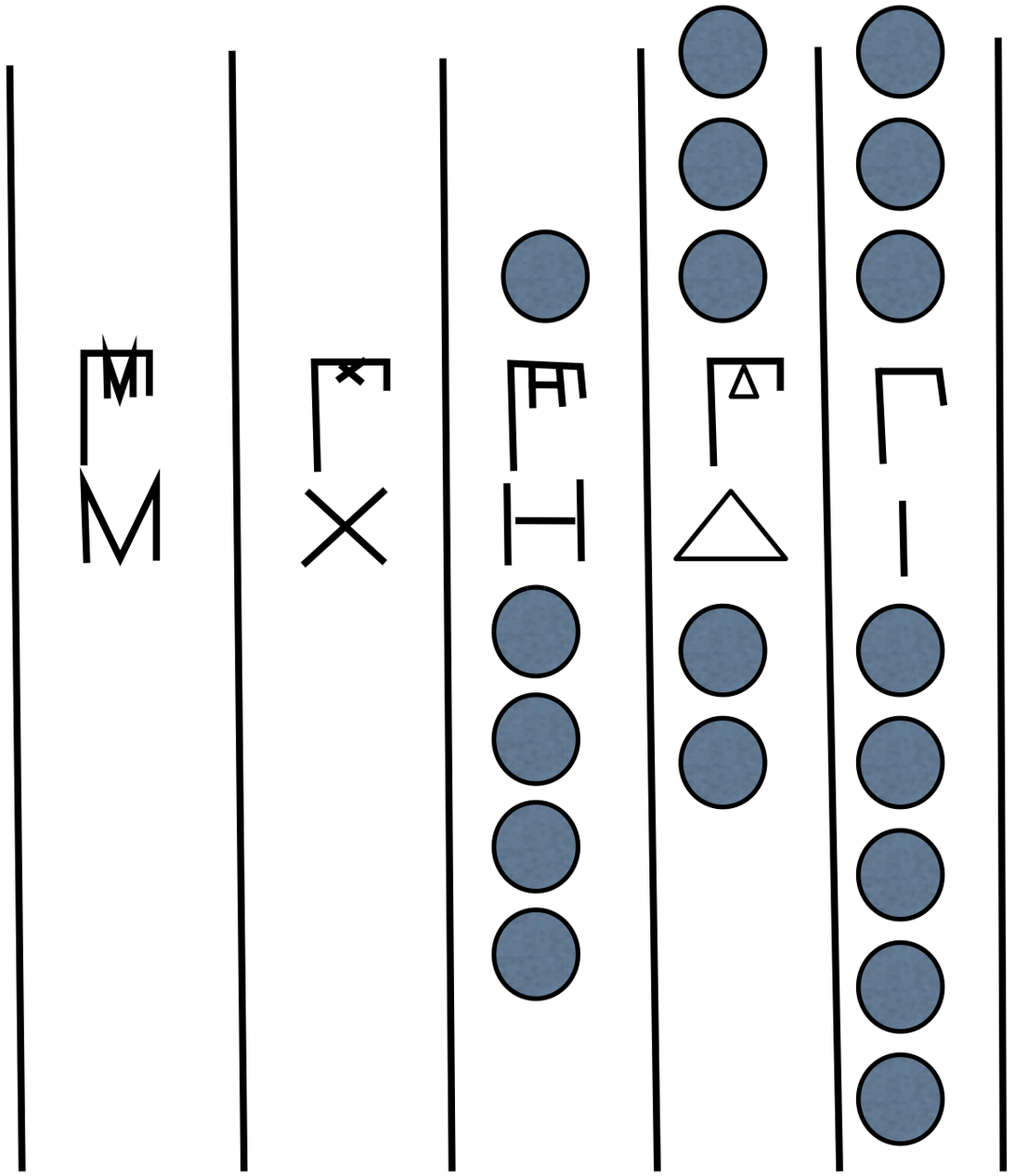

4760 - 3671

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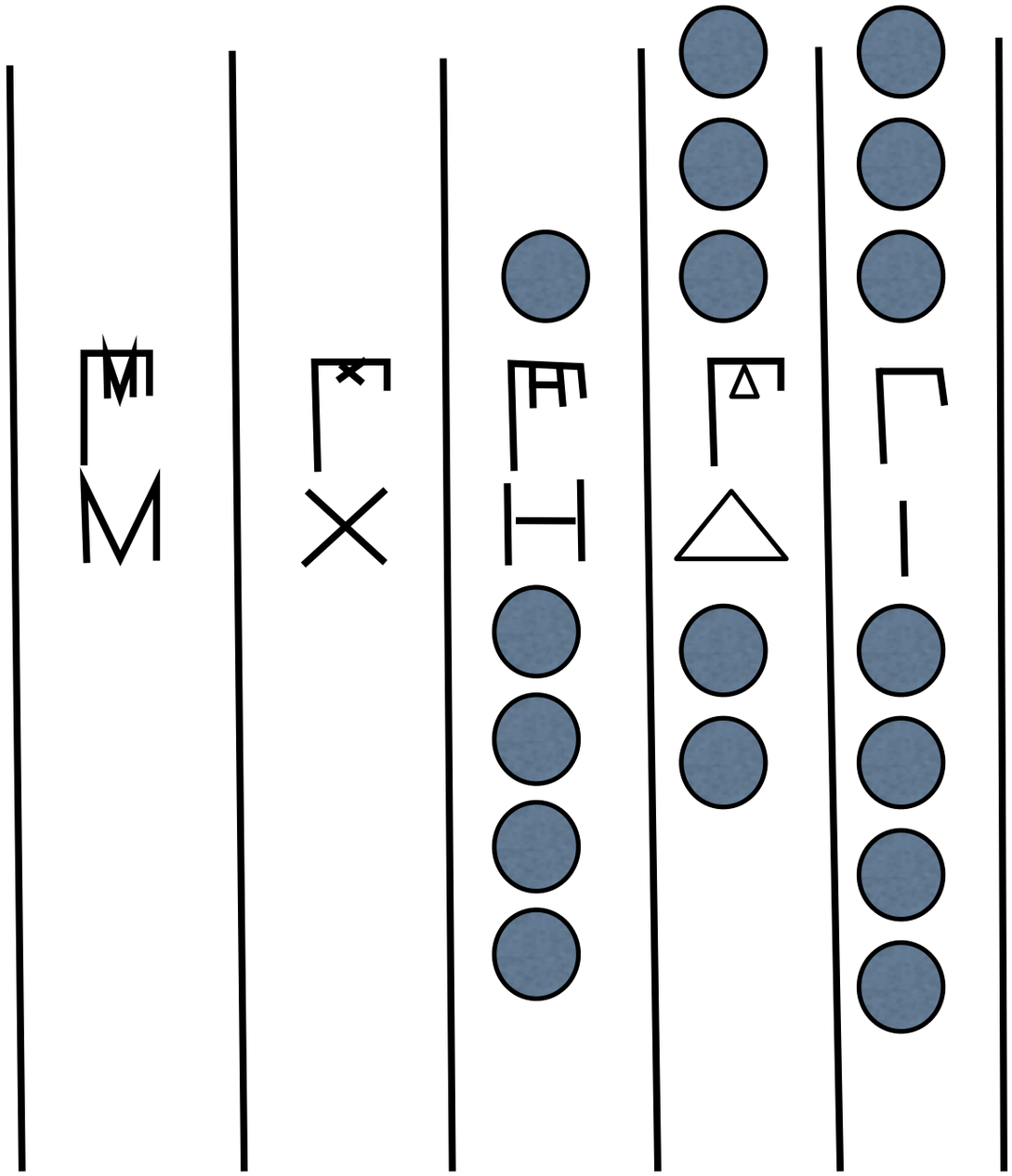
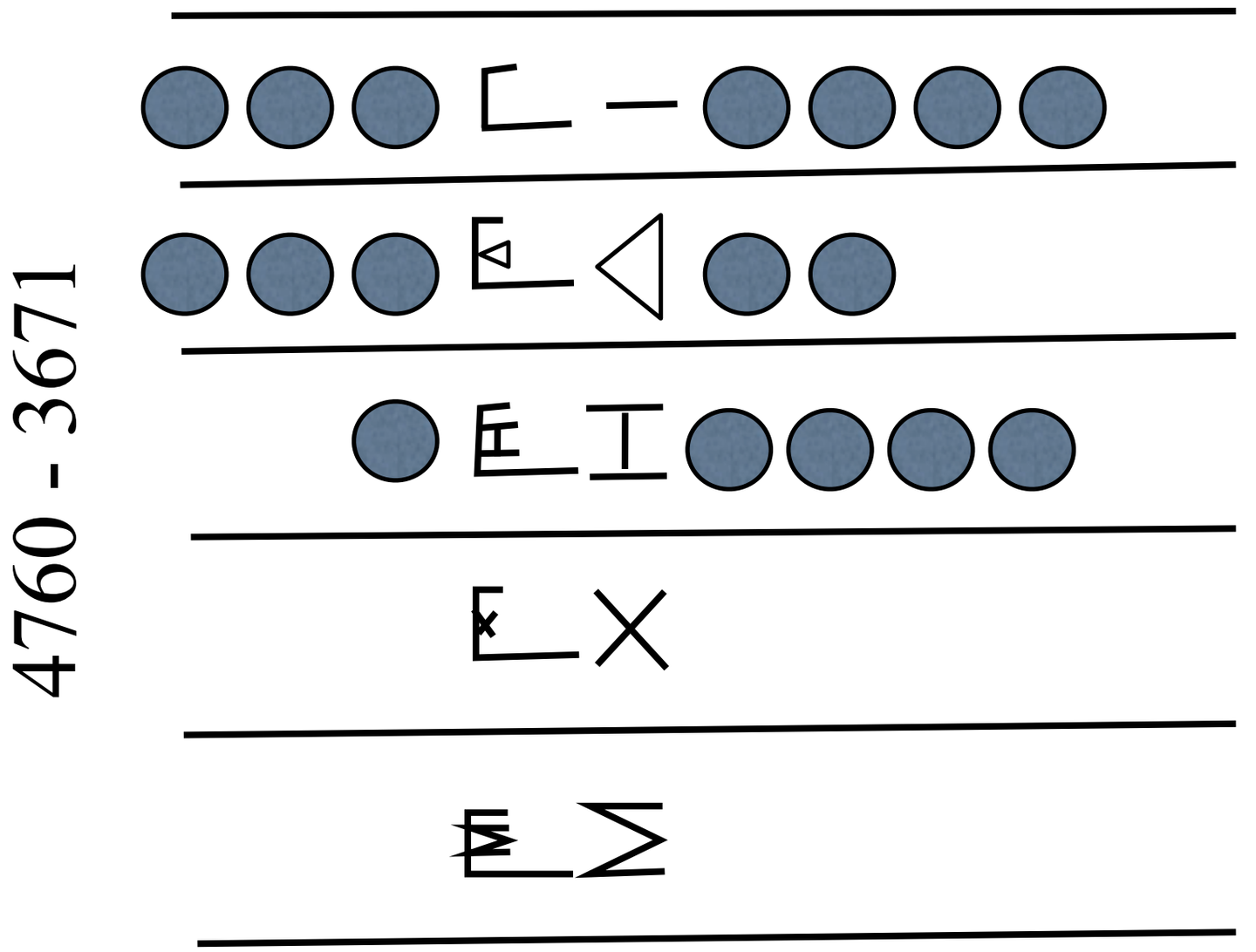
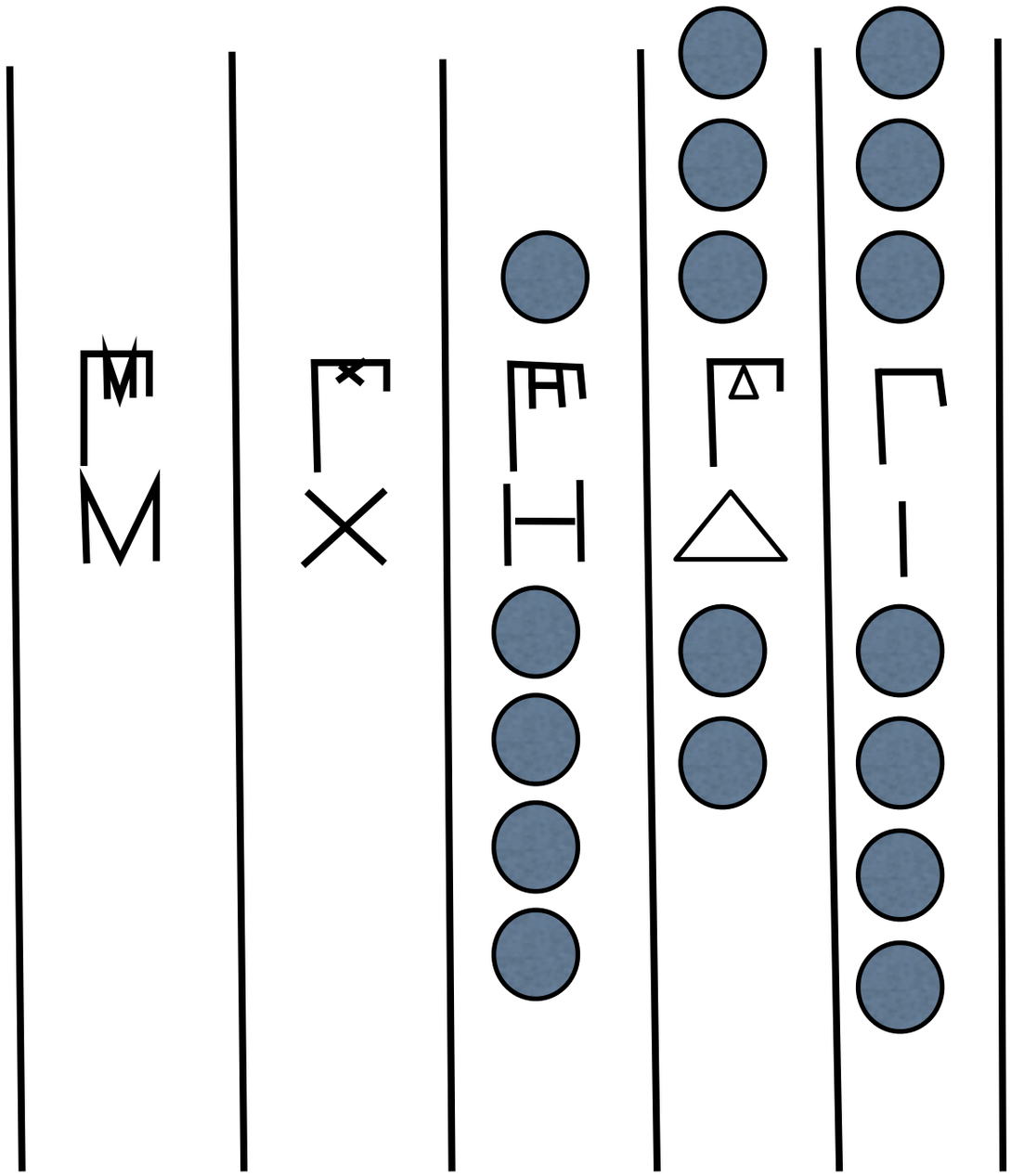




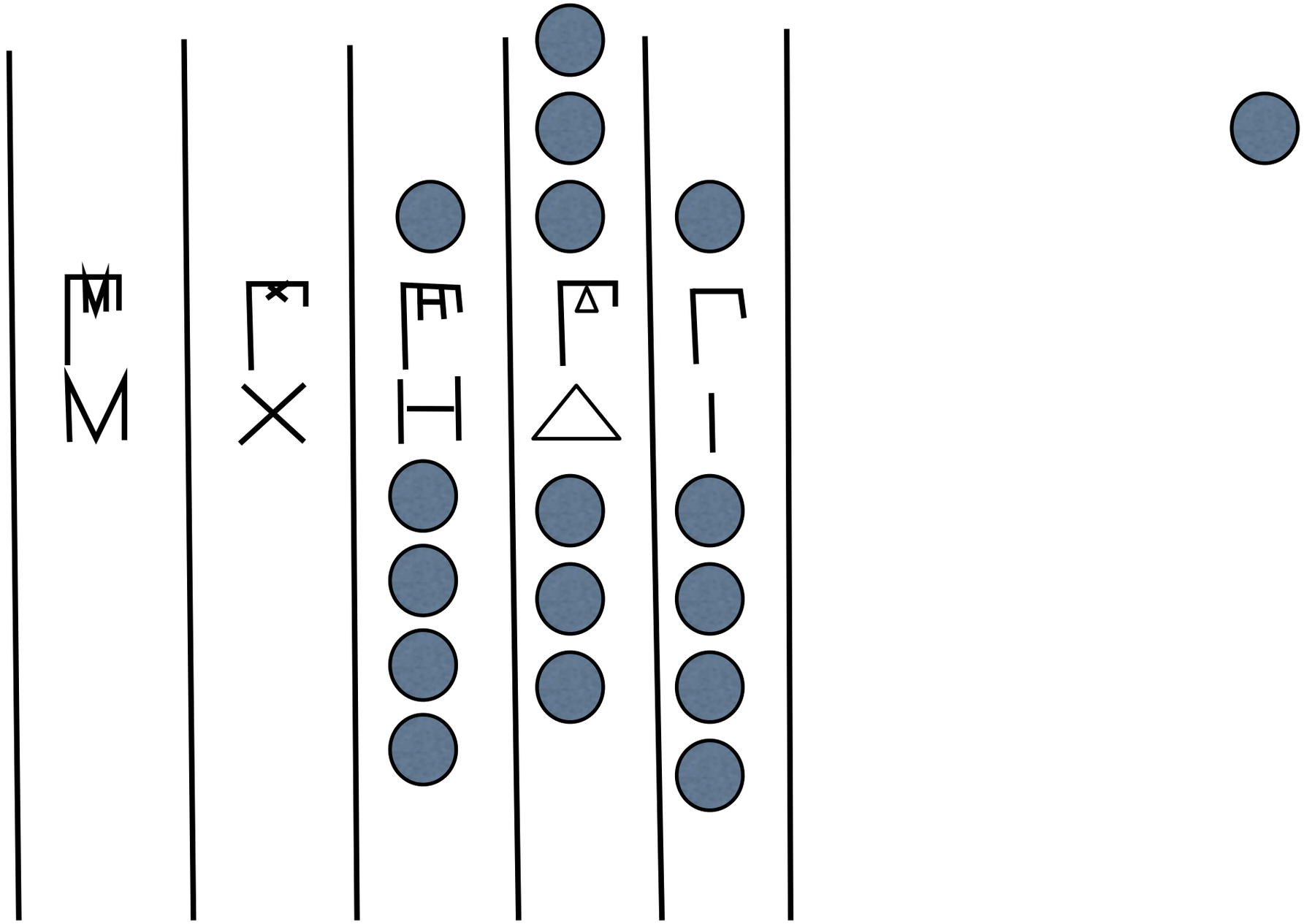
4760 - 3671

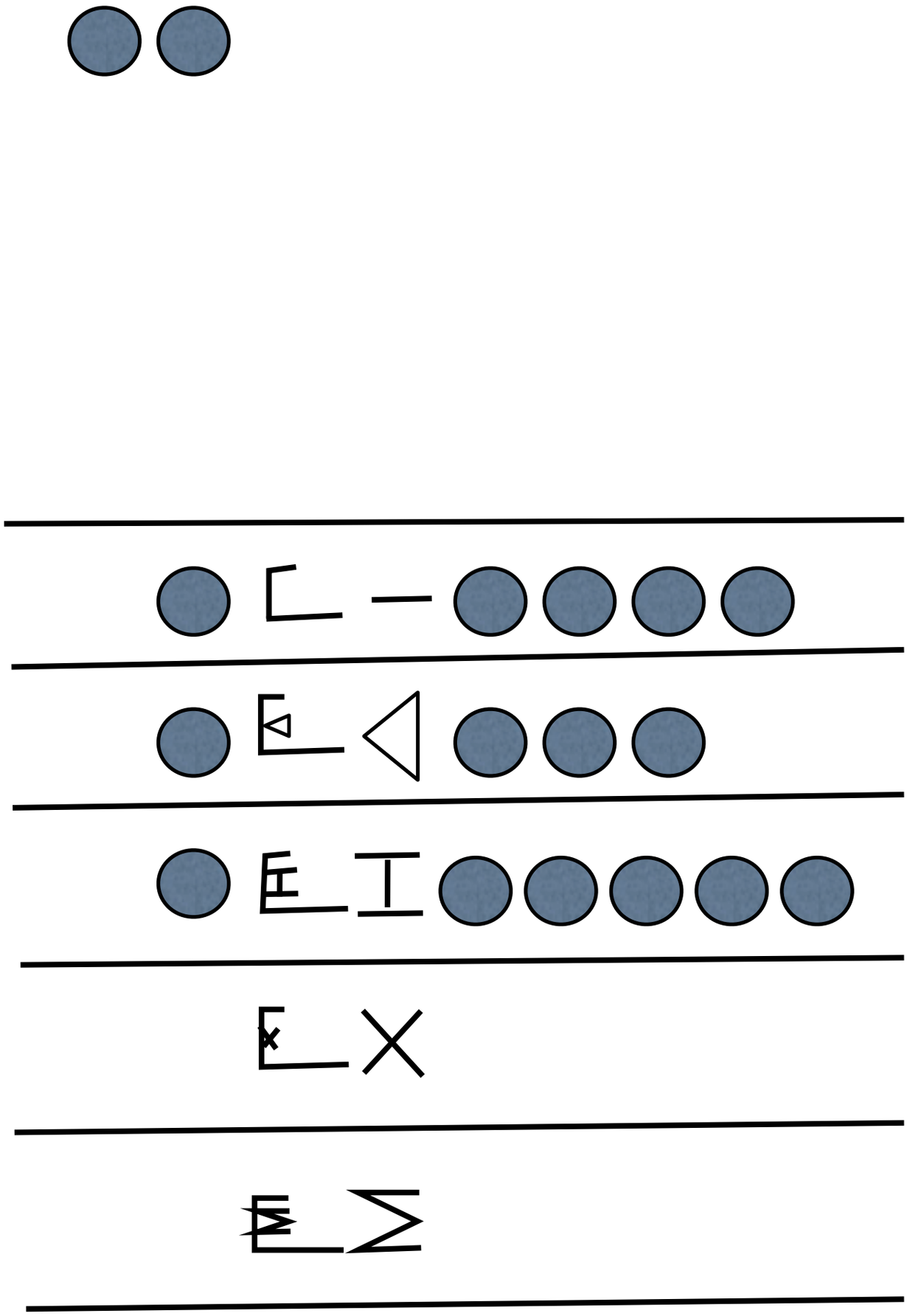
4760 - 3671

Agora temos de aplicar as regras de redução



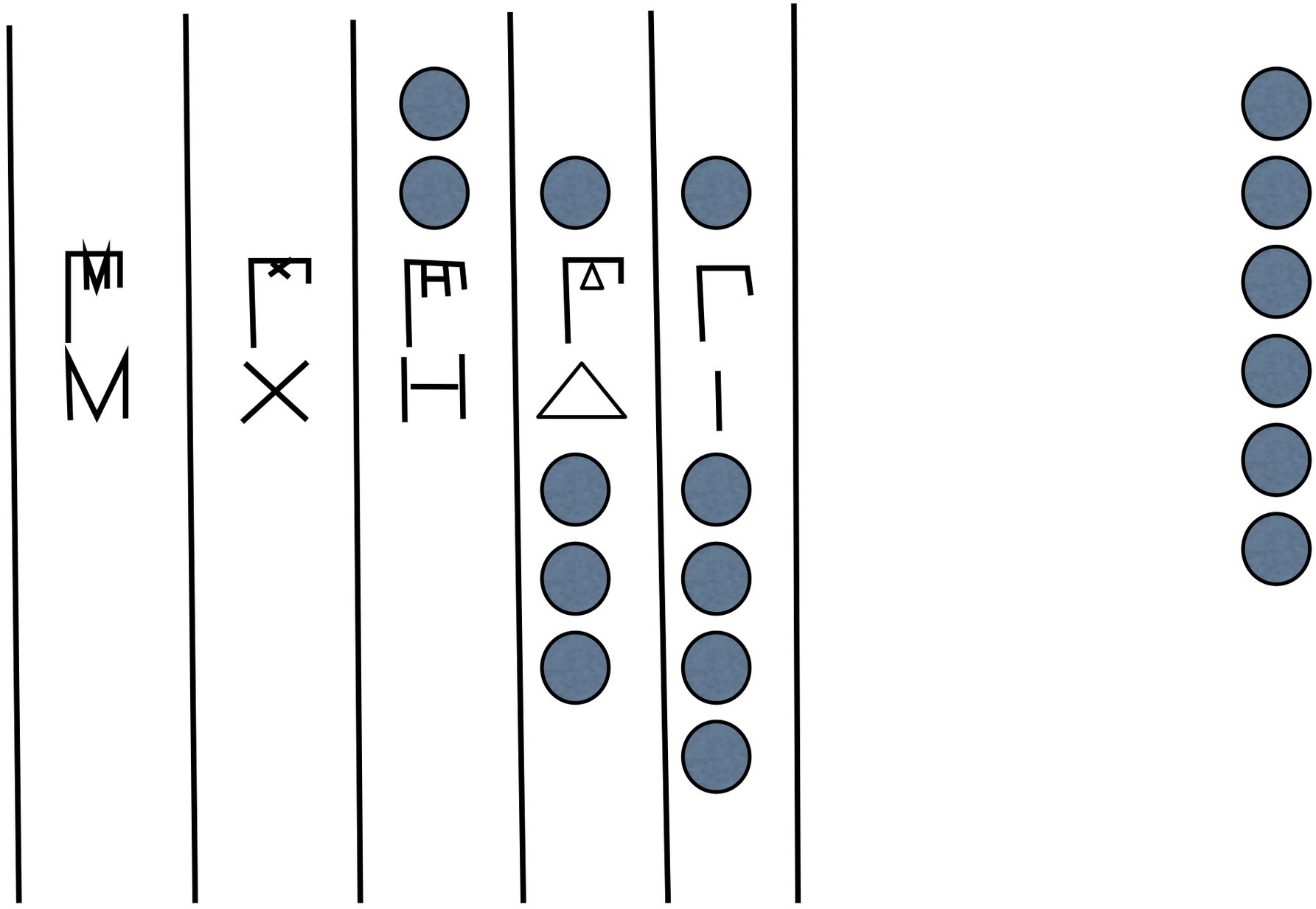
4760 - 3671



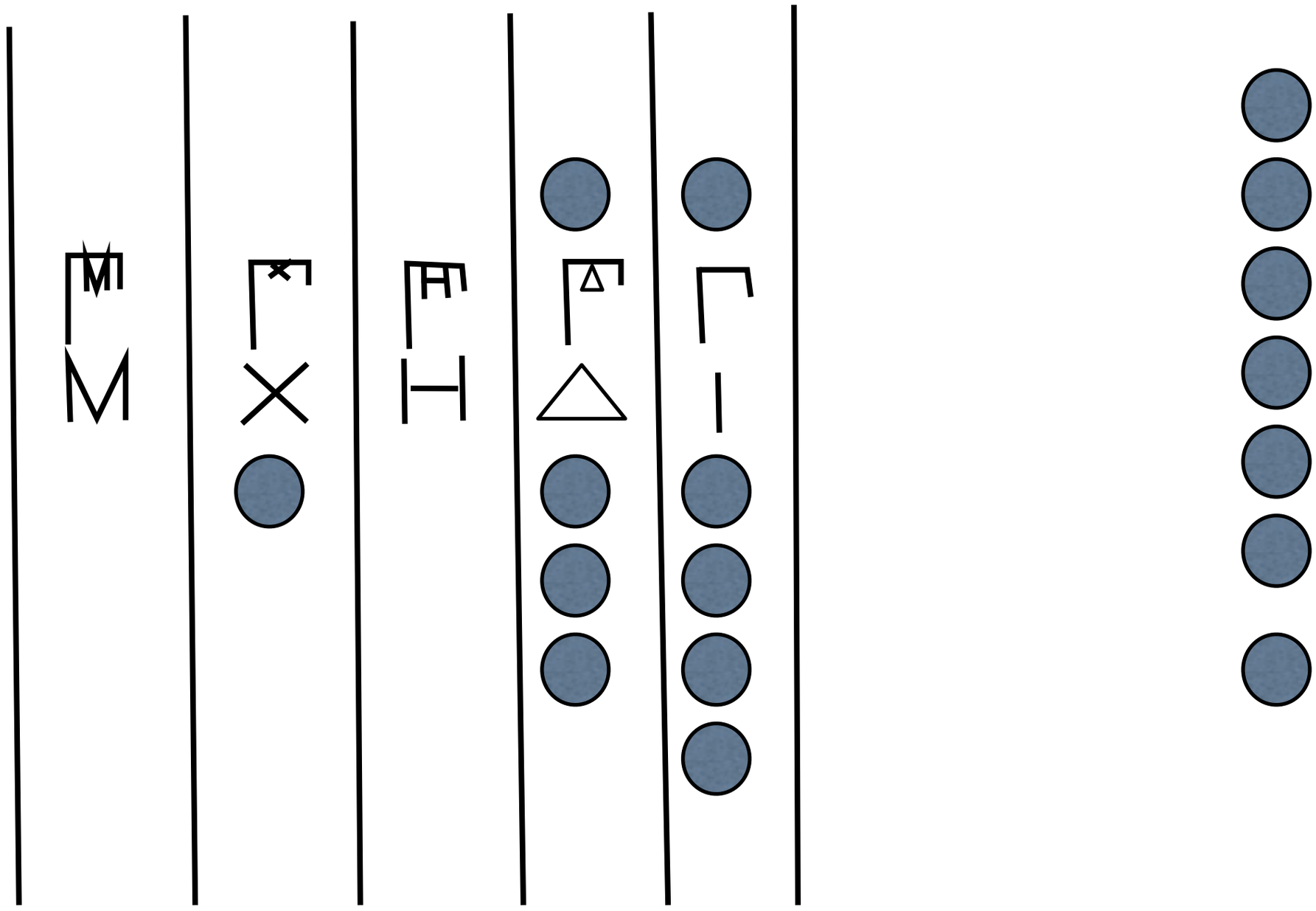


4760 - 3671

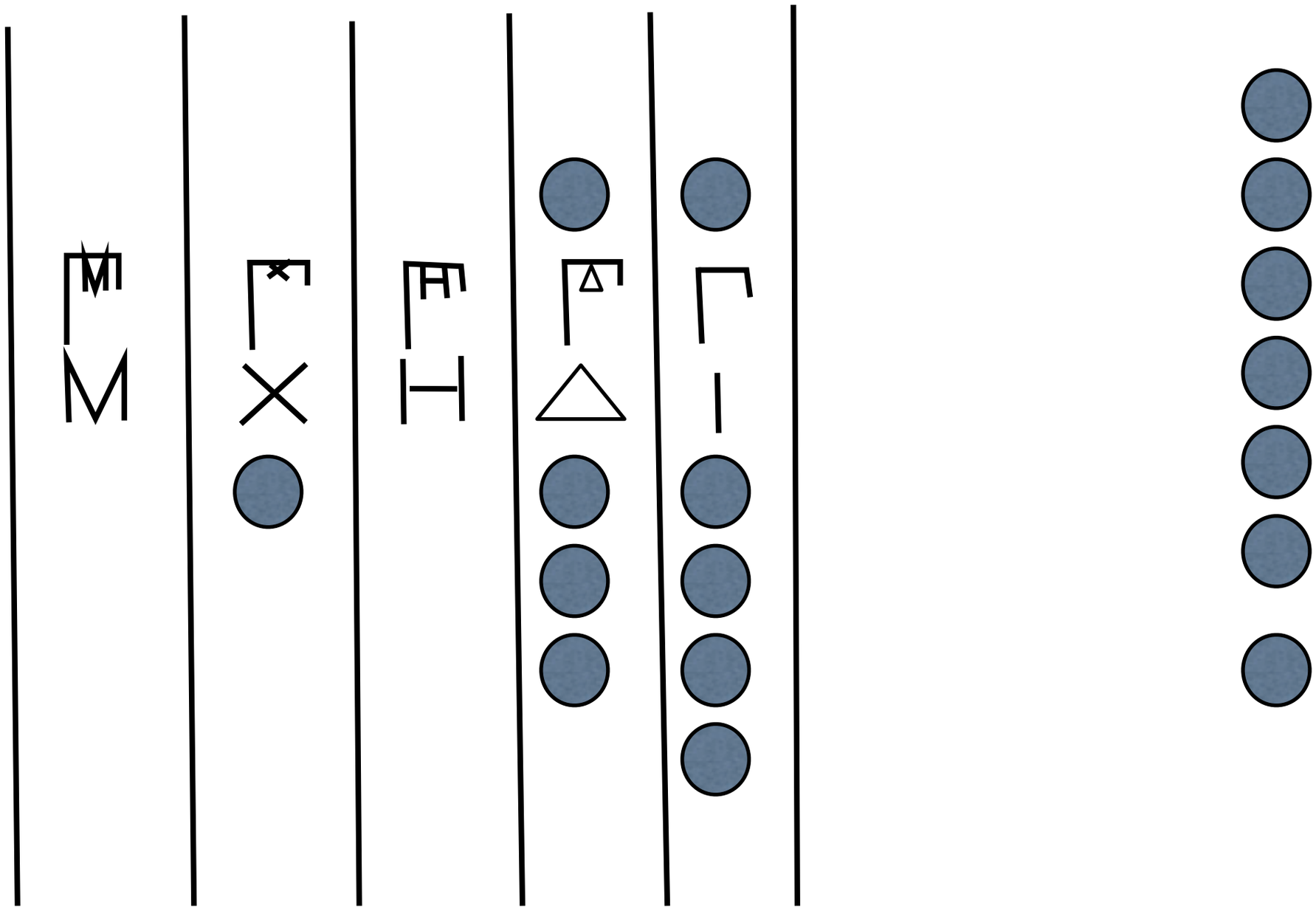
4760 - 3671



4760 - 3671



$$4760 - 3671 = 1089$$



A multiplicação pode ser implementada como adição iterada.

Os factores deviam ser registados algures e seria natural usar tabuadas. O ábaco permitiria então somar os produtos parciais.
Para efectuar 123x37 teríamos de somar 100x37, 20x37 e 3x37.

Estes ábacos não são úteis para as multiplicações (deixemos de parte as divisões!)

Roma

Roman Numeral Table							
1	I	**14**	XIV	**27**	XXVII	**150**	CL
2	II	**15**	XV	**28**	XXVIII	**200**	CC
3	III	**16**	XVI	**29**	XXIX	**300**	CCC
4	IV	**17**	XVII	**30**	XXX	**400**	CD
5	V	**18**	XVIII	**31**	XXXI	**500**	D
6	VI	**19**	XIX	**40**	XL	**600**	DC
7	VII	**20**	XX	**50**	L	**700**	DCC
8	VIII	**21**	XXI	**60**	LX	**800**	DCCC
9	IX	**22**	XXII	**70**	LXX	**900**	CM
10	X	**23**	XXIII	**80**	LXXX	**1000**	M
11	XI	**24**	XXIV	**90**	XC	**1600**	MDC
12	XII	**25**	XXV	**100**	C	**1700**	MDCC
13	XIII	**26**	XXVI	**101**	CI	**1900**	MCM

MathATube.com

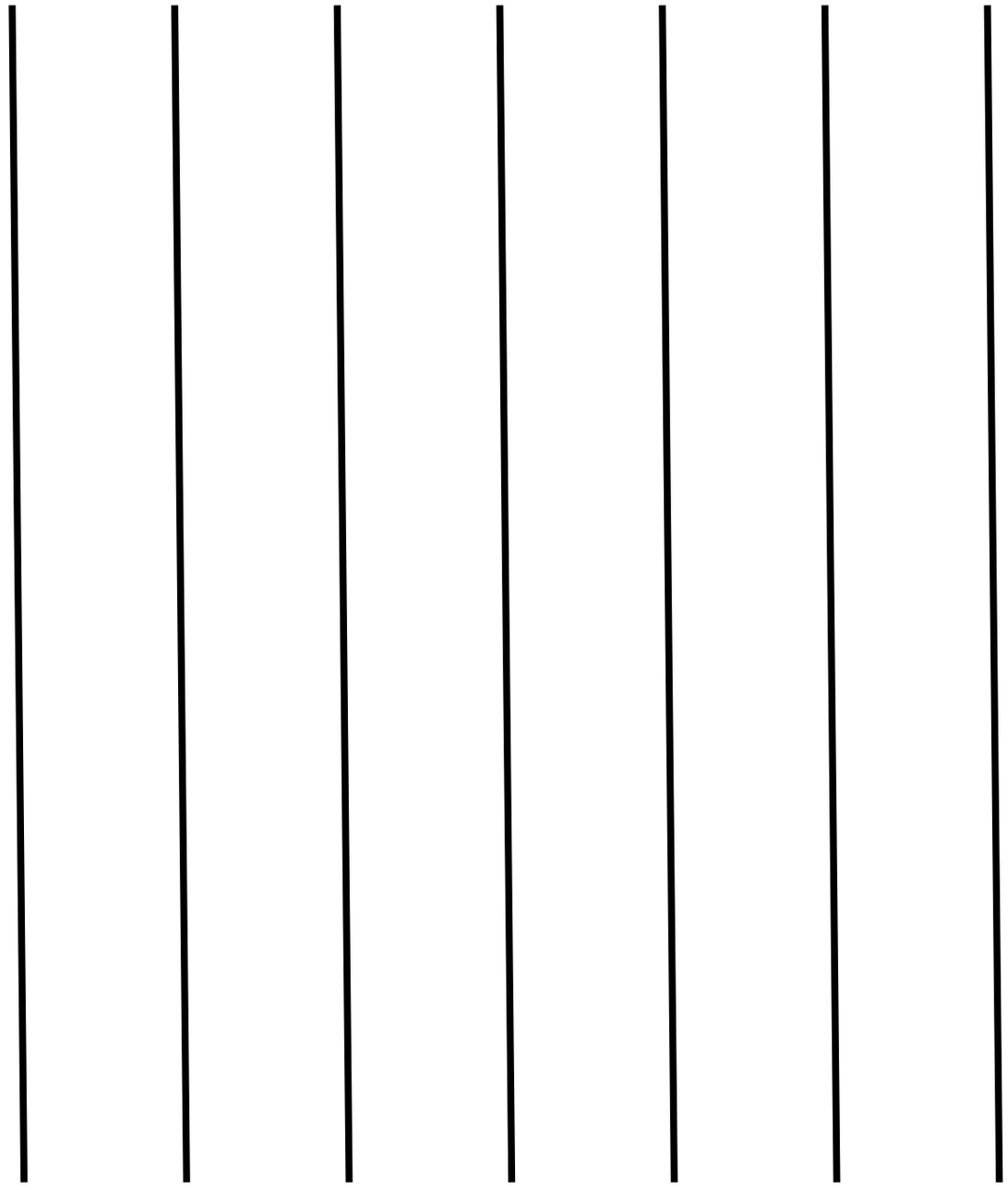
Temos o mesmo tipo de ábacos

Colunas alternadas

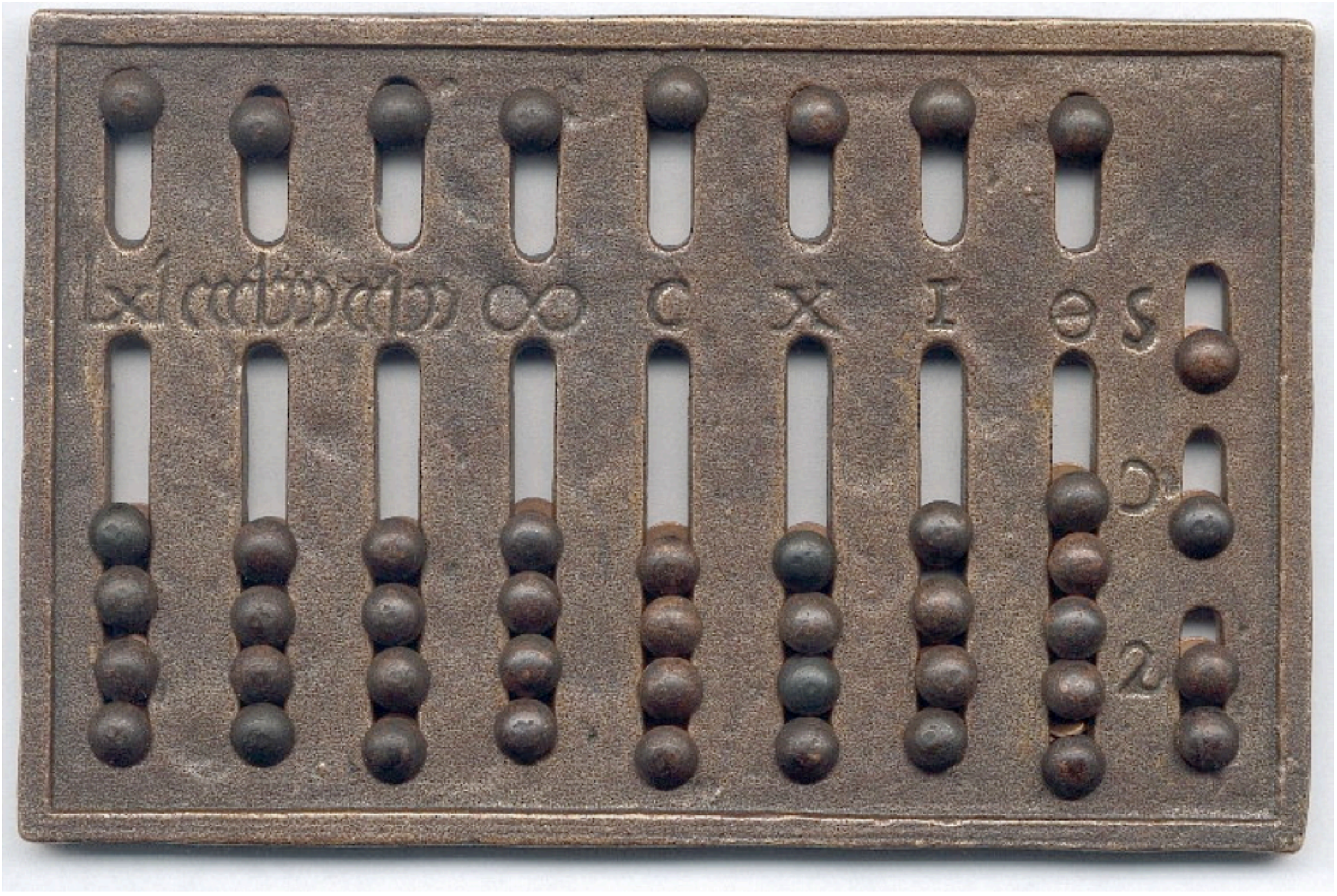
Colunas decimais

Mas também **ábacos portáteis**

D C L X V I



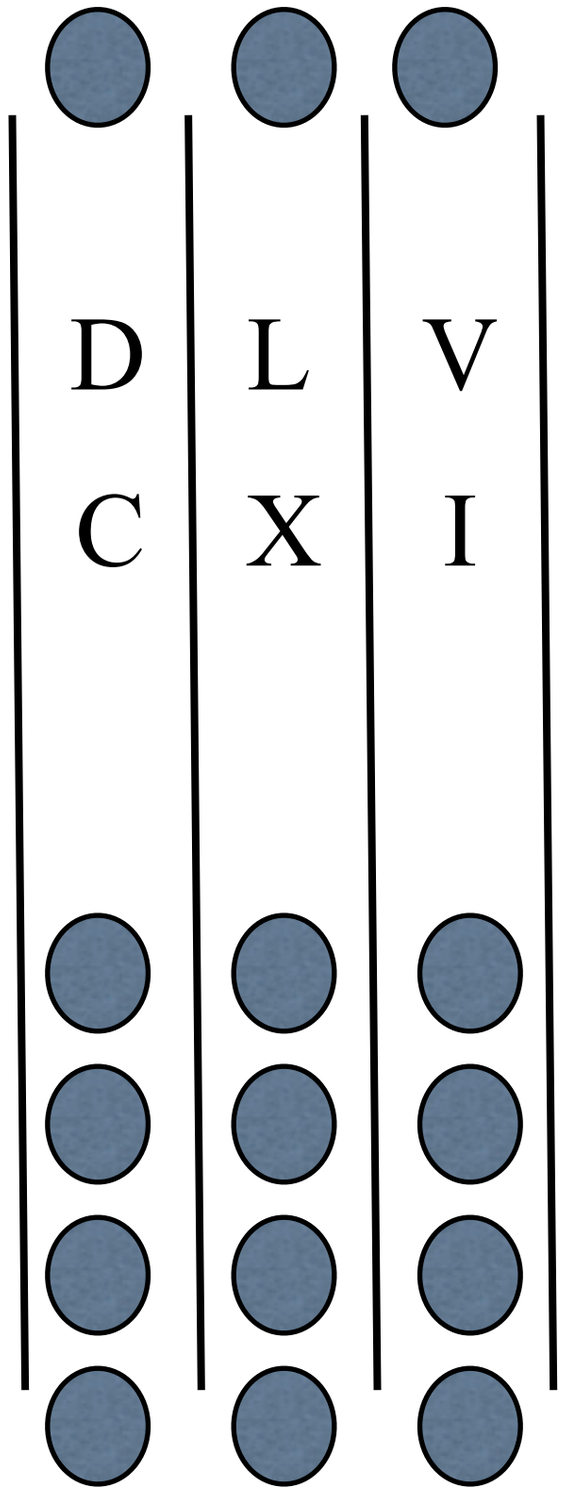
D	L	V
C	X	I



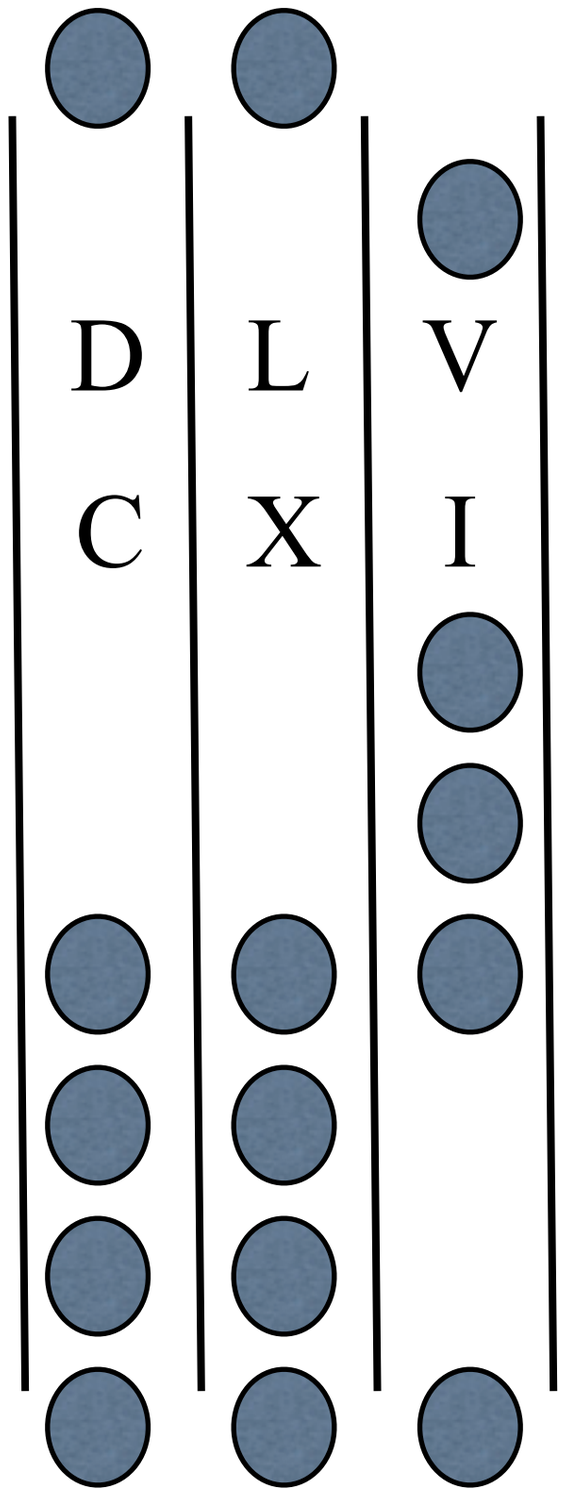
O facto de o número de peças em cada coluna ser fixo torna as operações mais difíceis

Nas somas devemos saber como “transportar”.

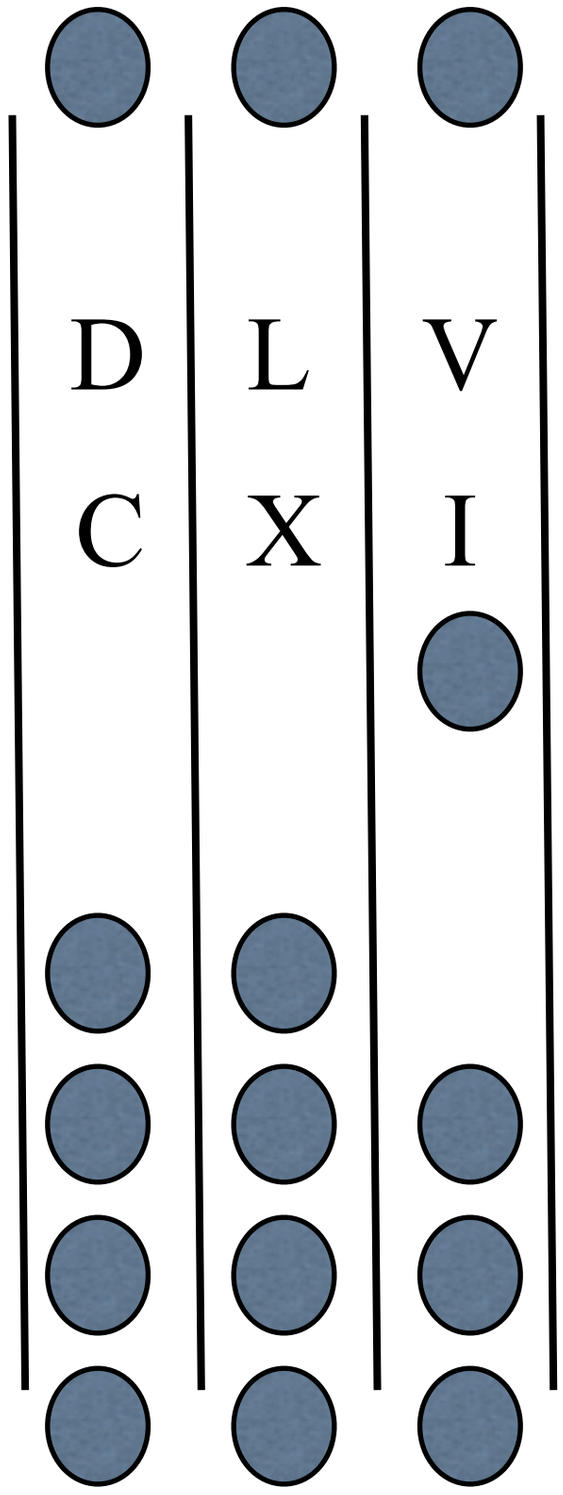
Nas subtrações, como as anti-reduções são limitadas, temos de ser engenhosos


D
L
V
C
X
I

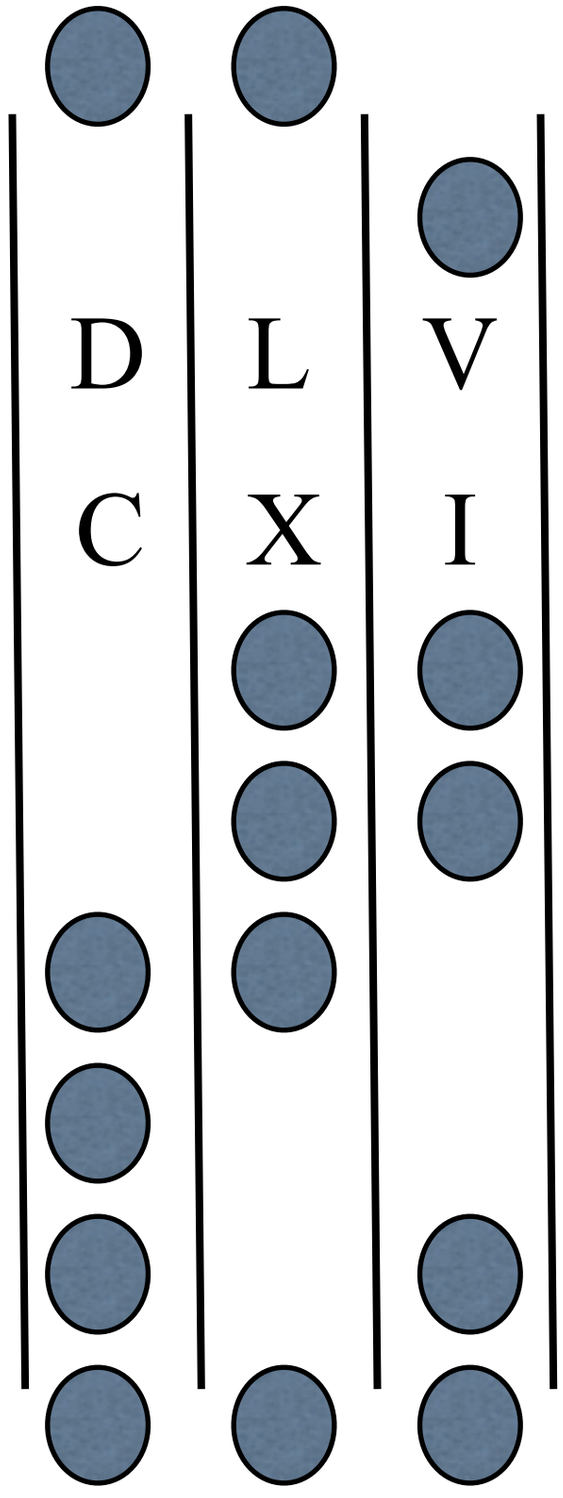
8 - 7



8 - 7 = 1

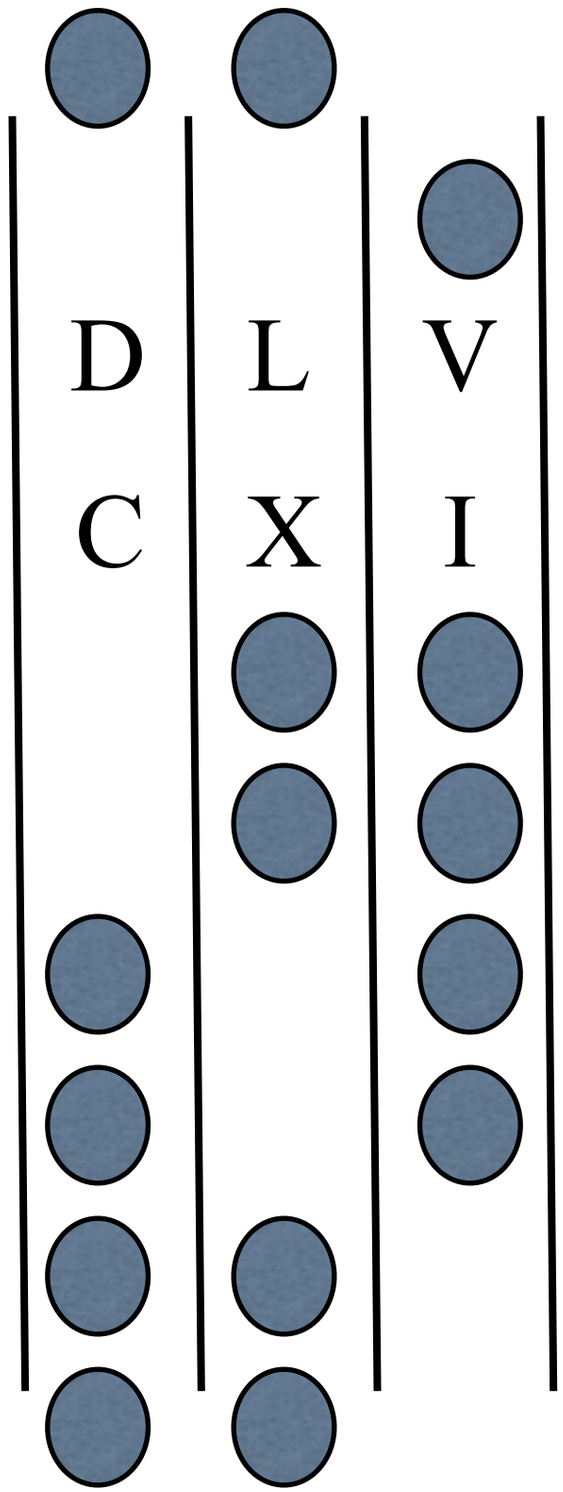


37 - 8

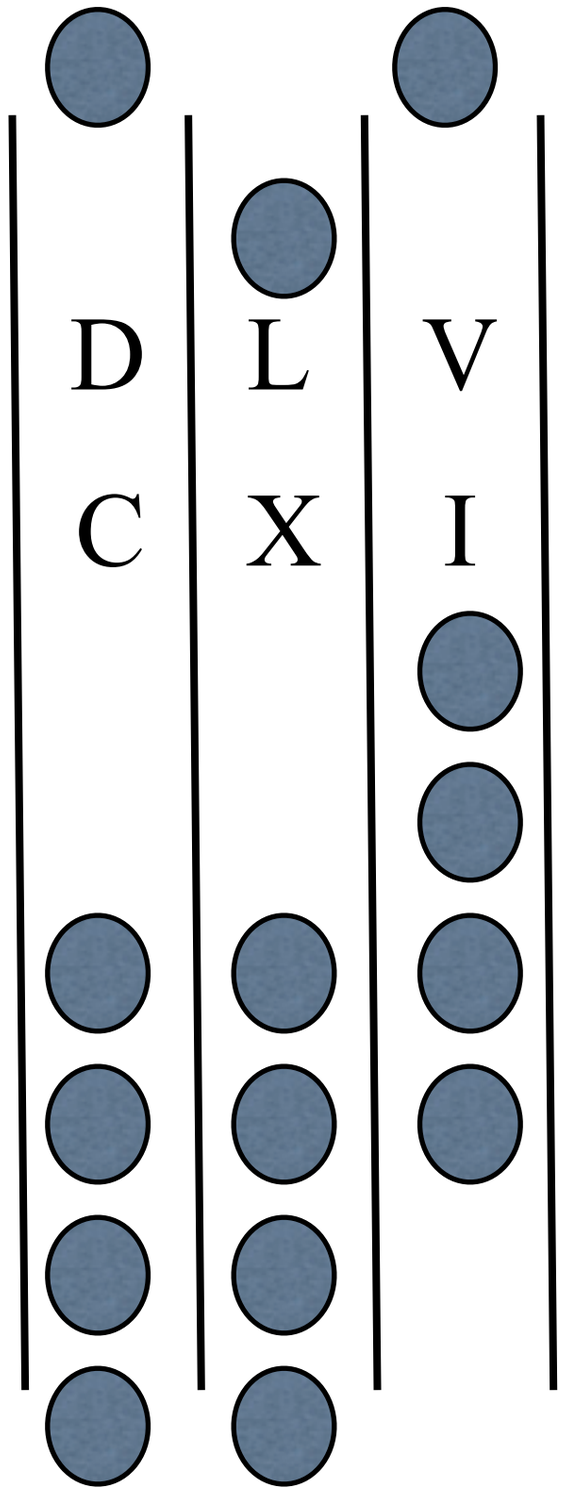


37 - 8 =

37 - 10 + 2

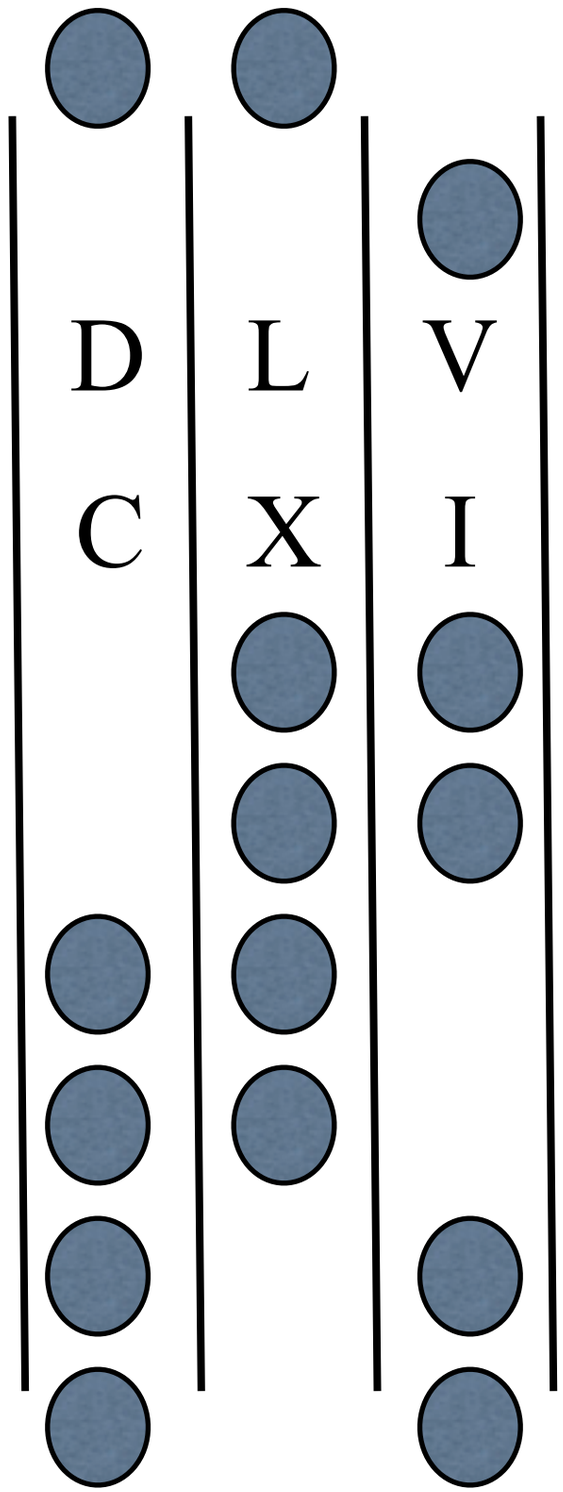


54 - 7

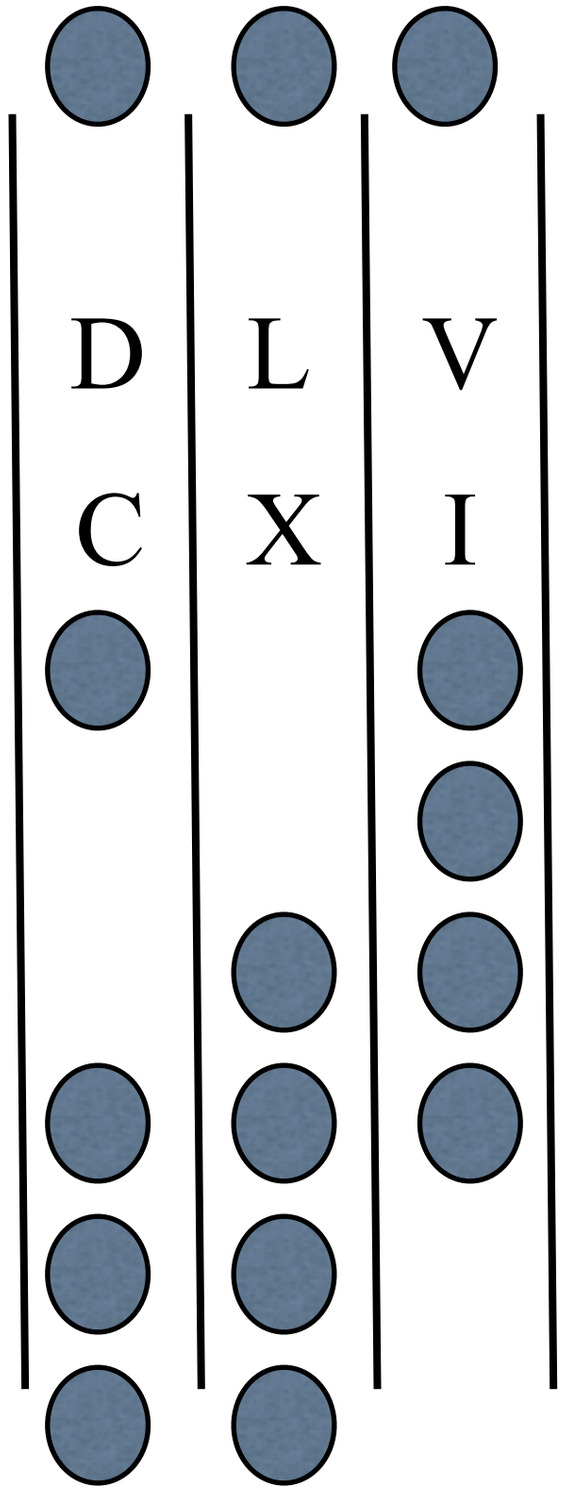


54 - 7 =

54 - 50 + 40 + 5 - 2

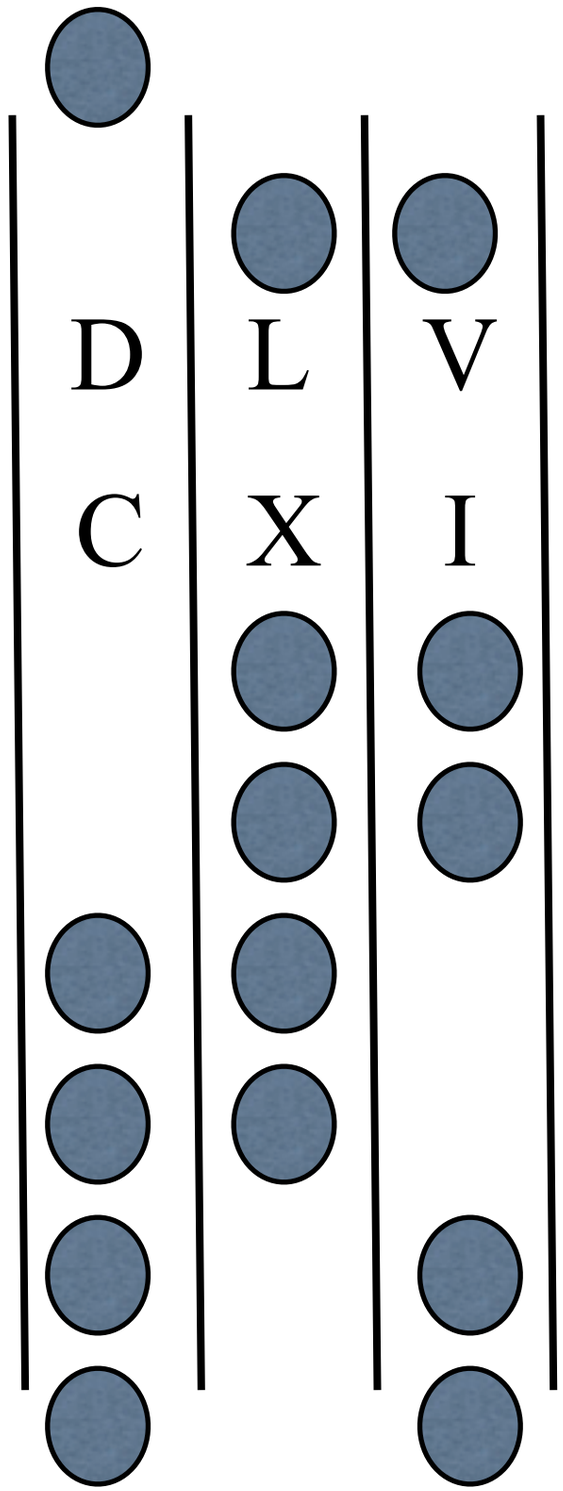


104 - 7



104 - 7 =

104 - 100 + 90 + 5 - 2



As multiplicações e as divisões eram ainda mais difíceis do que nos ábacos vulgares

Gerbert

13
87
4 019
400 520
539
100 065

Fig. 241. – Le principe de représentation des nombres entiers au moyen des apices *sur l'abaque perfectionné de Gerbert et ses disciples (sur cet abaque, comportant 27 colonnes réunies de trois en trois, les* apices *prenaient une valeur de position variant selon la colonne où ils étaient disposés ; de plus, l'absence d'unités d'un certain rang y était signifiée en laissant vide la colonne correspondante).*

(Apices - Limoges avant 1030)

As peças estão marcadas com numerais indo-árabes

Na representação de um número cada coluna pode conter, no máximo, uma peça

Os numerais destinam-se a ser lidos.

A fonte principal é Bernelin's *Liber Abaci* (999-1003).

Richer (991-998) escreveu umas linhas sobre o ábaco.

Gerbert descreve algumas regras de operação, muito confusas, numa carta de 980.

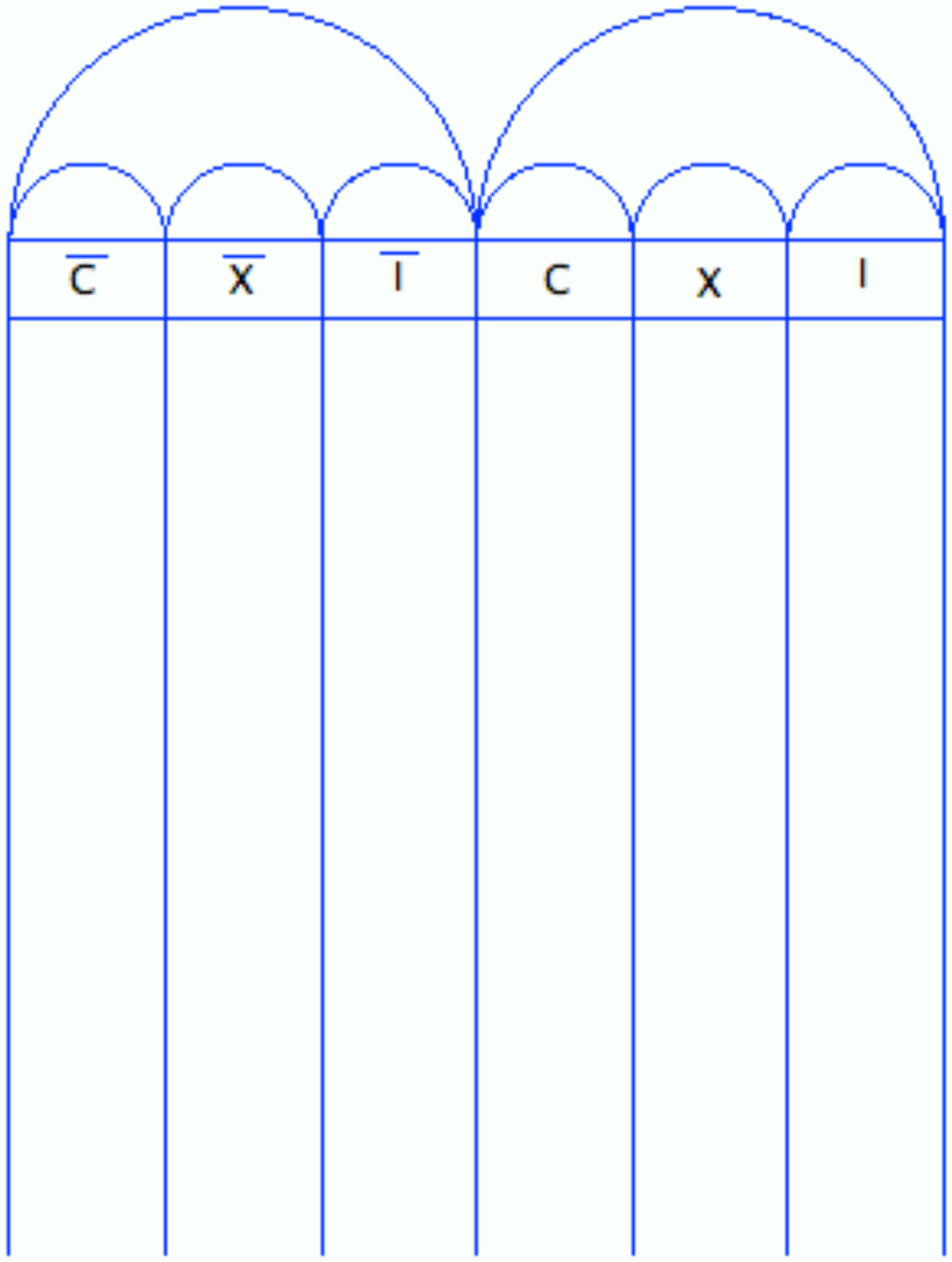
L'unité qui, ne progressant a partir d'aucun autre, est seule à posseder le principe naturel de tous les nombres [...] Comme l'unité est le principe dans la centaine, ansi le millier aussi parera être à un observateur attentif, d'une certaine façon, le principe des centaines de mille.

[...]

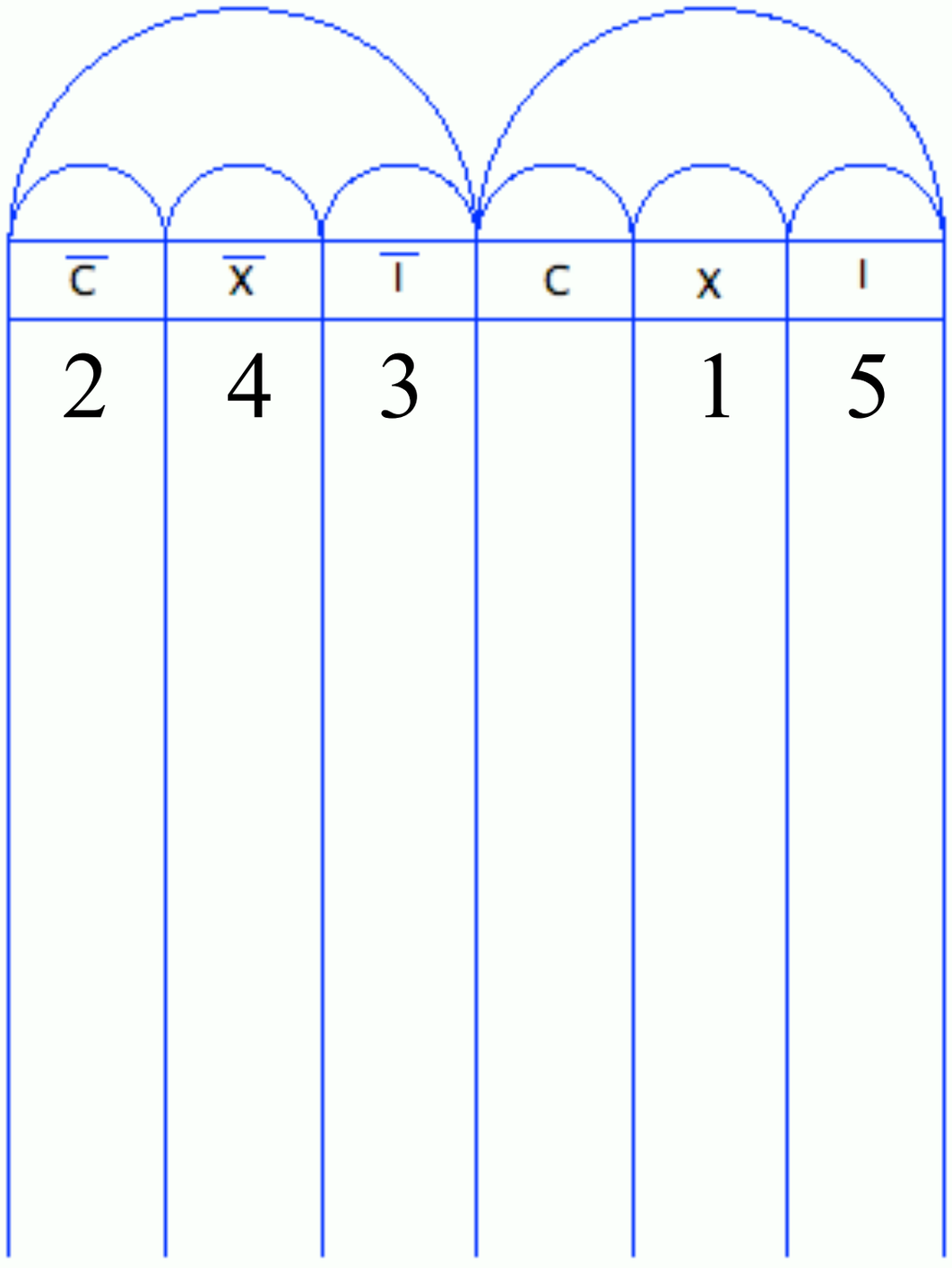
la dizaine se développe à partir de la multiplication de la première unité, ansi la muliplication de cette seconde unité fait surgir la troisième unité, c'est-à-dire la centaine. Et alors, d'une certaine façon, on revient à la première unité...

L’unité, qui est dite le premier nombre, est ansi figurée ..., soit par la lettre grecque alpha...

Tout nombre, multiplié par une unité, placera le doigt (digitum) dans la même colonne que l’unité et l’article (articulum) dans la seconde. Tout nombre multiplié par une dizaine placera le doigt dans la seconde colonne, et l’article dans la triosième....


C
X
I
C
X
I

243 015

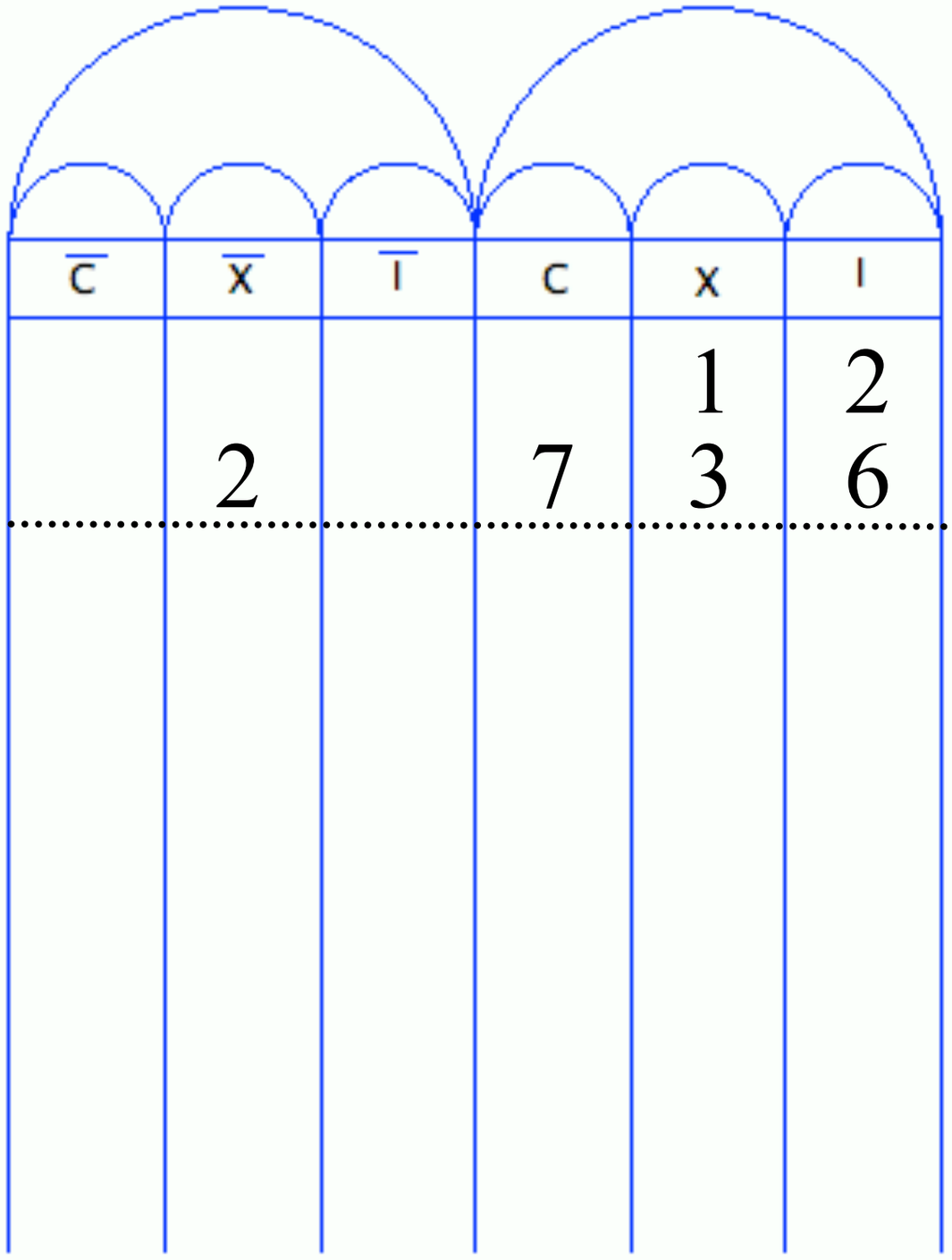


Vamos usar os nossos algarismos para representar os de Gerbert.

O problema recreativo de Bernelin: 12 quartos, 12 carpetes, 12 homens, 12 mulheres, 12 crianças. Quantas crianças?

12 x 12 x 12 x 12 x 12

12 x 20 736

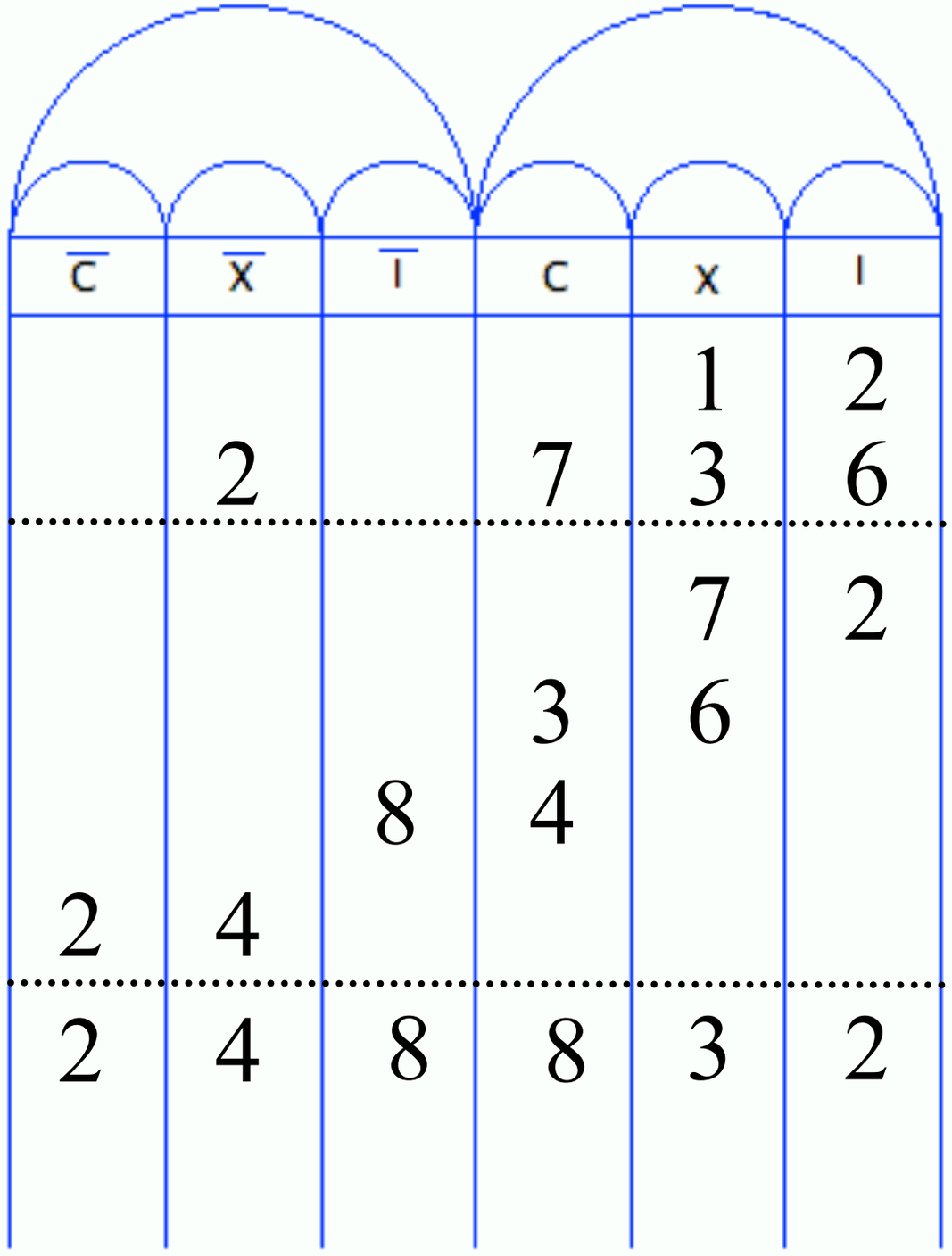

C̄ X̄ Ī C X I
1 2
2 7 3 6

C̄	X̄	Ī	C	X	I
				1	2
	2		7	3	6
				7	2
			3	6	
		8	4		
2	4				
2	4	8	8	3	2

Parece a nossa forma de calcular...

Isto é novo e estranho para quem estava habituado a usar os ábacos de peças neutras.

Queme estivesse familiarizado com os trabalhos de Al-Kwarizmi usaria papel e lápis...



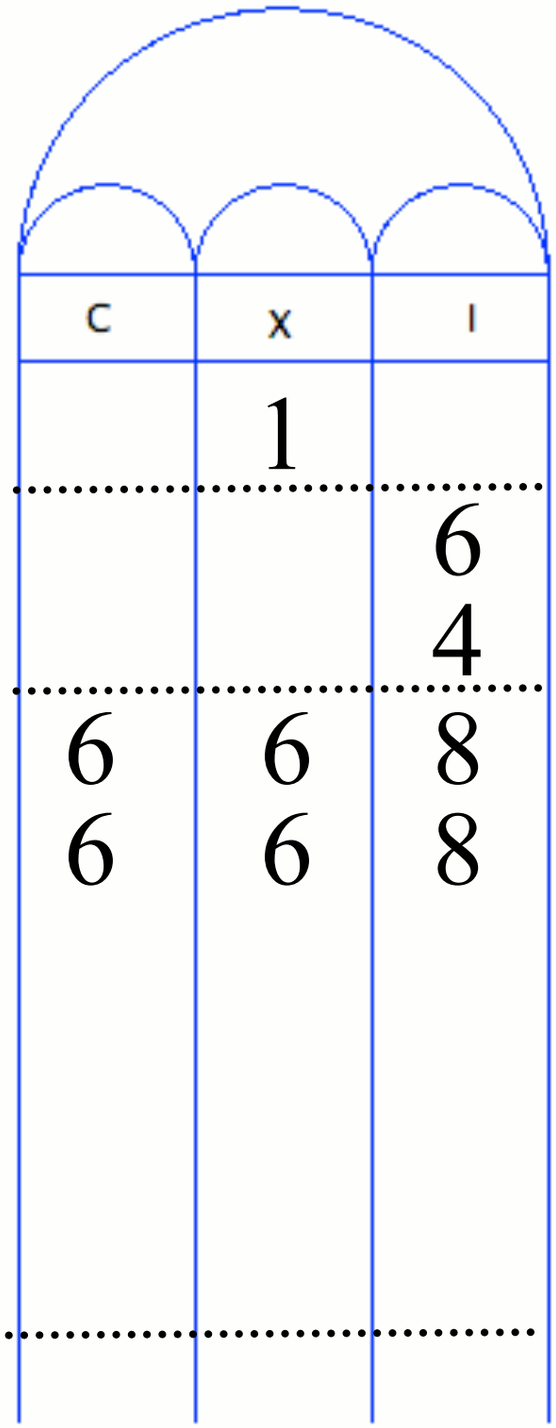
Este método deve-se a Gerbert

Divisio ferrea

Bernelin dá o exemplo 668 : 6

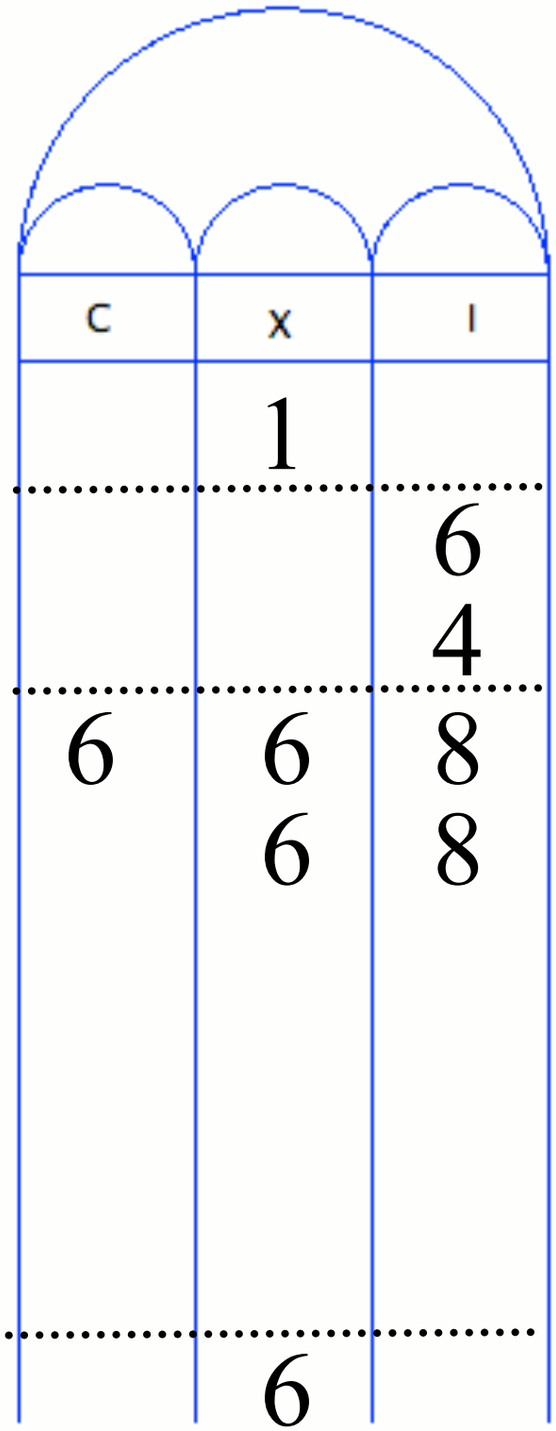
O truque: usar um divisor auxiliar, aqui 10, para tornar a conta mais fácil

Depois corrige-se o erro cometido...



Divisio ferrea

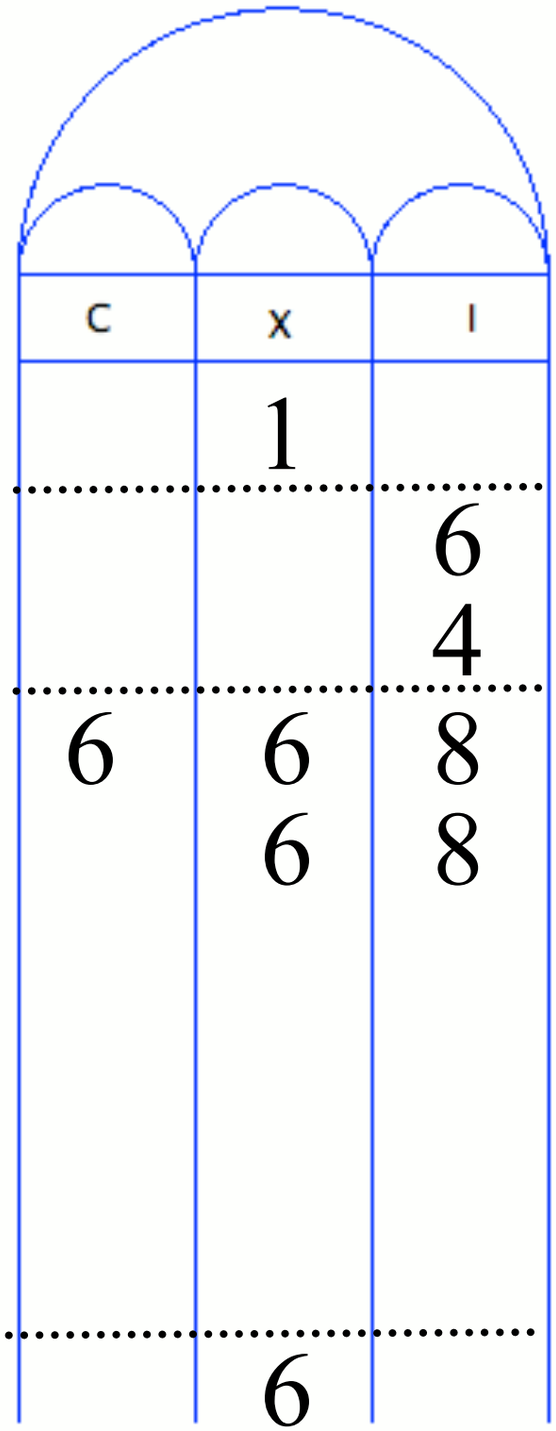
Divida-se 600 por 10



Divisio ferrea

Divida-se 600 por 10

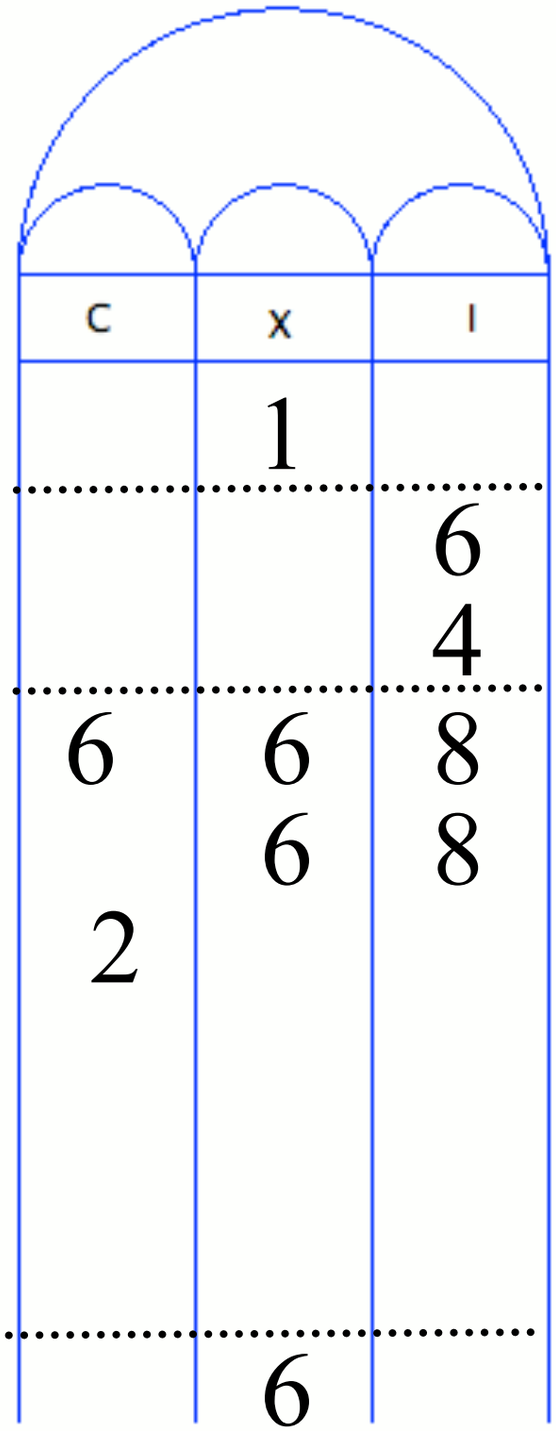
Multiplique-se 60 por 4 - a diferença - e some-se a 68



Divisio ferrea

Divida-se 600 por 10

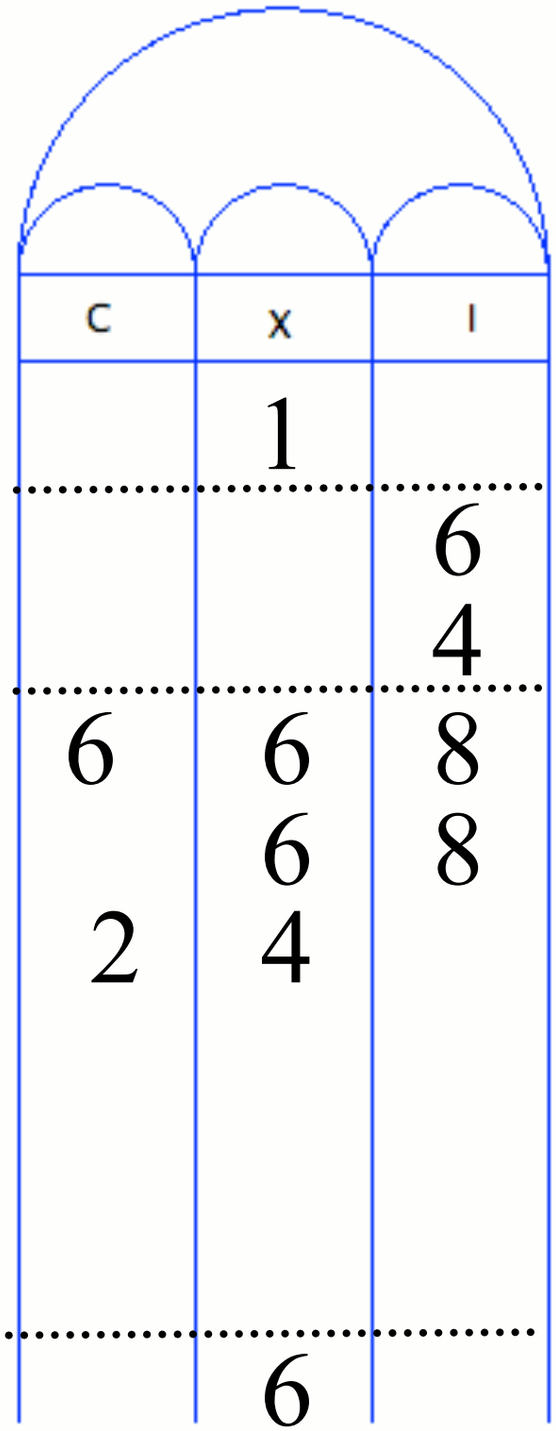
Multiplique-se 60 por 4 - a diferença - e some-se a 68



Divisio ferrea

Divida-se 600 por 10

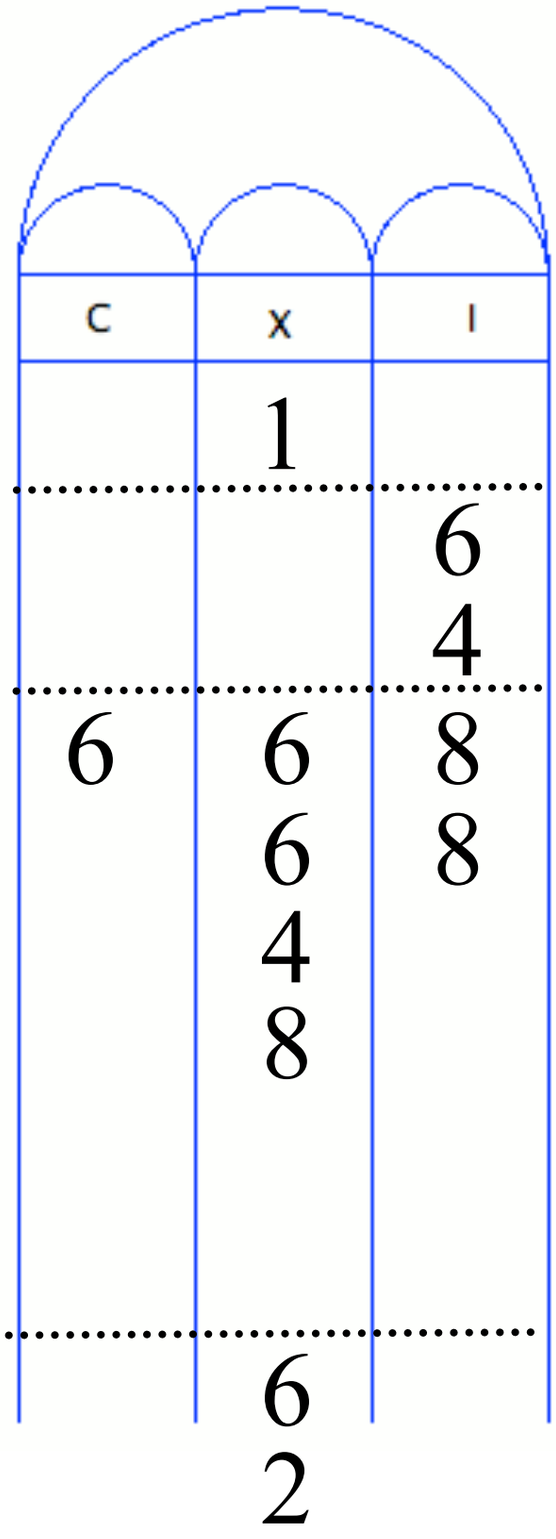
Multiplique-se 60 por 4 - a diferença - e some-se a 68



Divisio ferrea

Divida-se 200 por 10

Multiplique-se 20 por 4 e some-se a 68+40

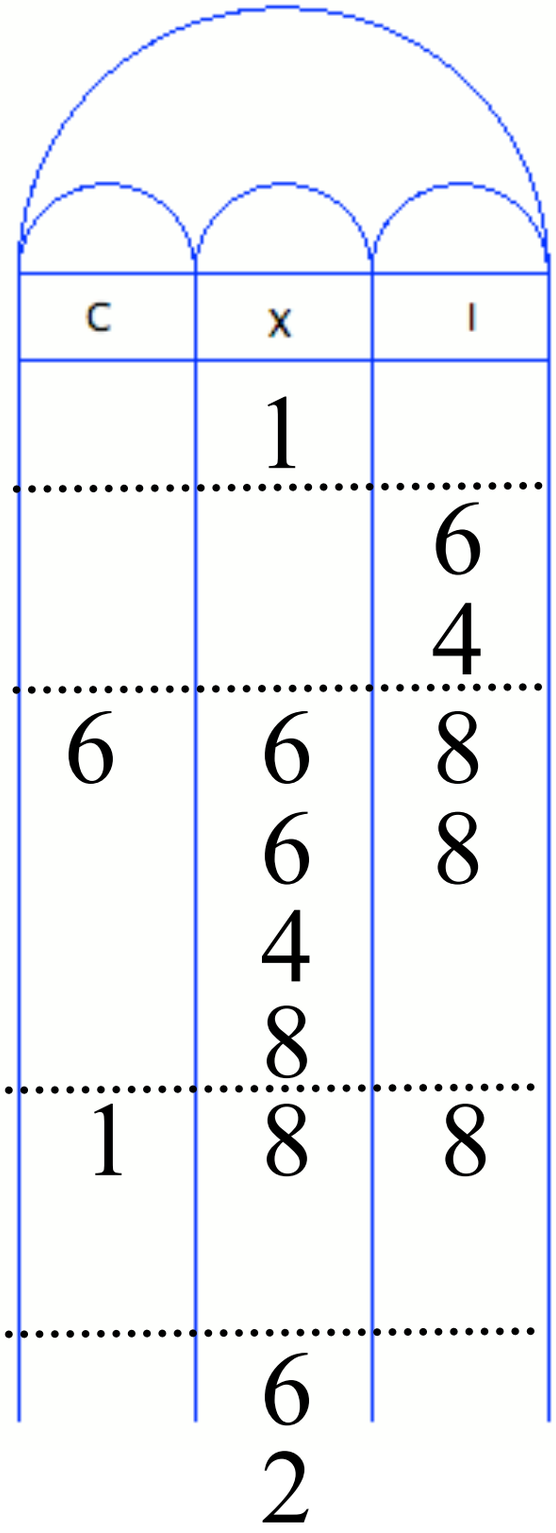


Divisio ferrea

Verifiquemos a situação actual...

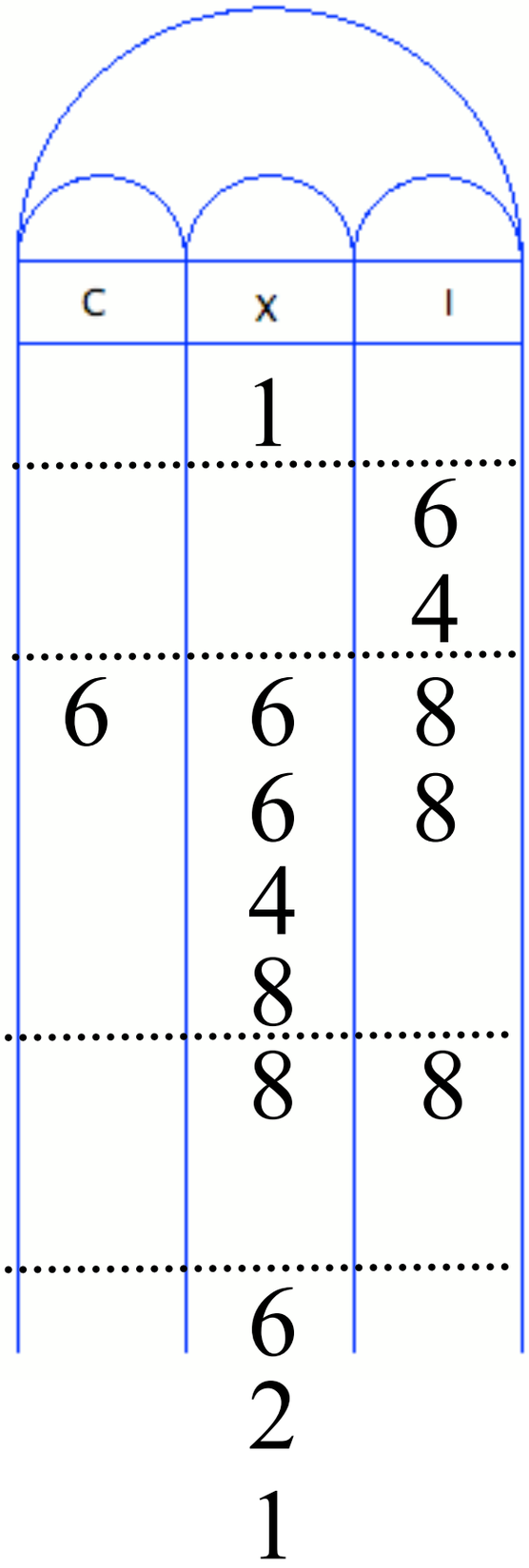
O nosso dividendo é agora 188, temos um quociente de 60+20. Continuemos...

Dividimos100 por 10...

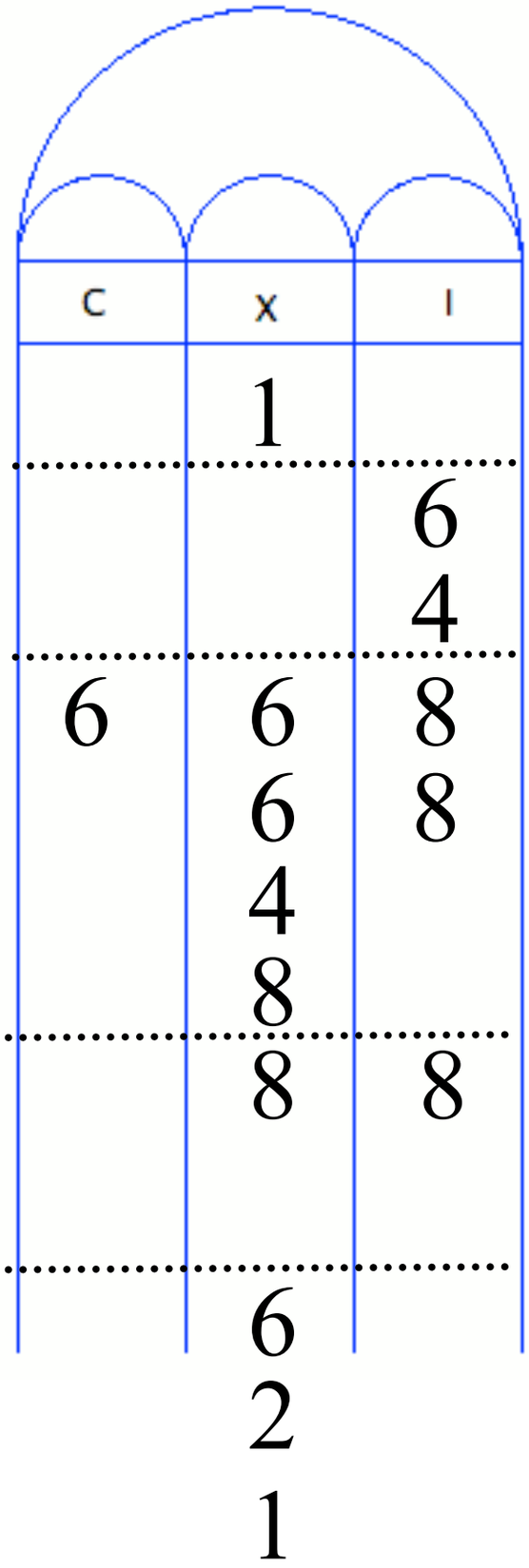


Divisio ferrea

Dividimos100 por 10...



Divisio ferrea



e corrigimos

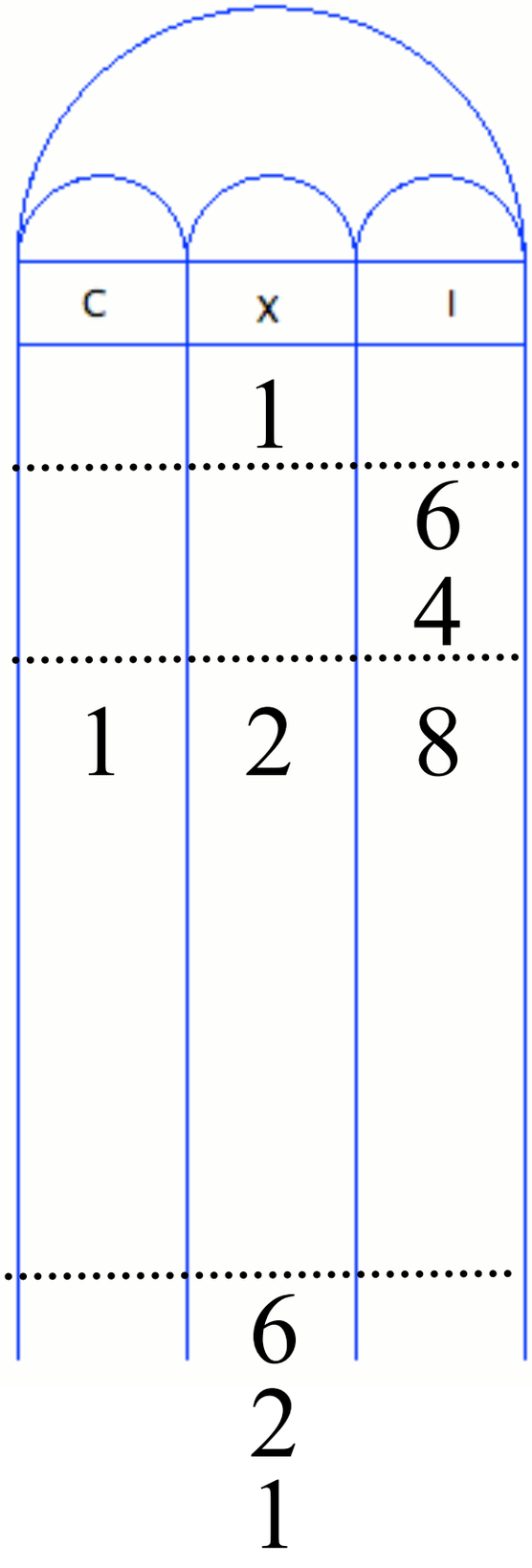
Divisio ferrea

e corrigimos

C	X	I
	1	
		6
		4
6	6	8
	6	8
	4	
	8	
	8	8
	4	
	6	
	2	
	1	

Divisio ferrea

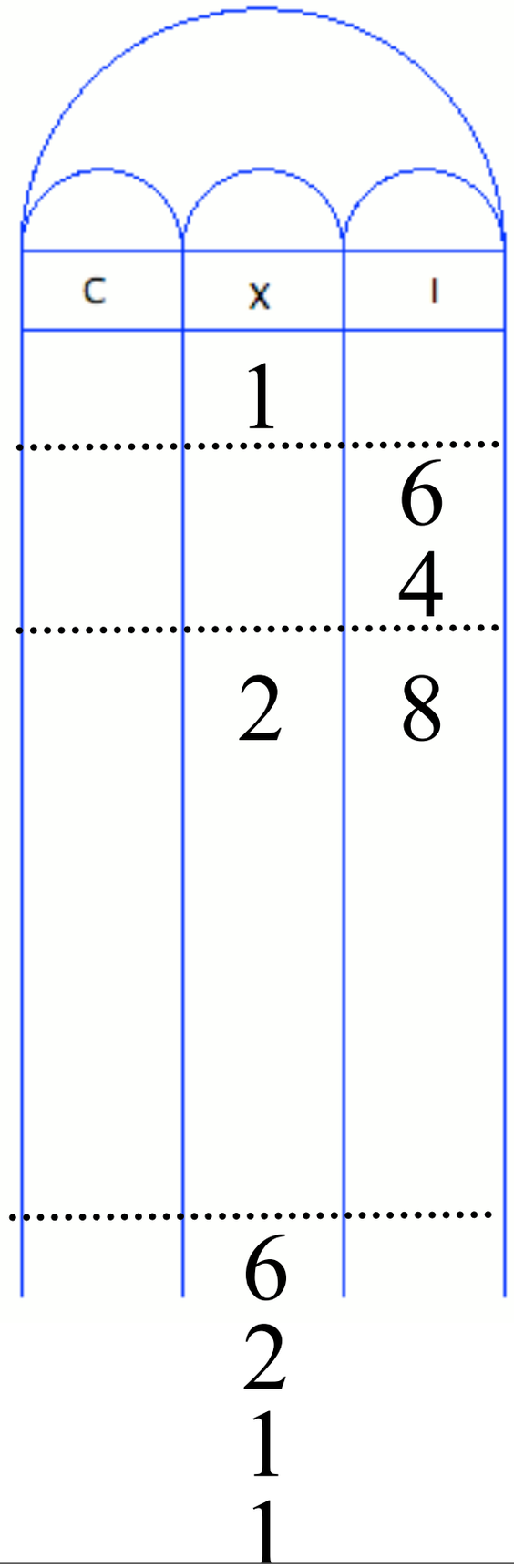
Verificando...



Divisio ferrea

Dividimos100 por 10

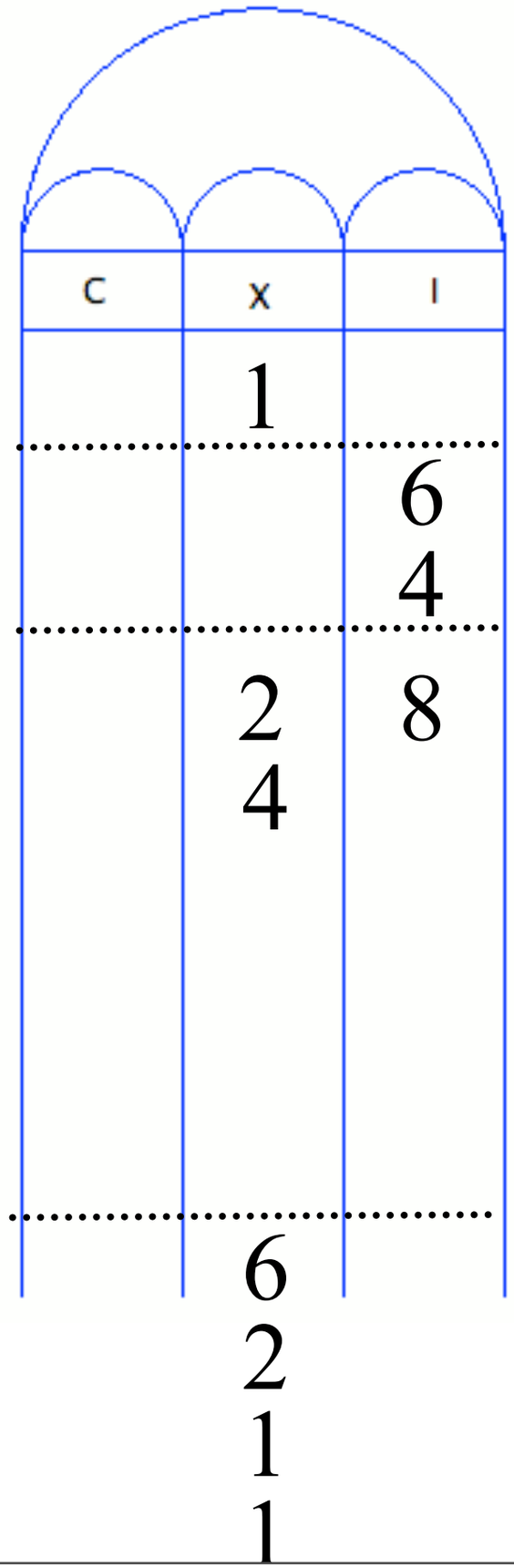
e somamos 10x4 ao quociente, para corrigir



Divisio ferrea

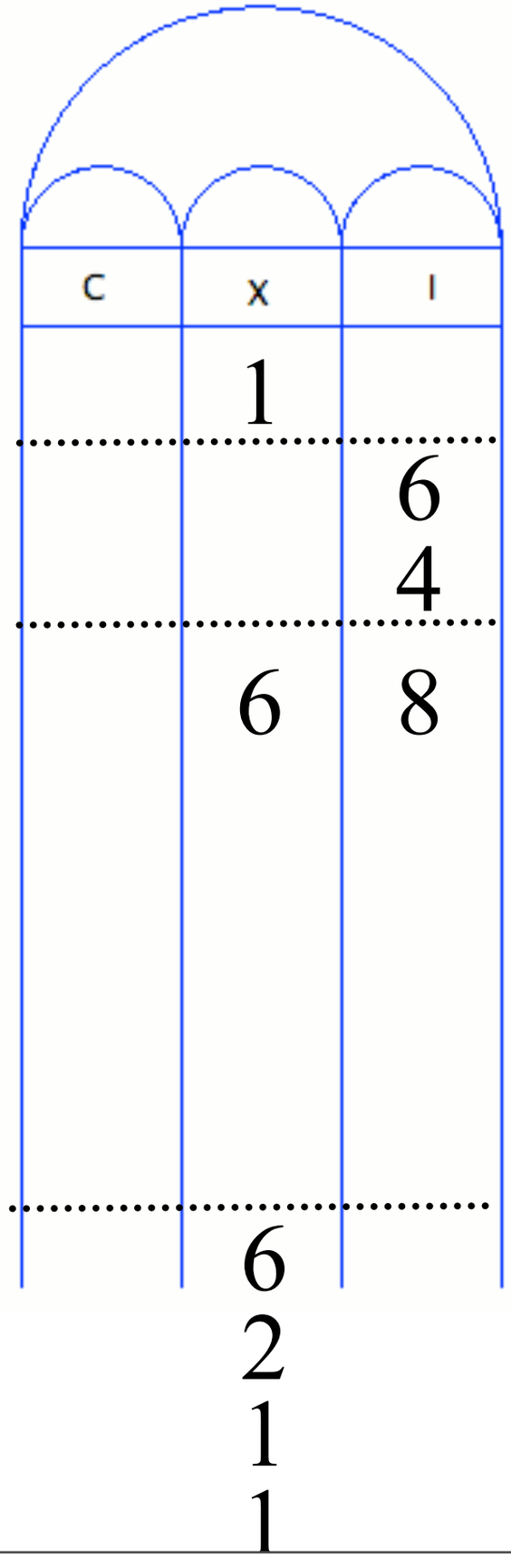
Dividimos100 por 10

e somamos 10x4 ao quociente, para corrigir



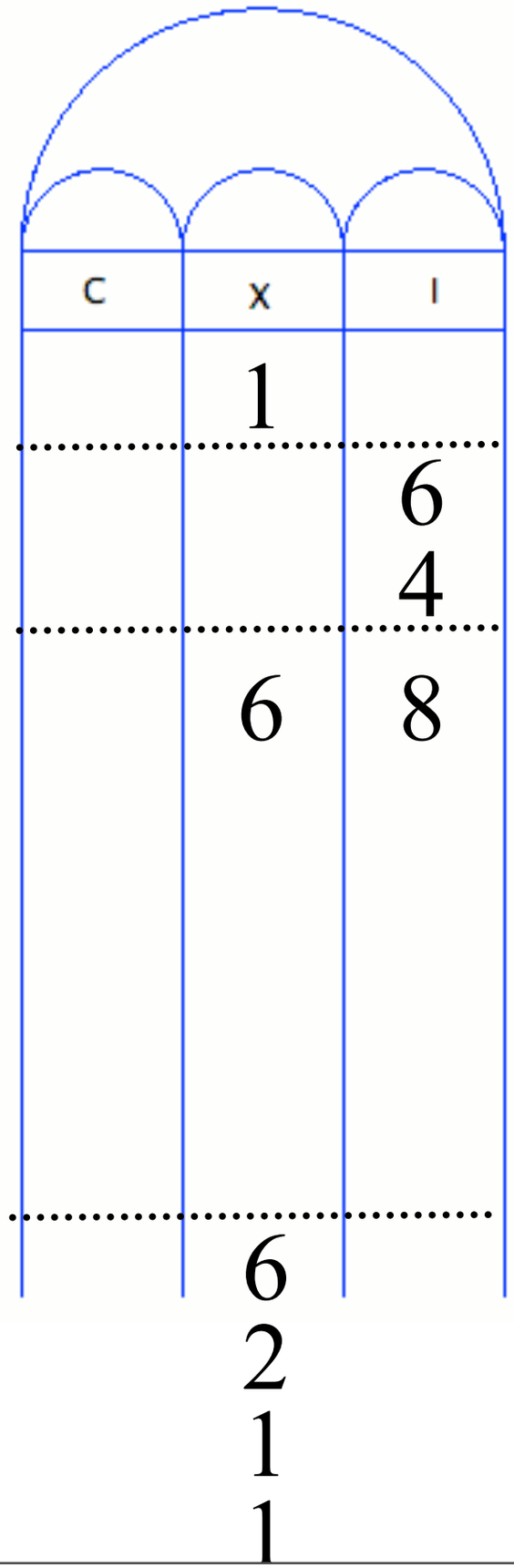
Divisio ferrea

Verificando...



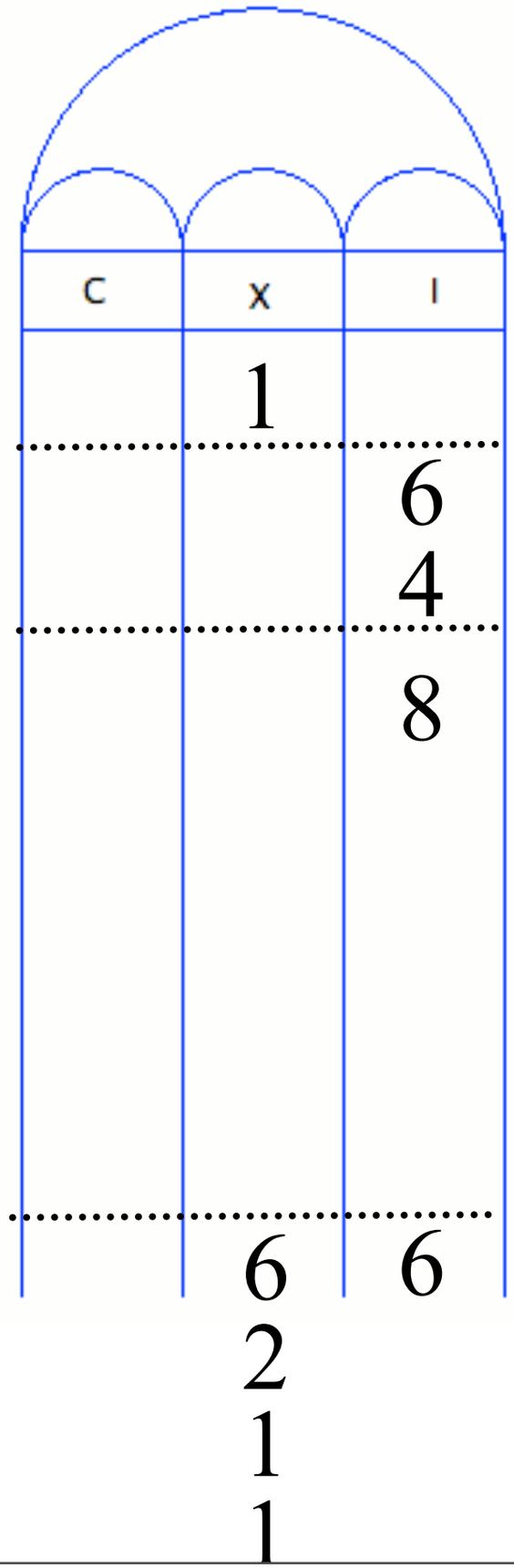
Divisio ferrea

Dividimos 60 por 10



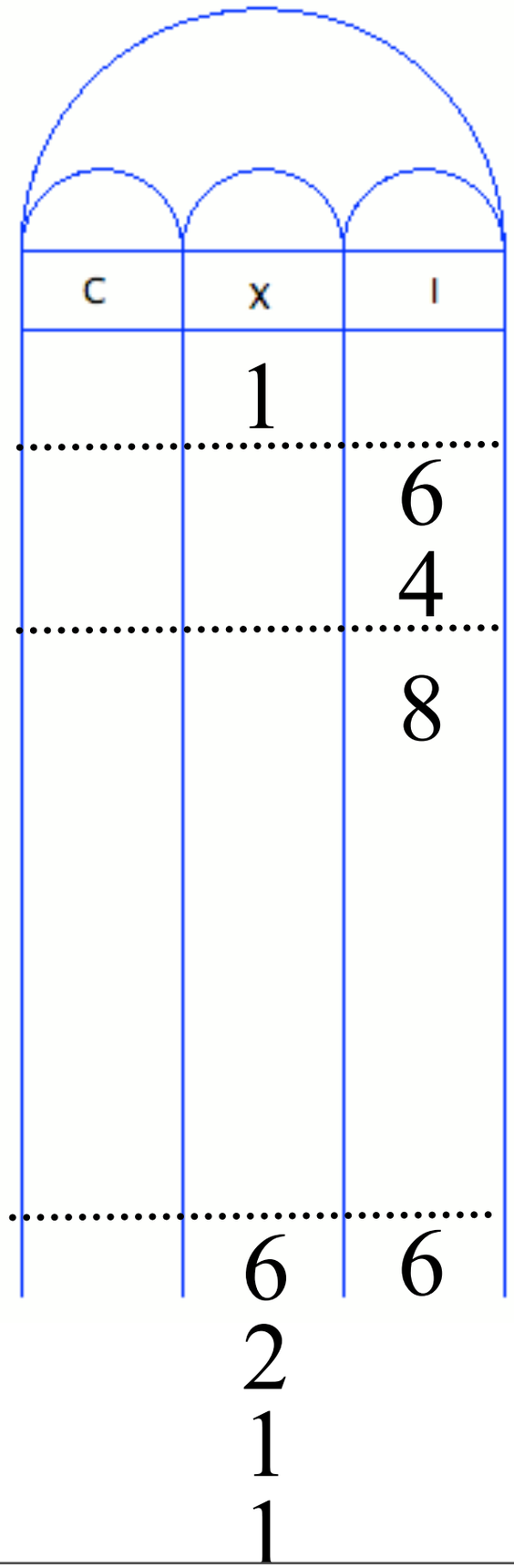
Divisio ferrea

Multiplicamos 4 por 6 e somamos a 8



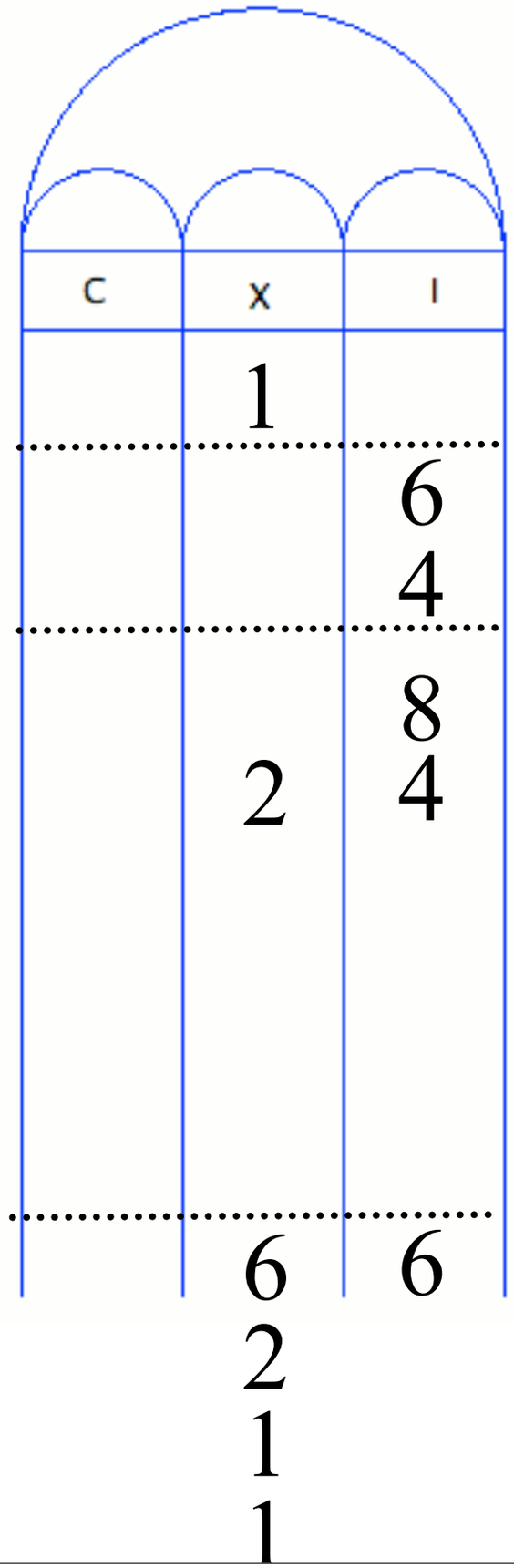
Divisio ferrea

Multiplicamos 4 por 6 e somamos a 8



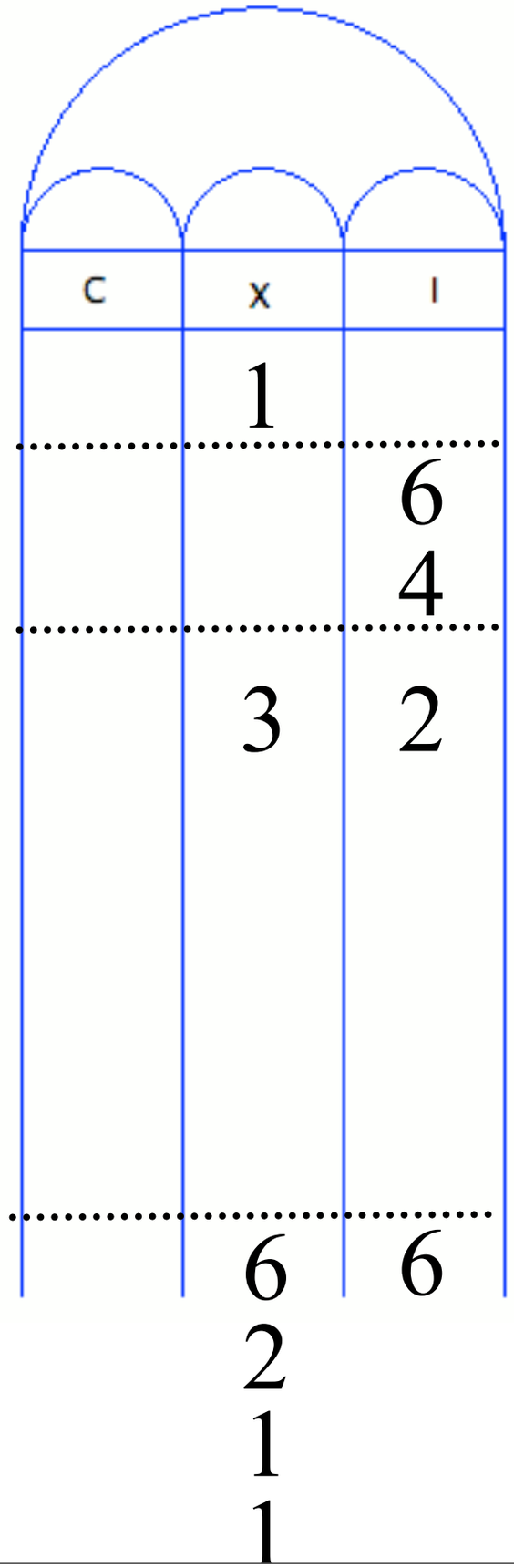
Divisio ferrea

Multiplicamos 4 por 6 e somamos a 8



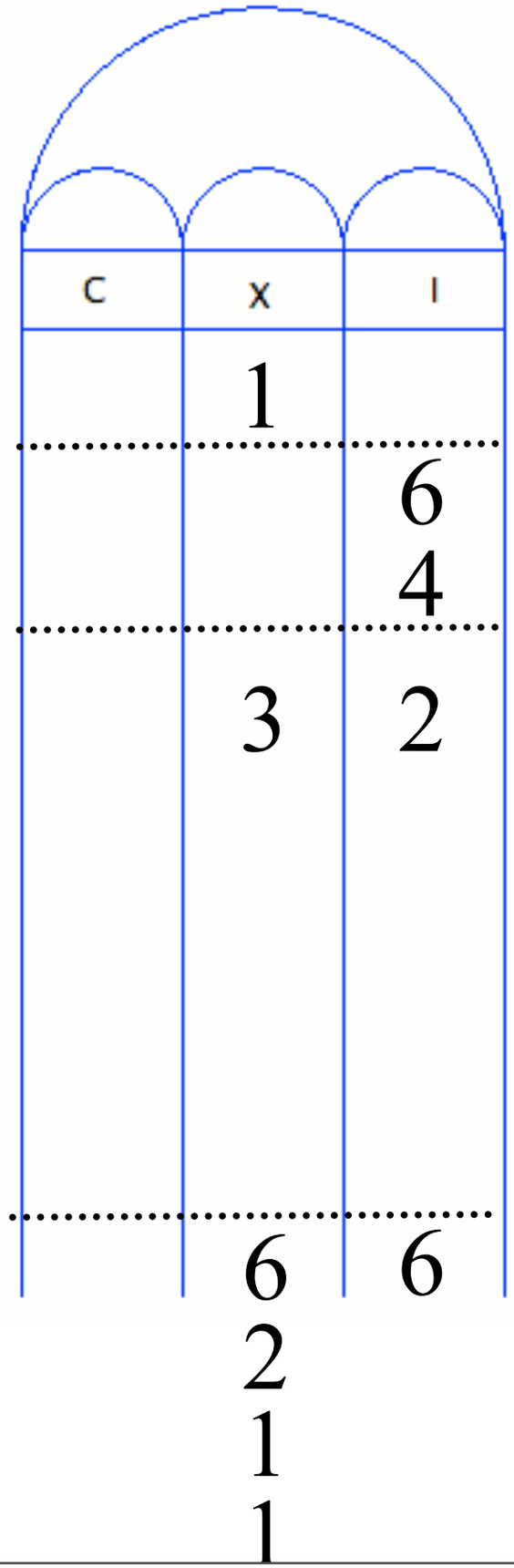
Divisio ferrea

Verificando...



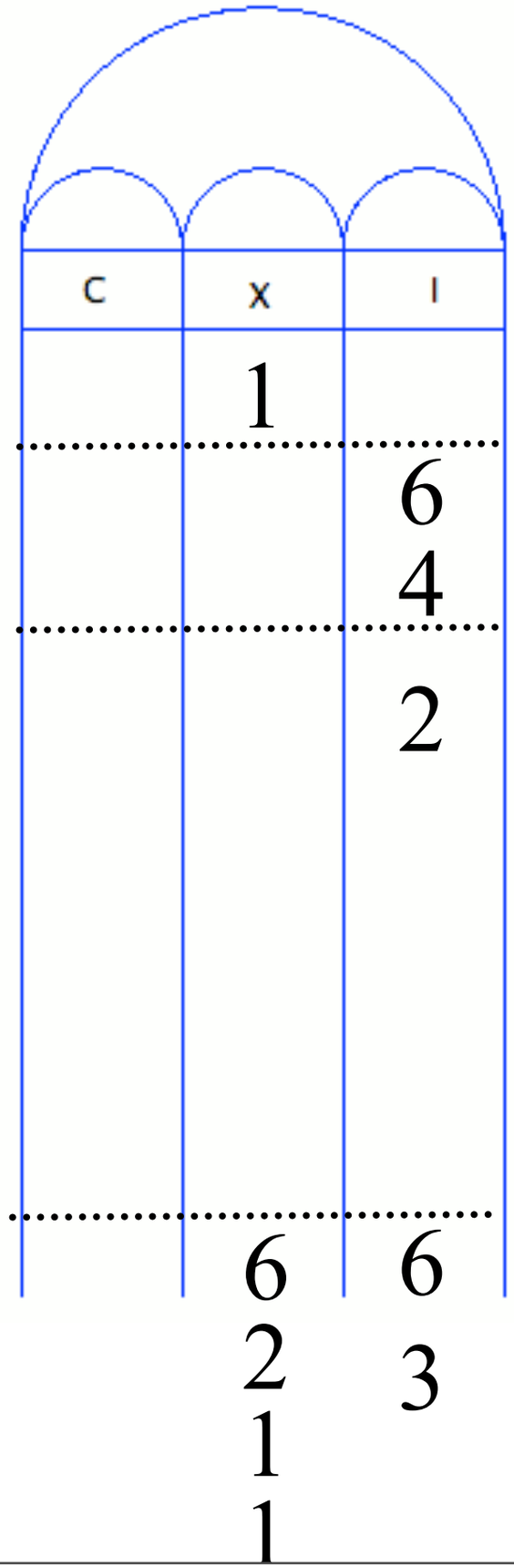
Divisio ferrea

Dividimos 30 por 10



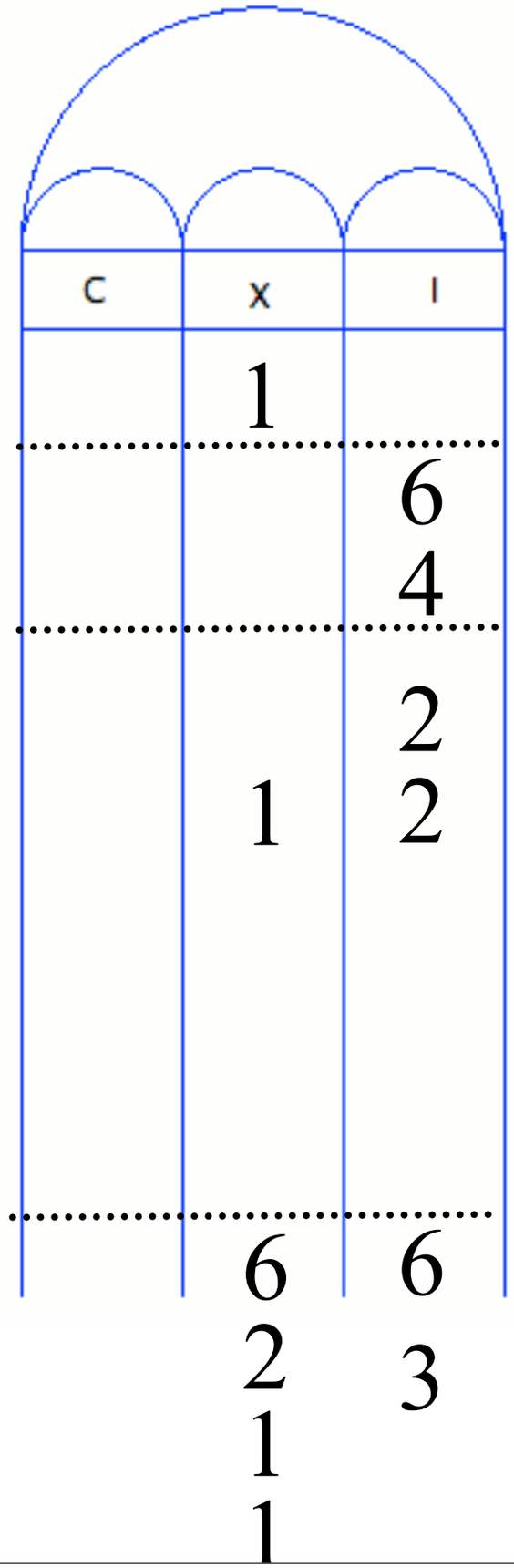
Divisio ferrea

corrigimos somando ao quaociente 4x3



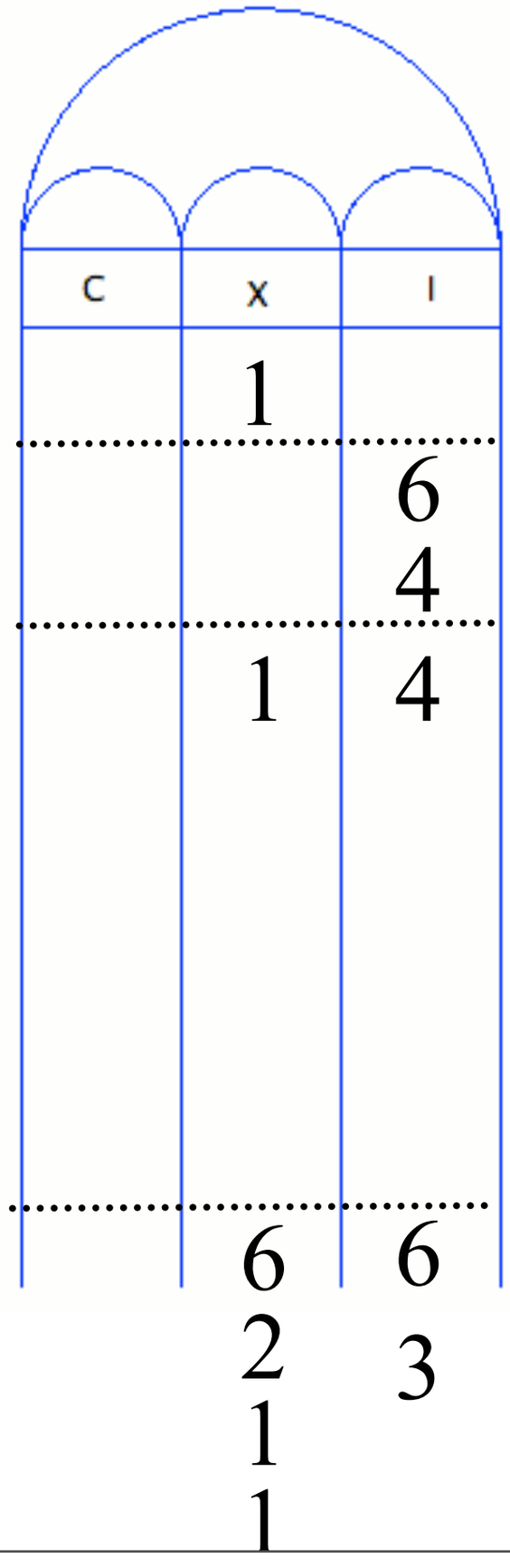
Divisio ferrea

corrigimos somando ao quaociente 4x3



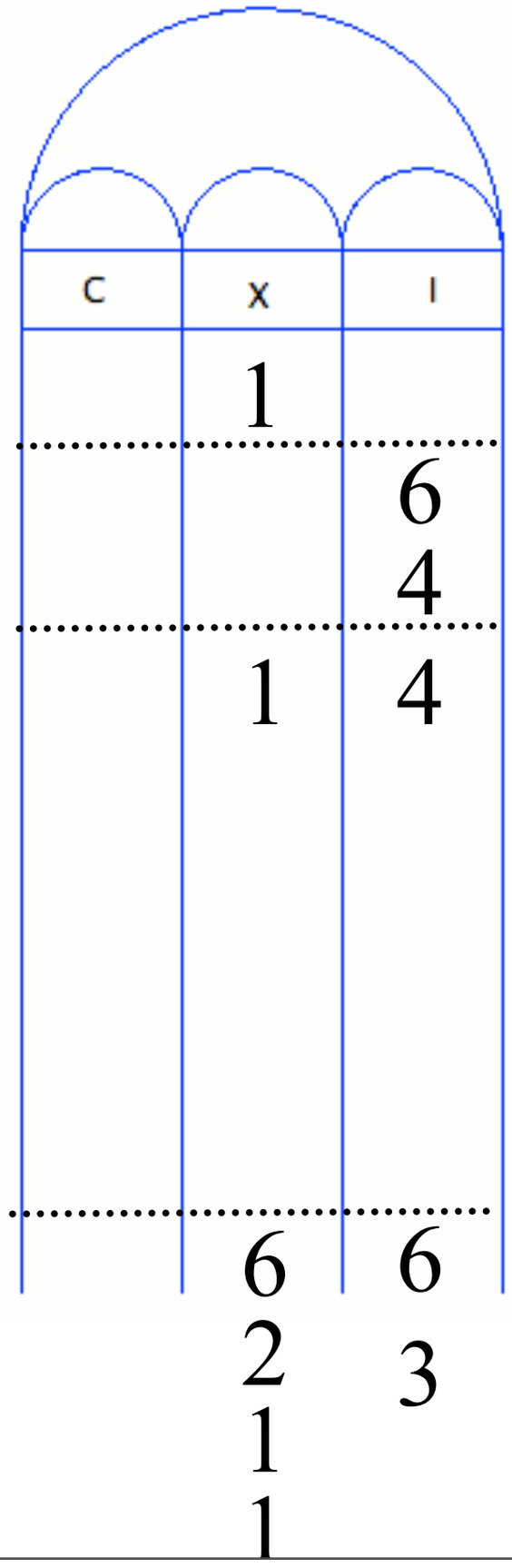
Divisio ferrea

Verificando...



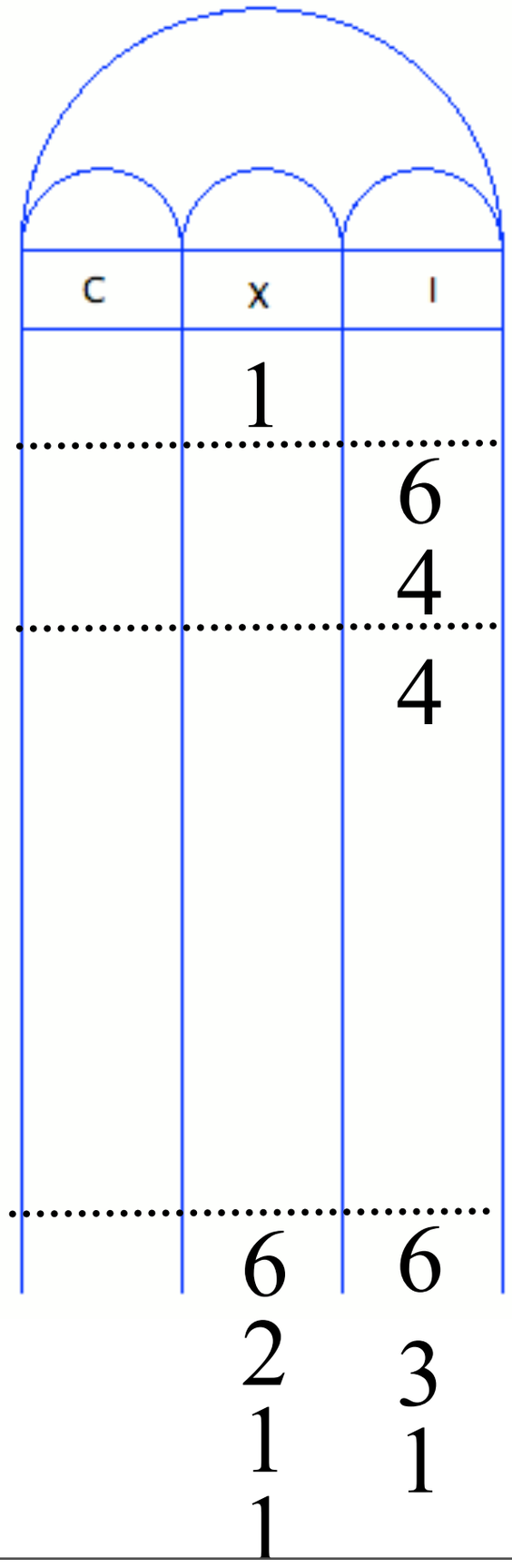
Divisio ferrea

Dividir 10 por 10



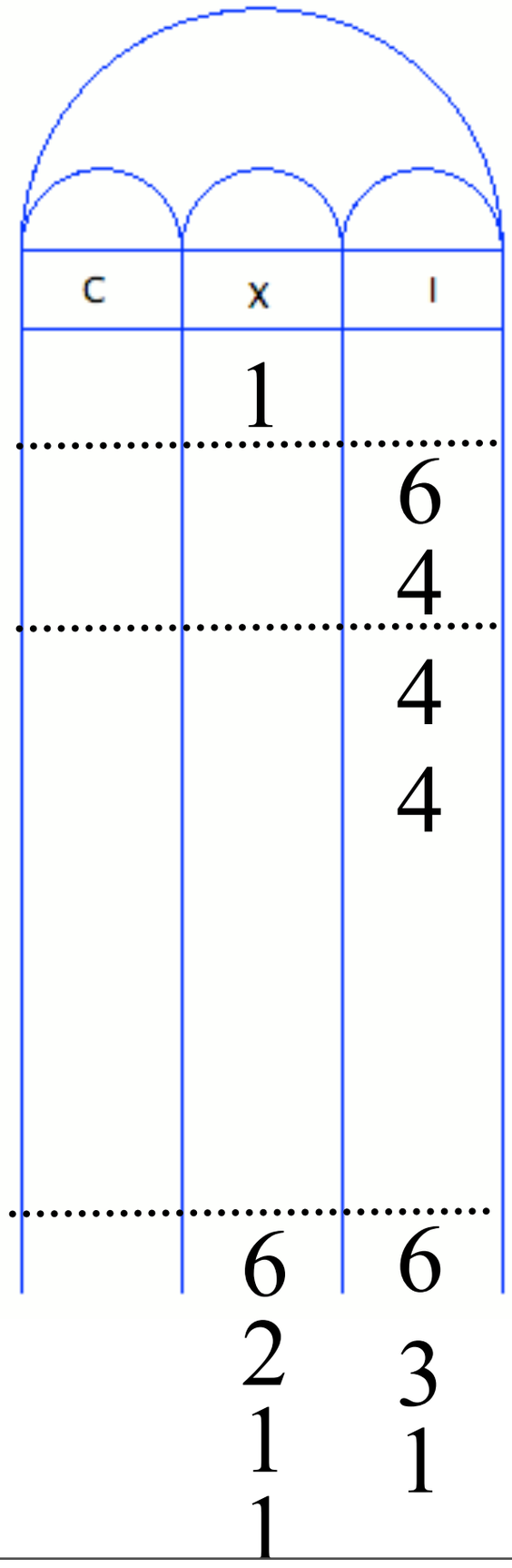
Divisio ferrea

Somar 4x1



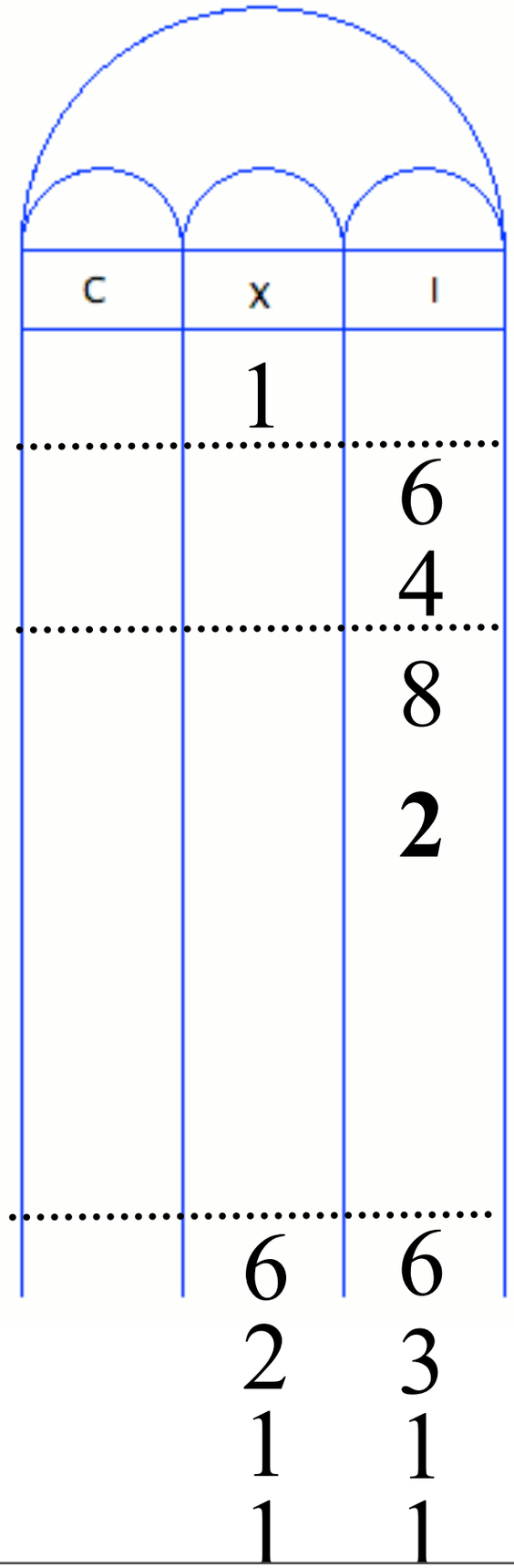
Divisio ferrea

Verificar



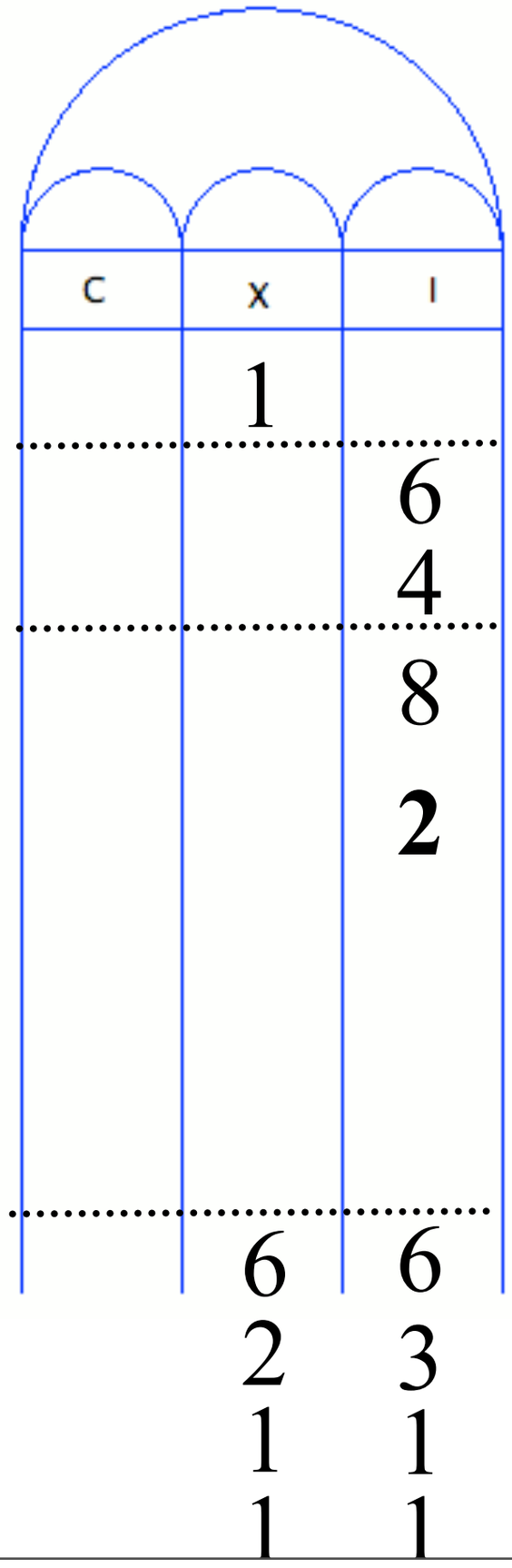
Divisio ferrea

Não podemos dividir 8 por 10.
Dividimos por 6 e terminamos.



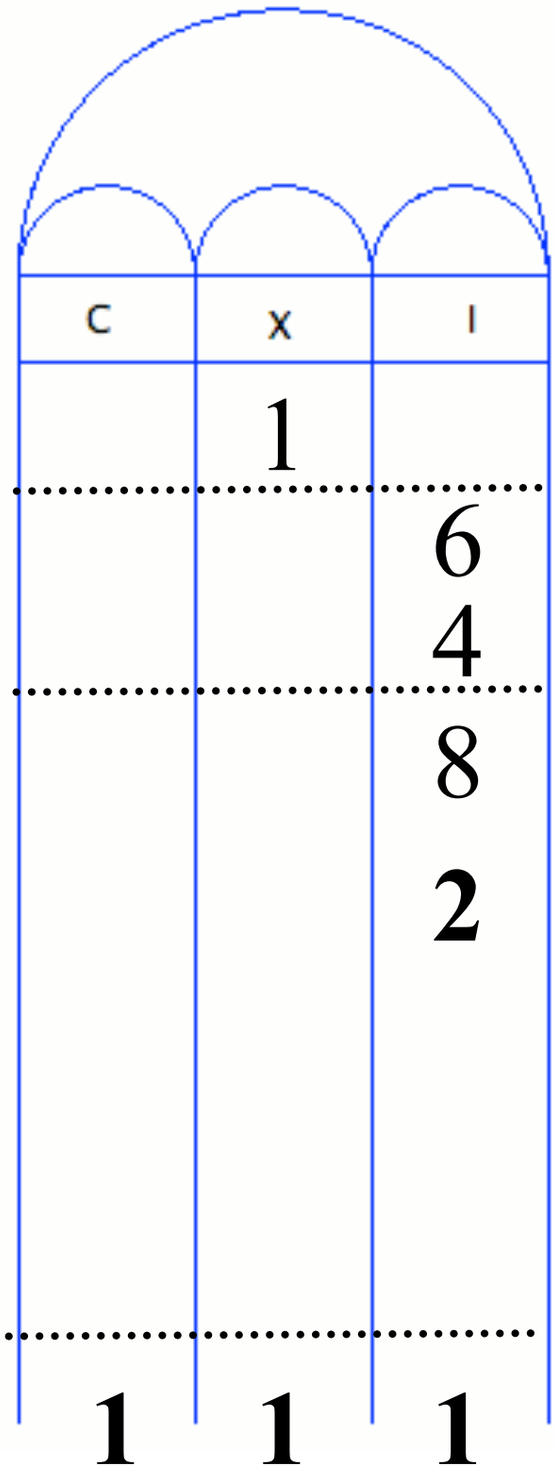
Divisio ferrea

Somar 60+20+10+10+6+3+1+1
e temos o resultado: 111. Resto 2.



Divisio ferrea

Somar: 60+20+10+10+6+3+1+1
e temos o resultado: 111. Resto 2.



Este método prescinde de subtracções e tentativas. Uma vez escolhido o divisor auxiliar, tudo é automático.

Uma boa escolha do divisor auxiliar simplifica as contas e induz regras simples (dividir por 10 corresponde a mover uma peça do dividendo para o quociente, etc)

Não há menções à *Divisio Ferrea* anteriores a Bernelin. Gerbert adaptou algum procedimento dos ábacos antigos, ou inventou este método.

Por que funciona:

Ao dividir ***d*** por ***b*** obtemos

$$d = q \times b + r$$

que equivale a

$$d + q \times z = q \times (b + z) + r$$

portanto é importante escolher **z** de forma a que **b** + **z** seja “simpático”.

“Simpático” significa uma potência de 10, ou um dígito multiplicado por uma potência de 10 (estes números só têm um dígito aqui).

77068 : 6807

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		7			
		6	8 1	9	7 3
	7	7		6	8
	7	7		6	8

77068 : 6807

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		7			
		6	8 1	9	7 3
	7	7		6	8
		7		6	8
				1	

77068 : 6807

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		7			
		6	8		7
			1	9	3
	7	7		6	8
		7		6	8
		1	9	3	
				1	

77068 : 6807

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		7			
		6	8		7
			1	9	3
	7	7		6	8
		7		6	8
		1	9	3	
		8	9	9	8
				1	

77068 : 6807

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		7			
		6	8		7
			1	9	3
	7	7		6	8
		7		6	8
		1	9	3	
		1	9	9	8
				1	

77068 : 6807

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		7			
		6	8		7
			1	9	3
	7	7		6	8
		7		6	8
		1	9	3	
		1	9	9	8
				1	1

77068 : 6807

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		7			
		6	8		7
			1	9	3
	7	7		6	8
		7		6	8
		1	9	3	
		1	9	9	8
			1	9	3
				1	1

77068 : 6807

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		7			
		6	8 1	9	7 3
	7	7		6	8
		2	1	9	1
				1	1

77068 =

11 x 6807 + 2191

986022 : 507041

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
9					
5 3	9	7 2	9	4 5	1 9
9	8	6			2
9	8	6			2

986022 : 507041

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
9					
5 3	9	7 2	9	4 5	1 9
9	8	6			2
	8	6			2

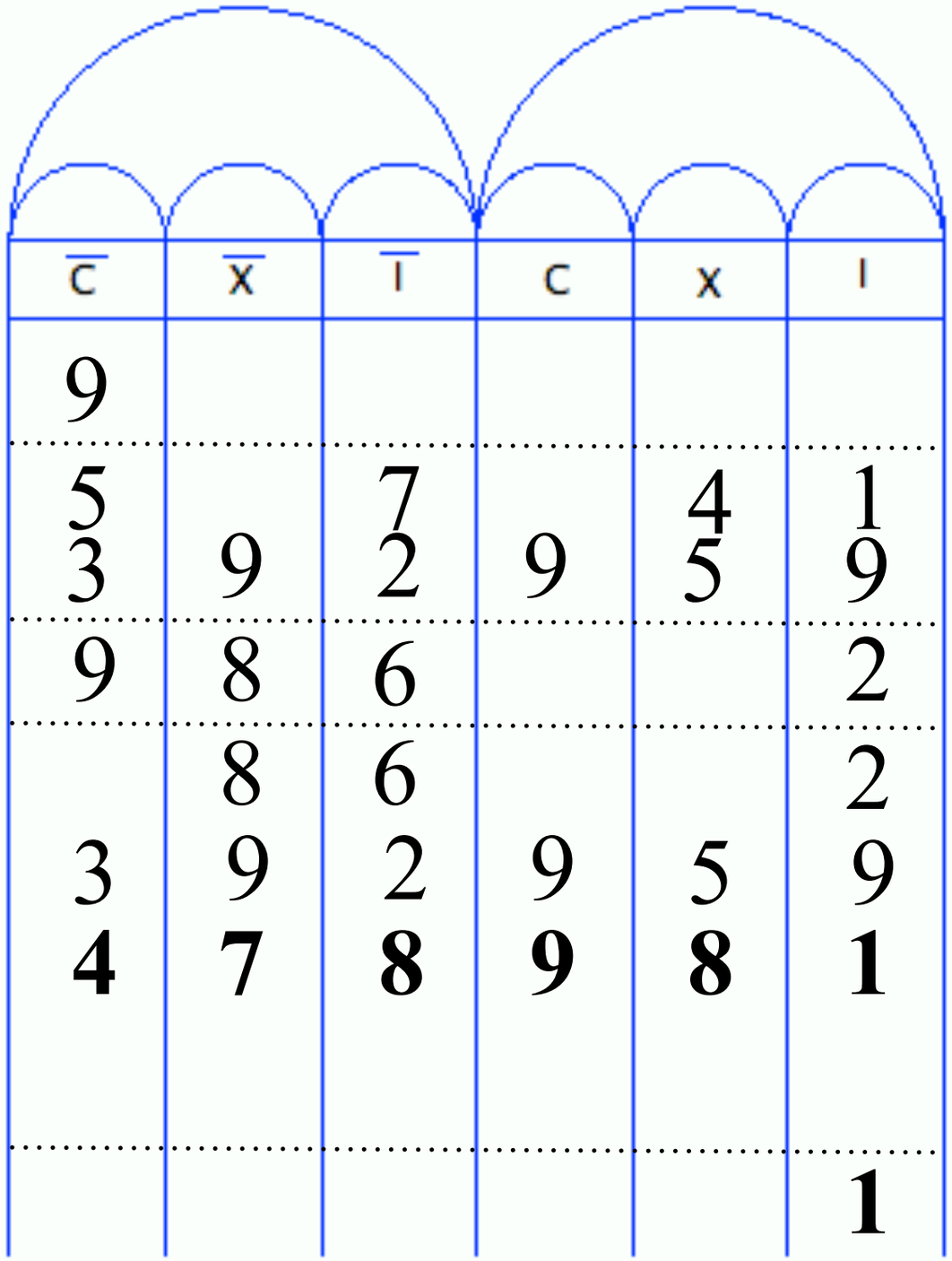
986022 : 507041

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
9					
5 3	9	7 2	9	4 5	1 9
9	8	6			2
					1

986022 : 507041

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
9					
5 3	9	7 2	9	4 5	1 9
9	8	6			2
	8 9	6 2	9	5	2 9
					1

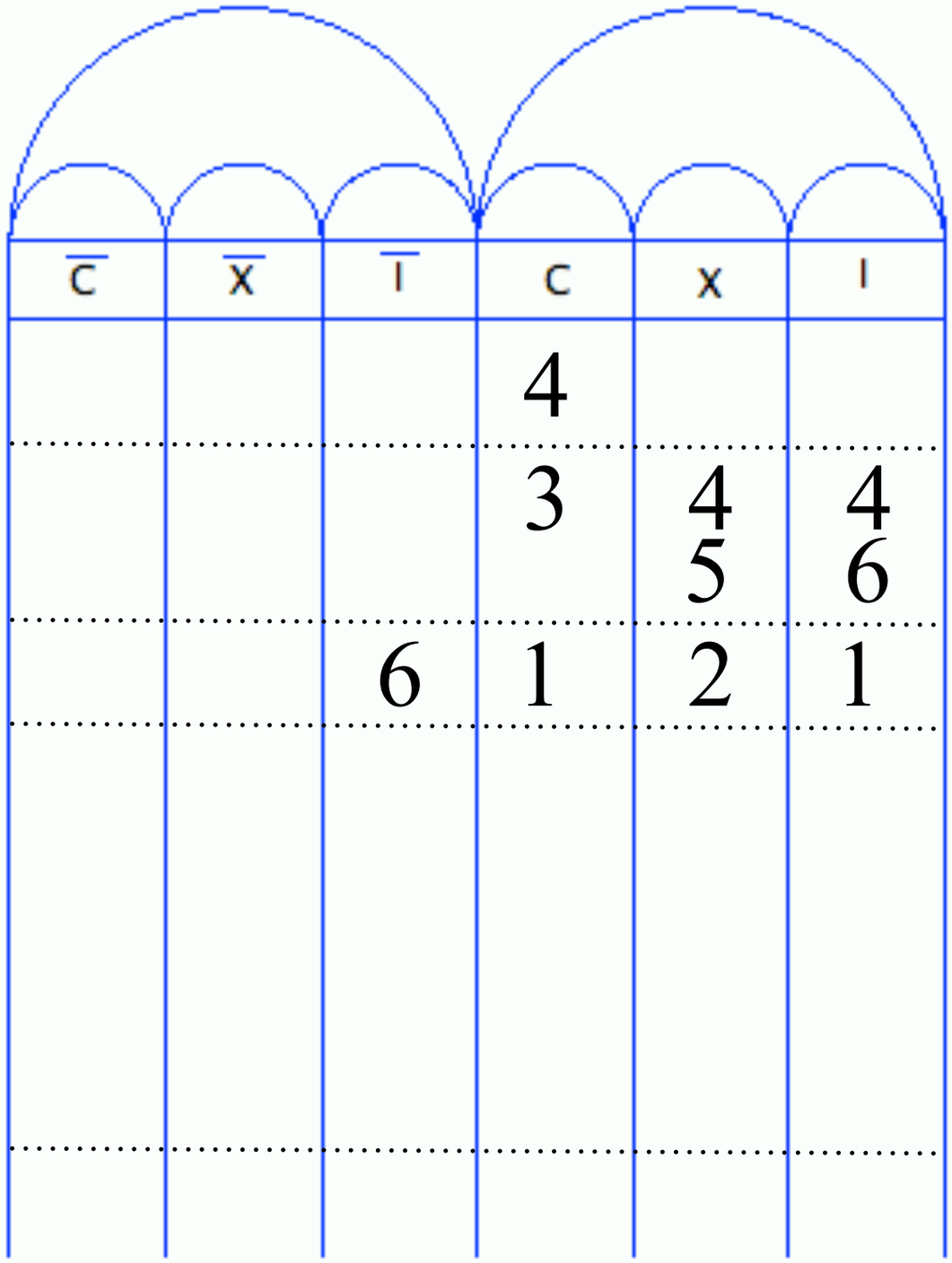
986022 : 507041



6121 : 344

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
			4		
			3	4 5	4 6
		6	1	2	1
		6	1	2	1

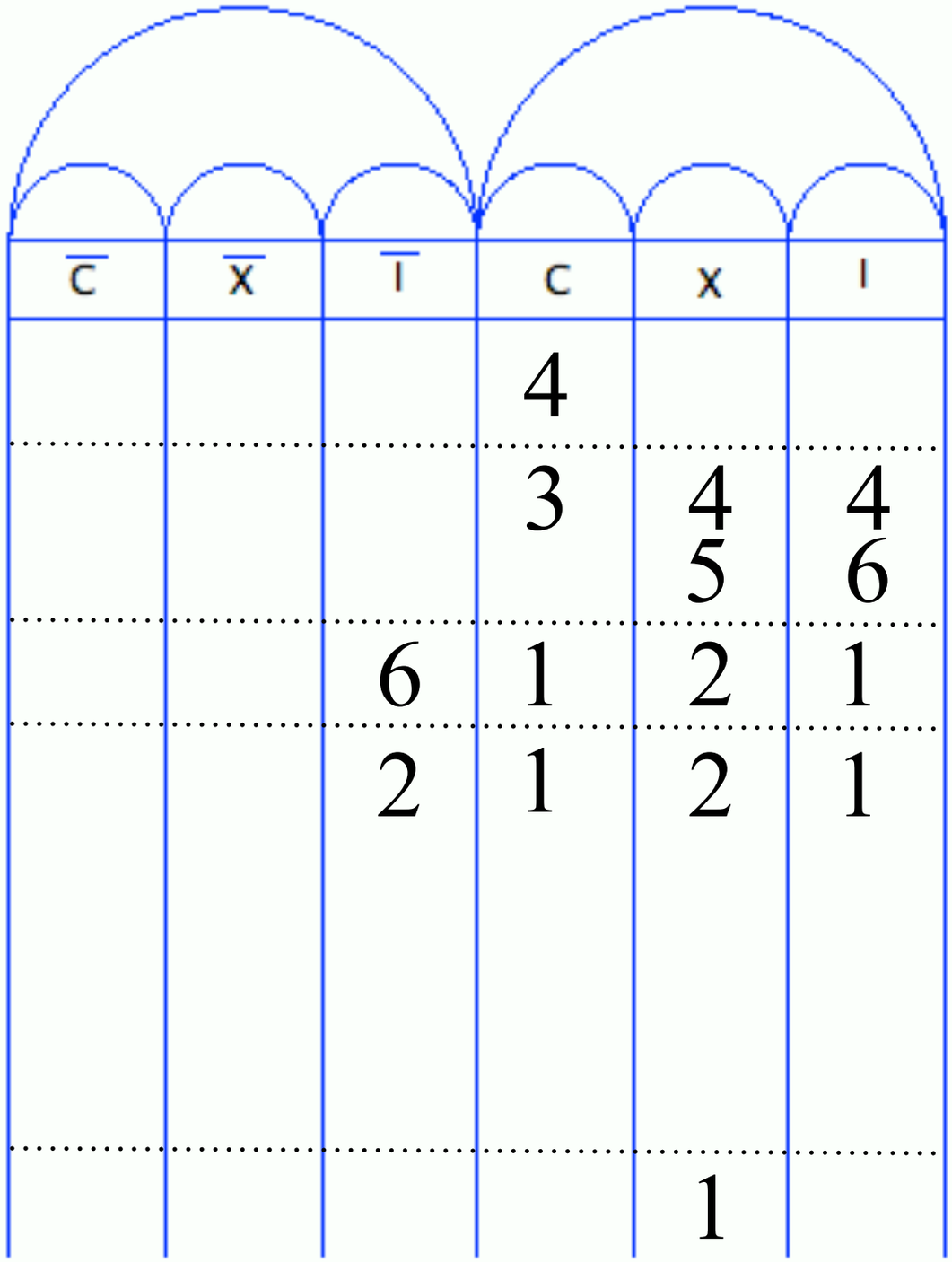
6121 : 344



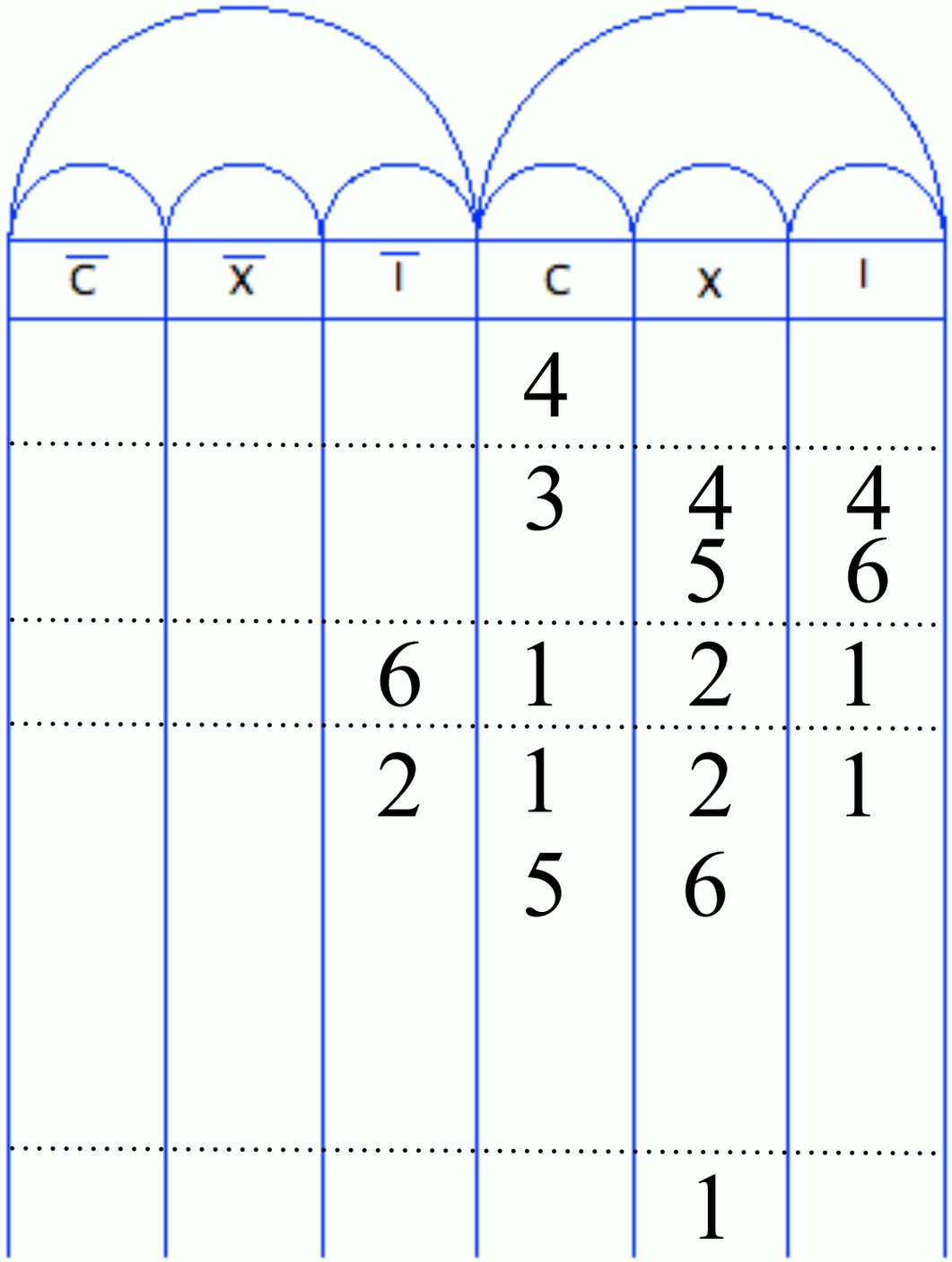
6121 : 344

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
			4		
			3	4 5	4 6
		6	1	2	1
		2	1	2	1

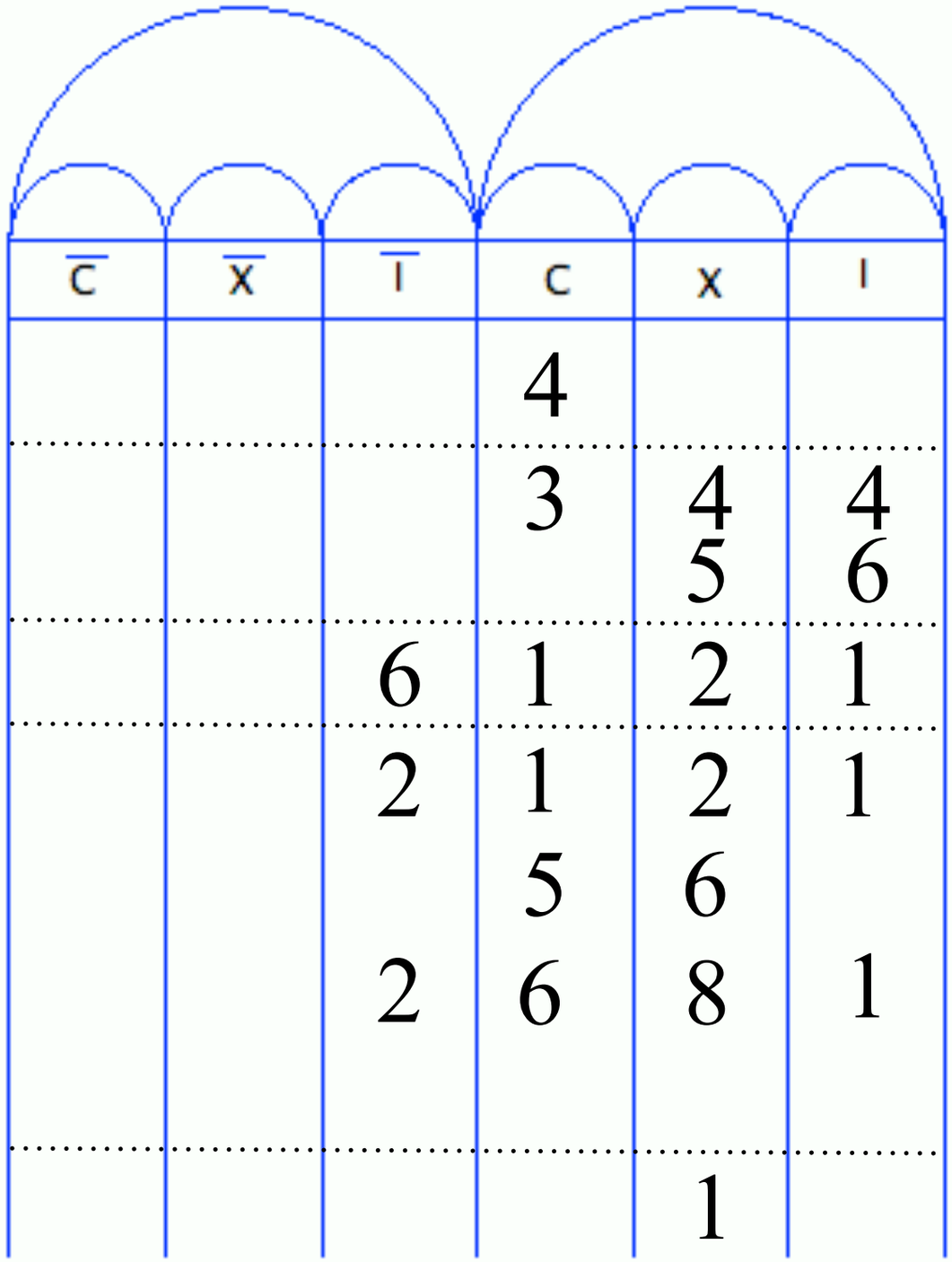
6121 : 344



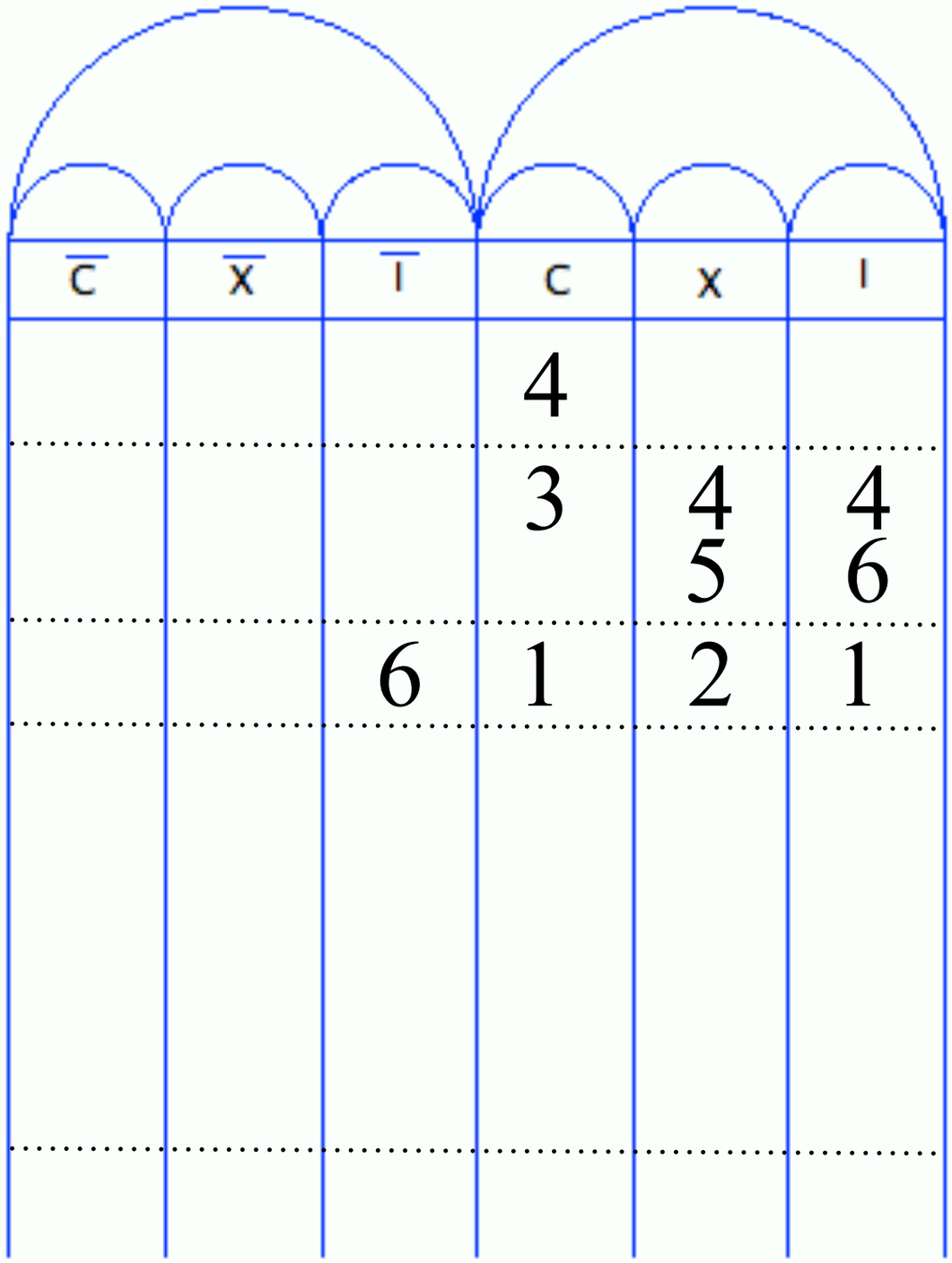
6121 : 344



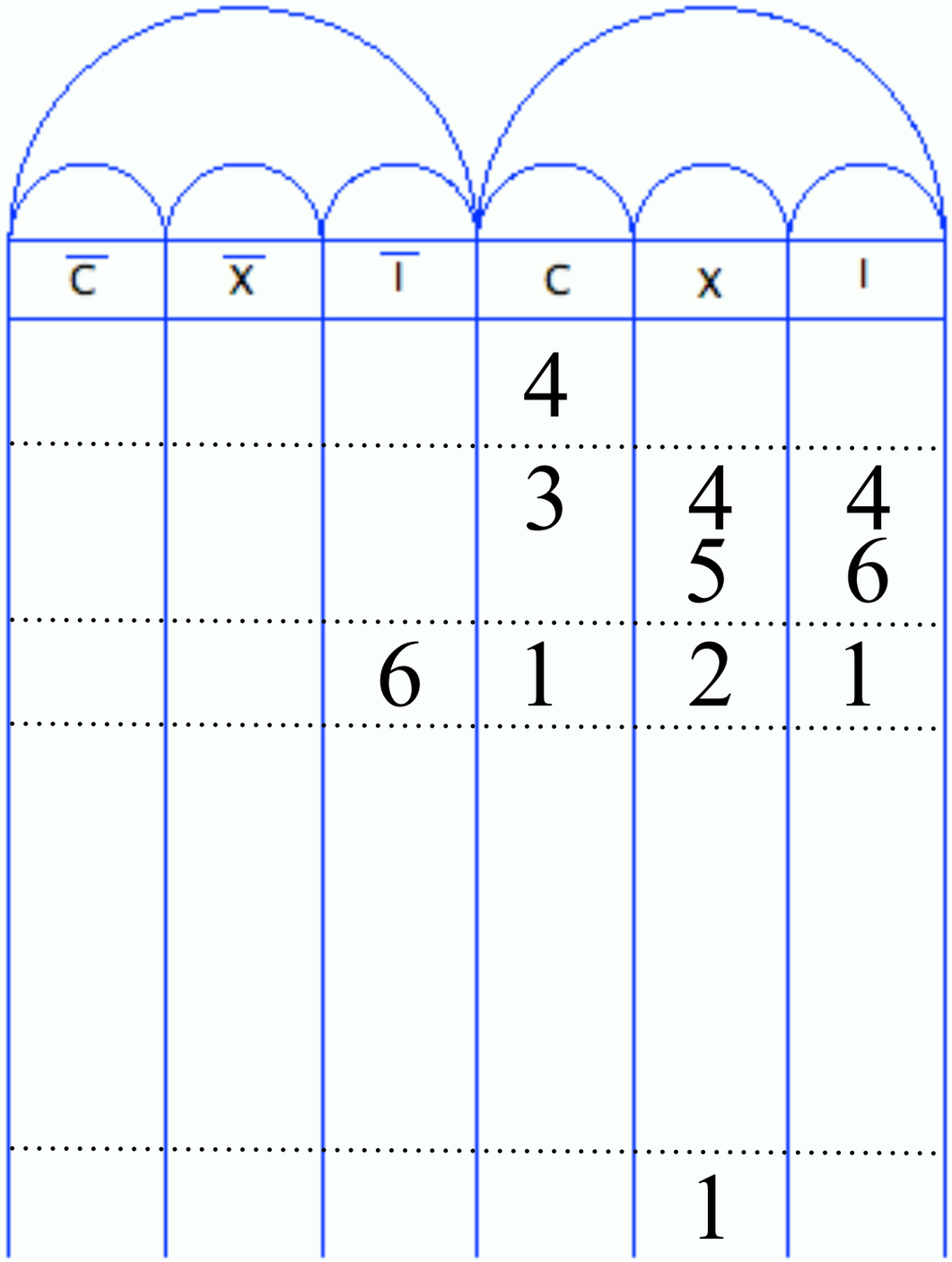
6121 : 344



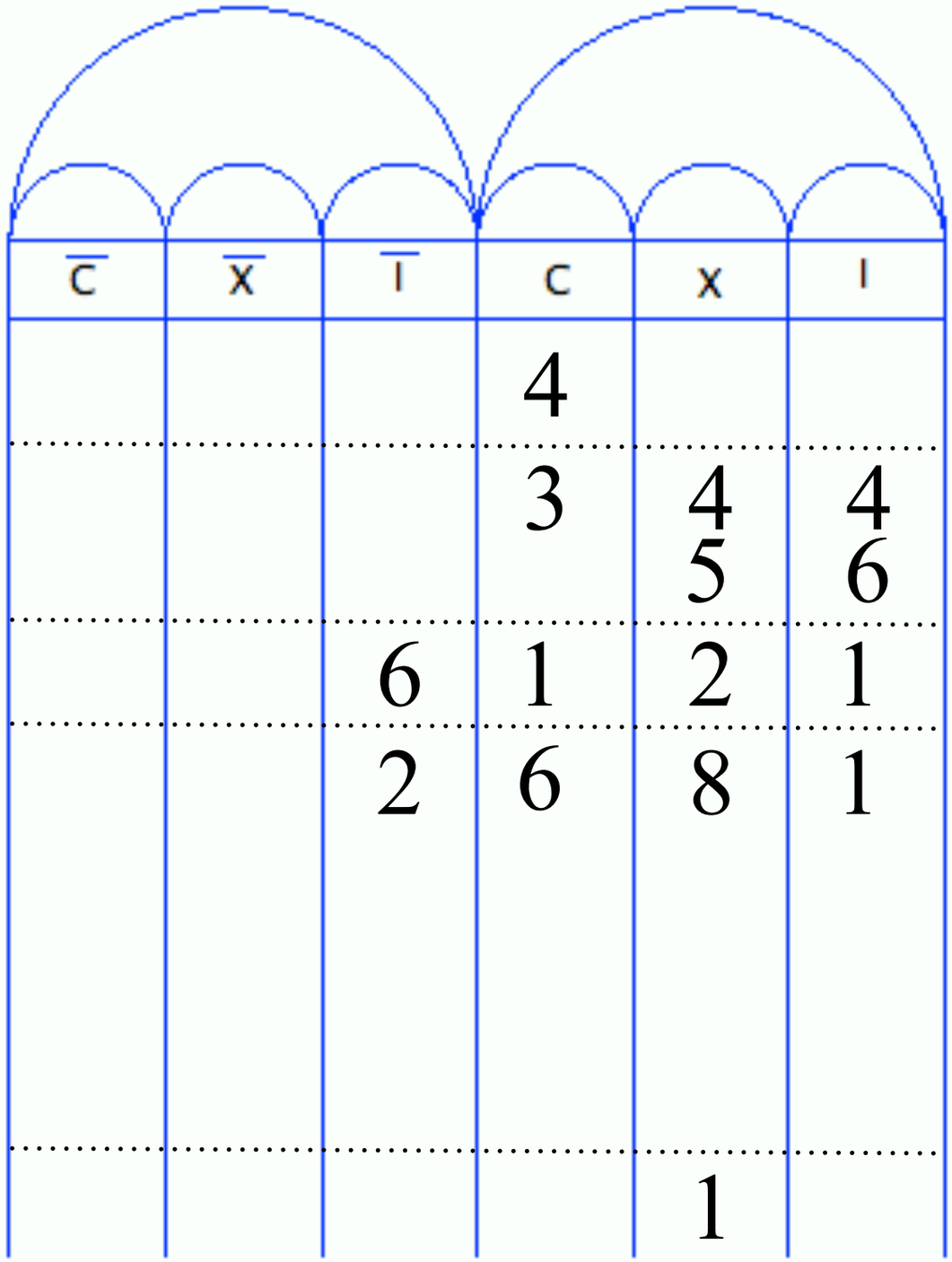
6121 : 344



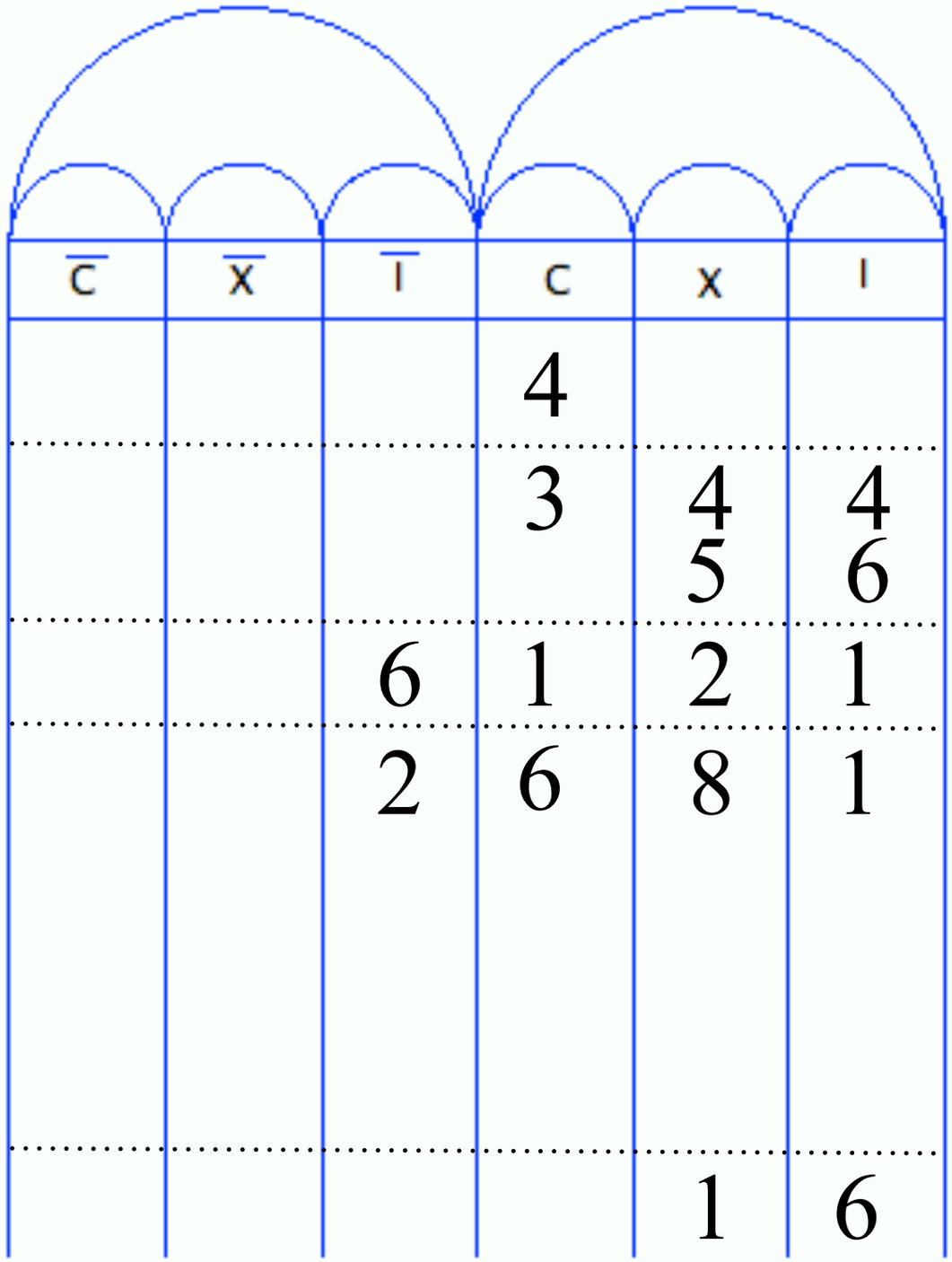
6121 : 344



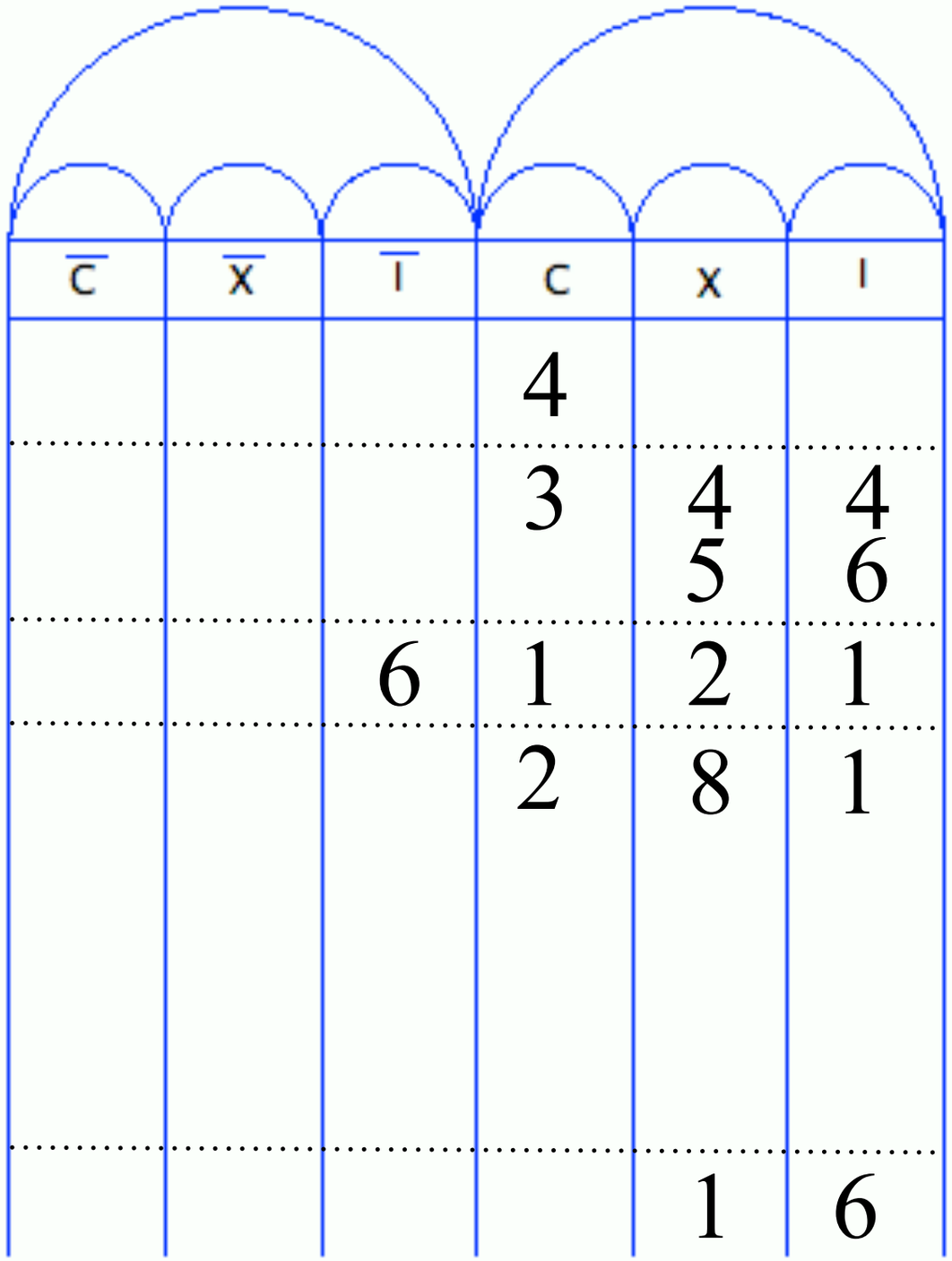
6121 : 344



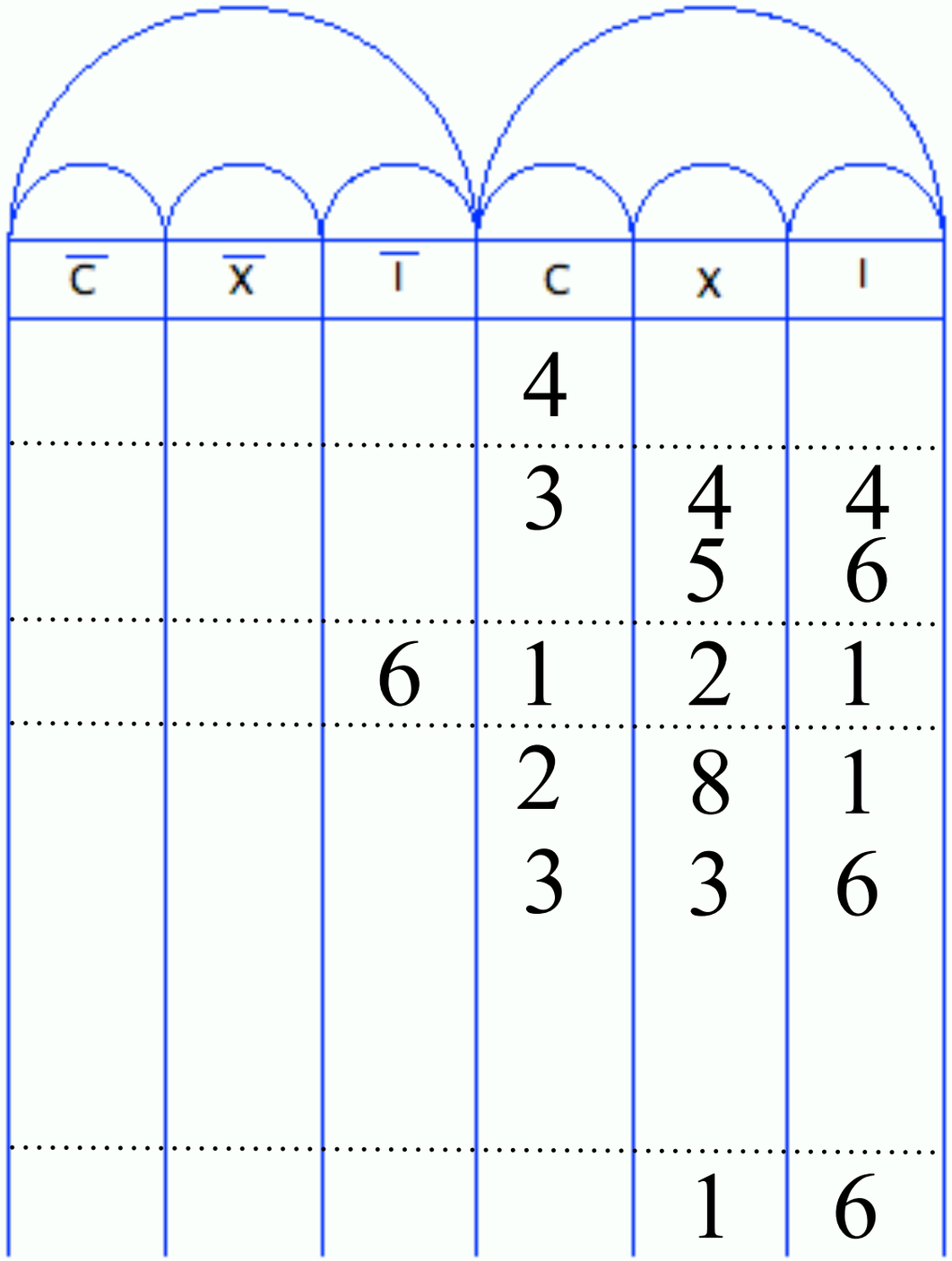
6121 : 344



6121 : 344



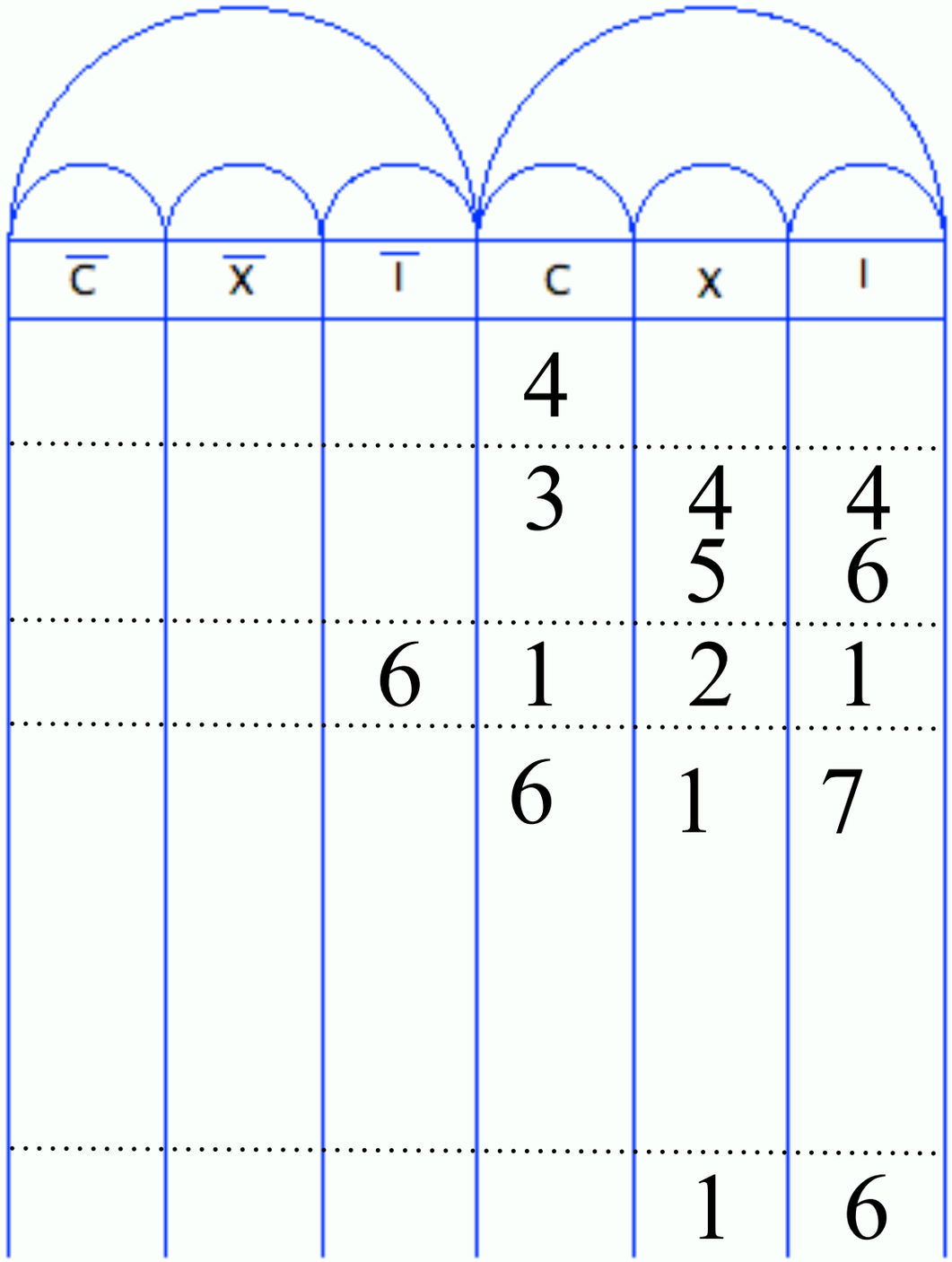
6121 : 344



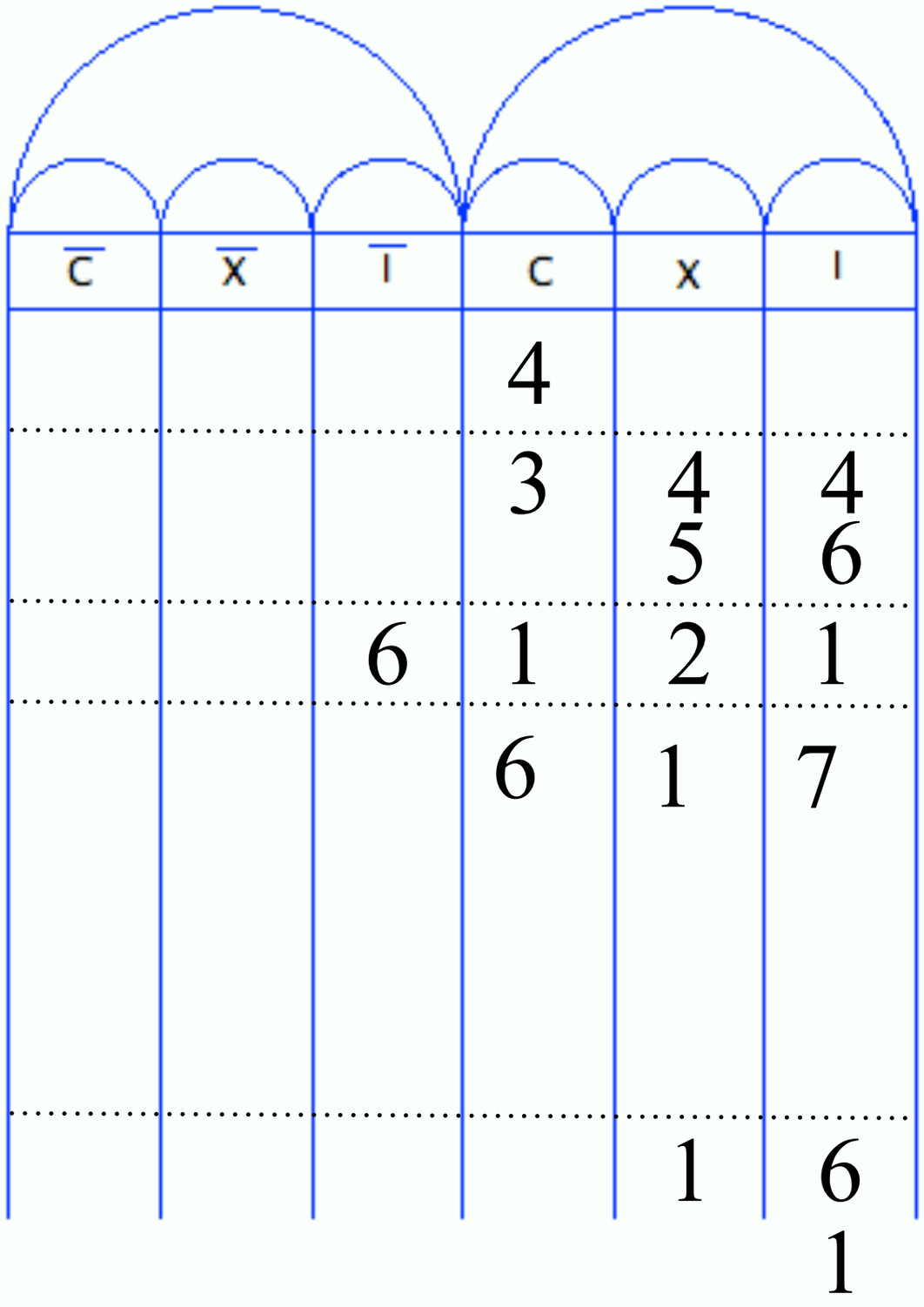
6121 : 344

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
			4		
			3	4 5	4 6
		6	1	2	1
			2 3 6	8 3 1	1 6 7
				1	6

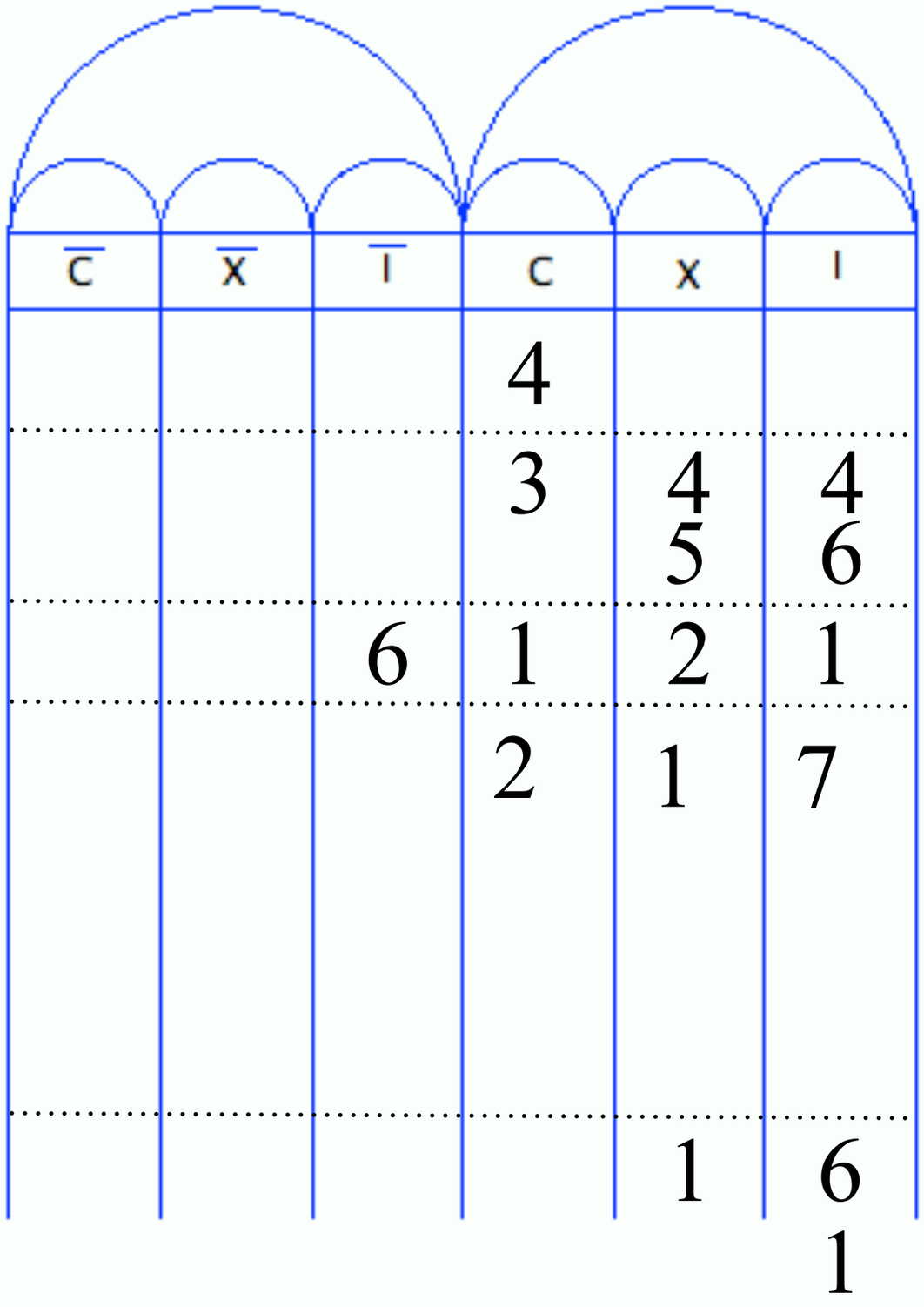
6121 : 344



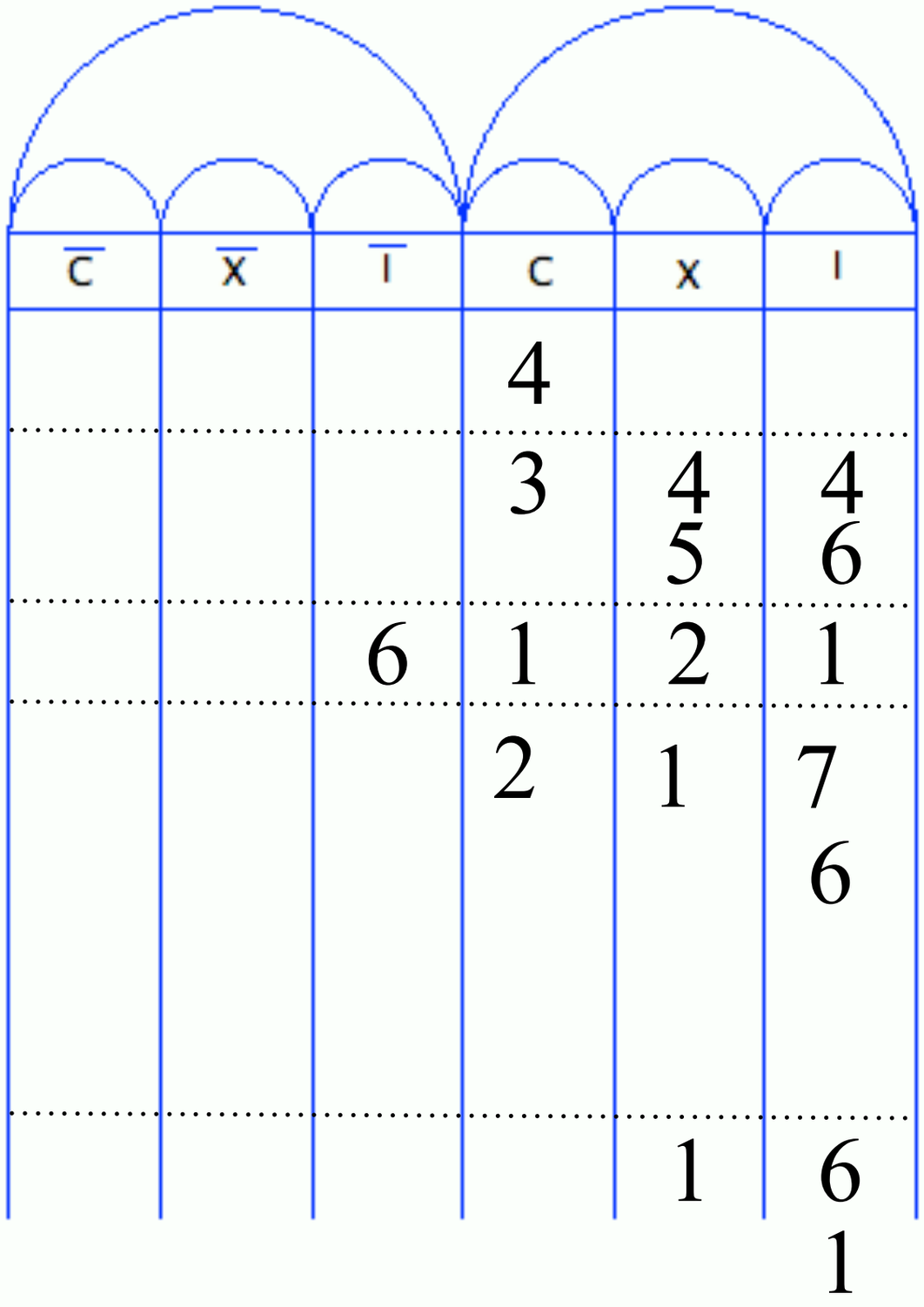
6121 : 344



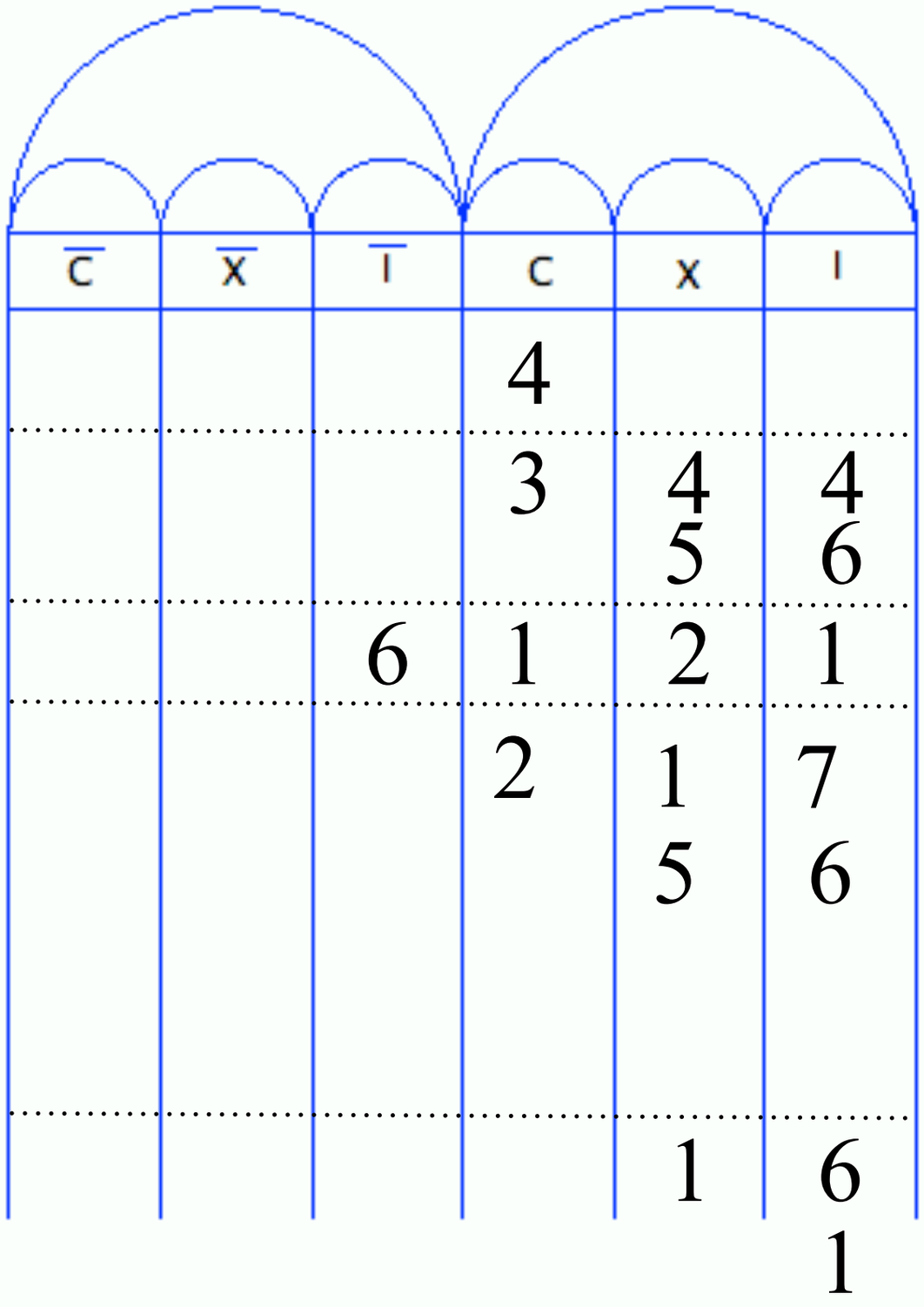
6121 : 344



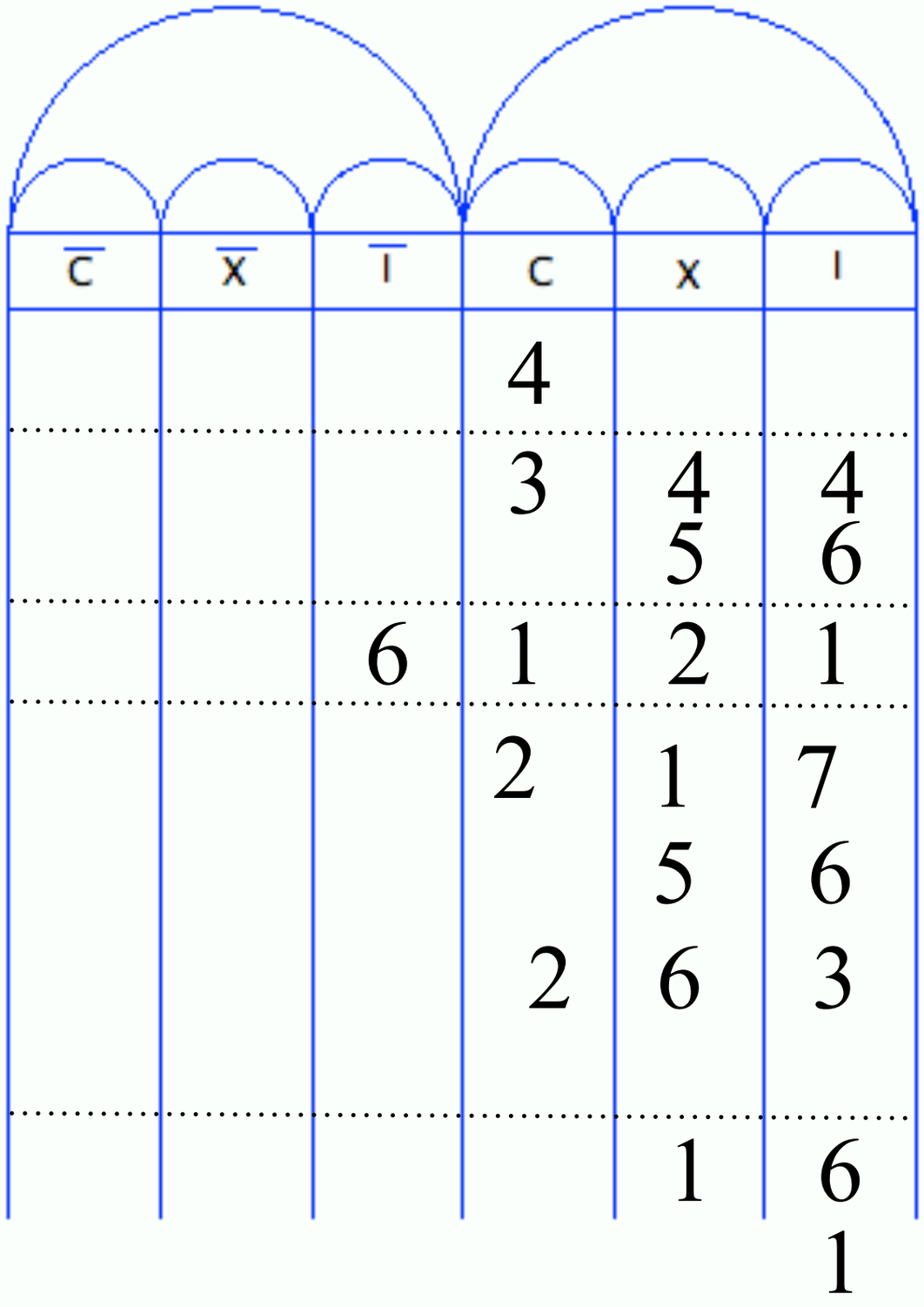
6121 : 344



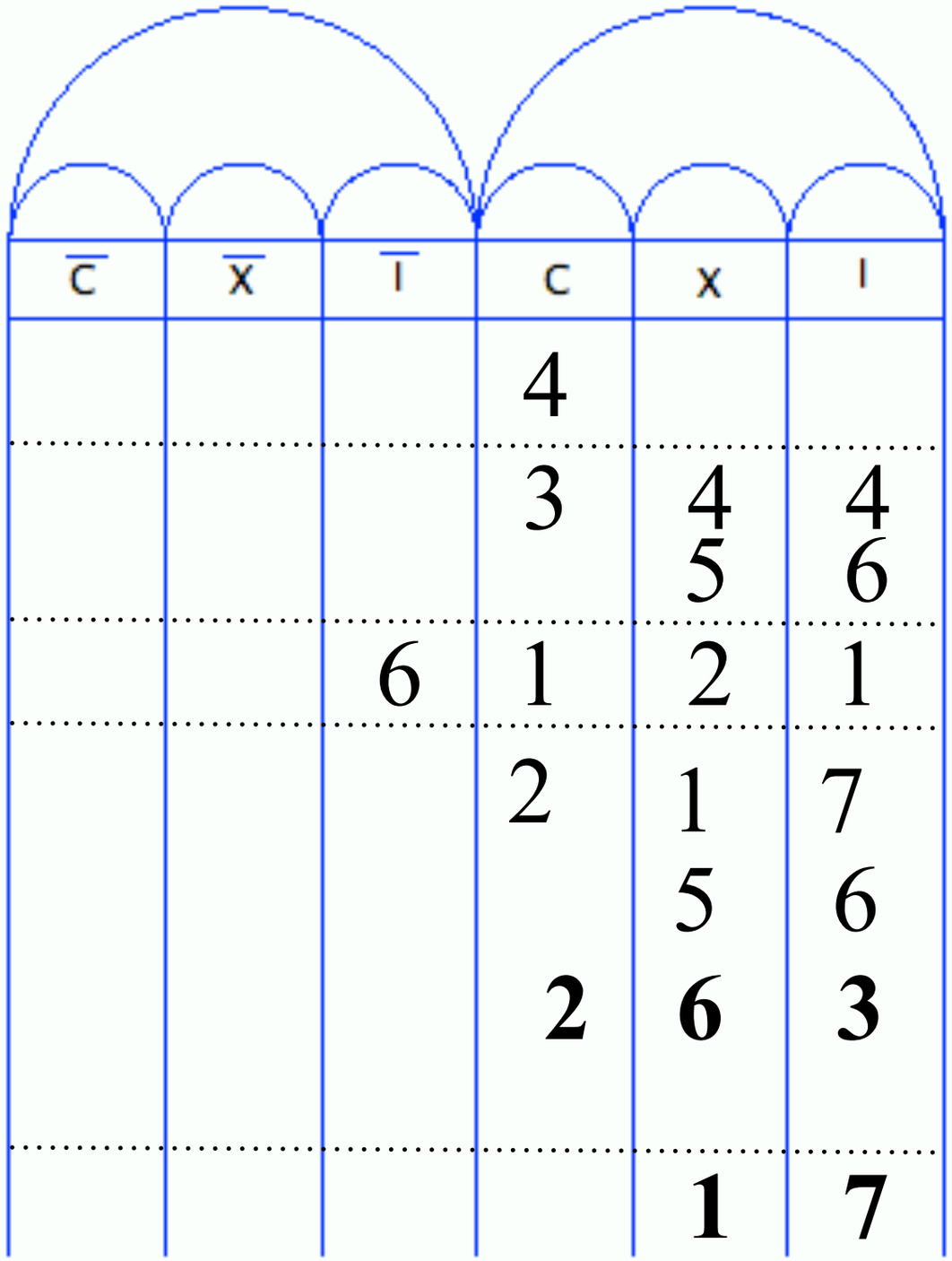
6121 : 344



6121 : 344



6121 : 344



Gerbert usava Astrolábio,
Esfera Armilar,
Tubos de observação astronómica,
Ábaco novo.

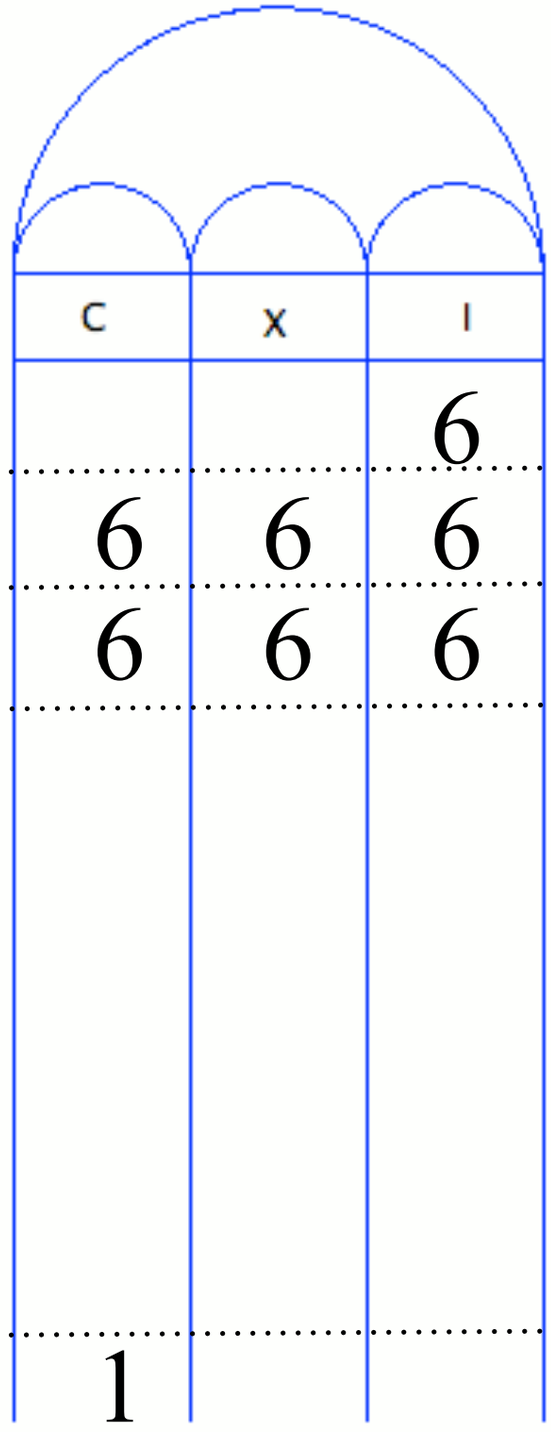
Homem cultíssimo, grande professor, homem de ciência.

Os seus adversários acusaram-no de ter feito um pacto com o Diabo para obter tal sabedoria... A lenda sobreveveu vários séculos.

Divisio aurea

666 : 6

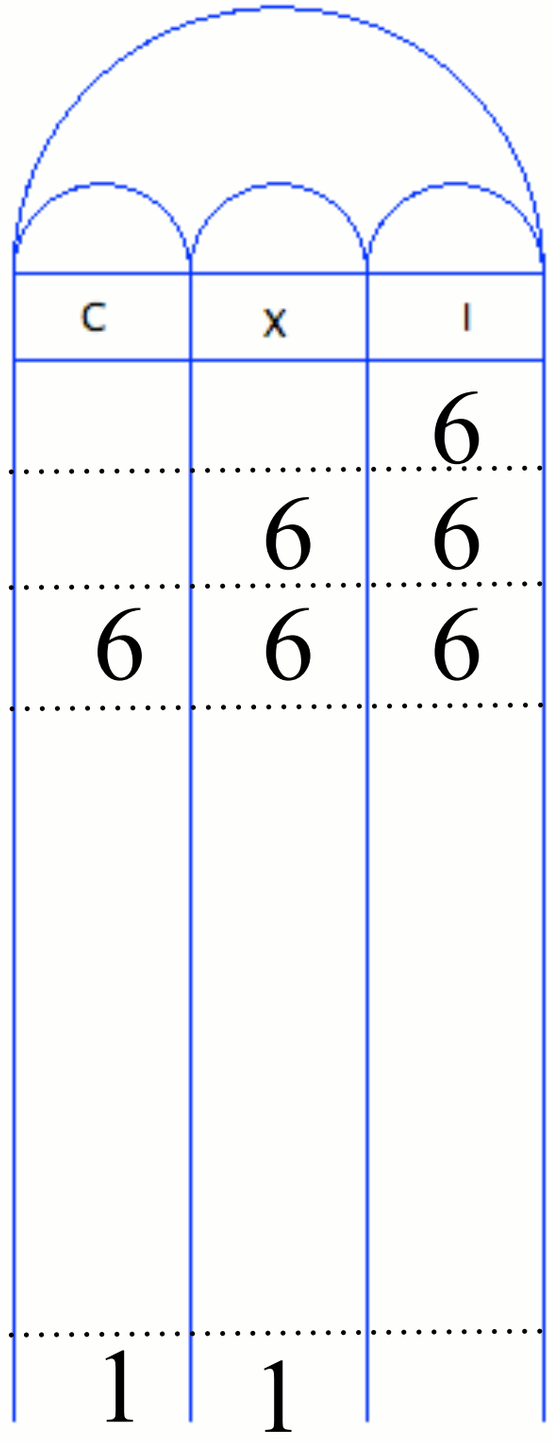
Quantos 6s em 6?...



Divisio aurea

666 : 6

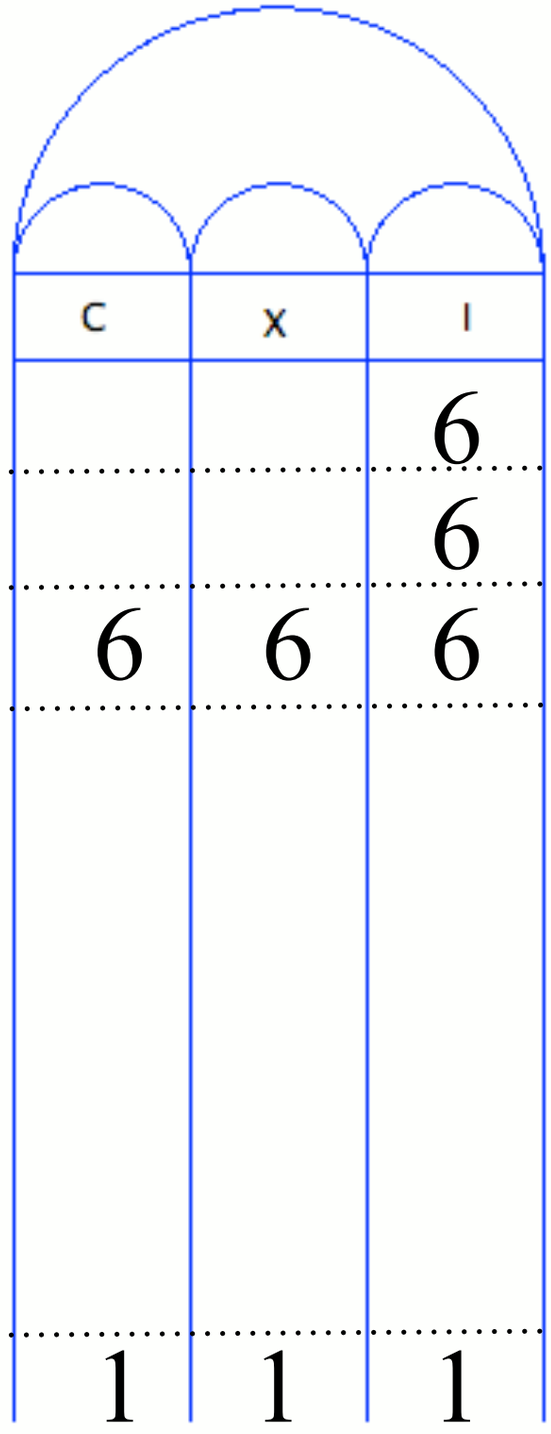
Quantos 6s em 6?...



Divisio aurea

666 : 6

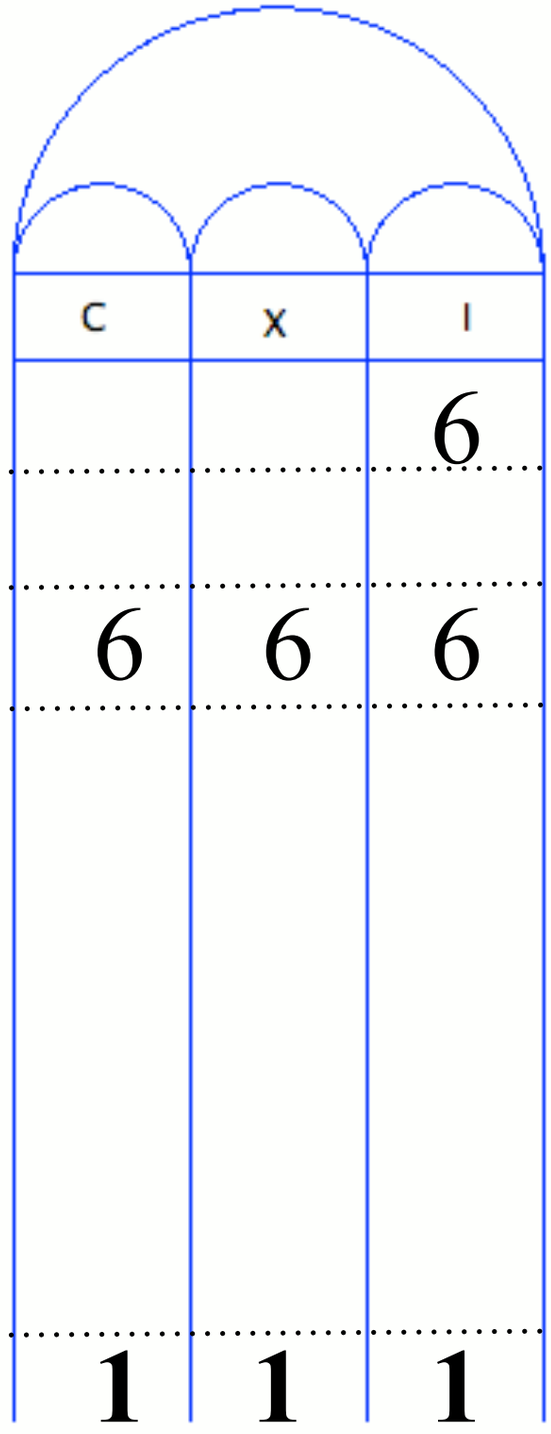
Quantos 6s em 6?...



Divisio aurea

666 : 6

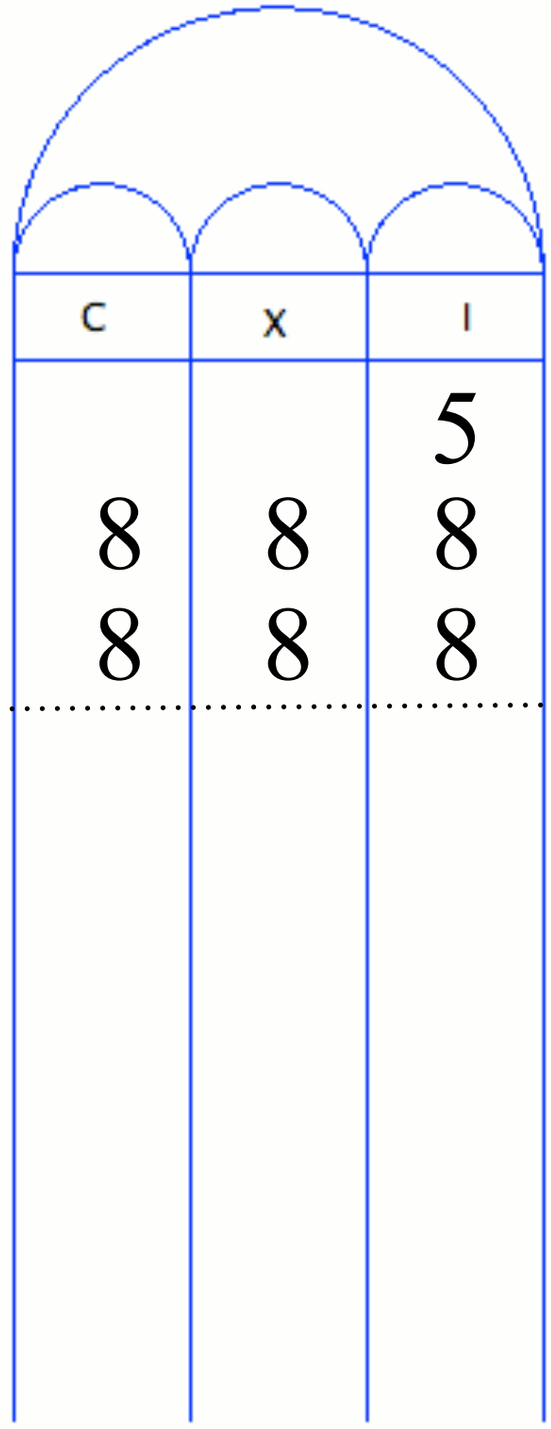
666 = 111 x 6



Divisio aurea

888 : 5

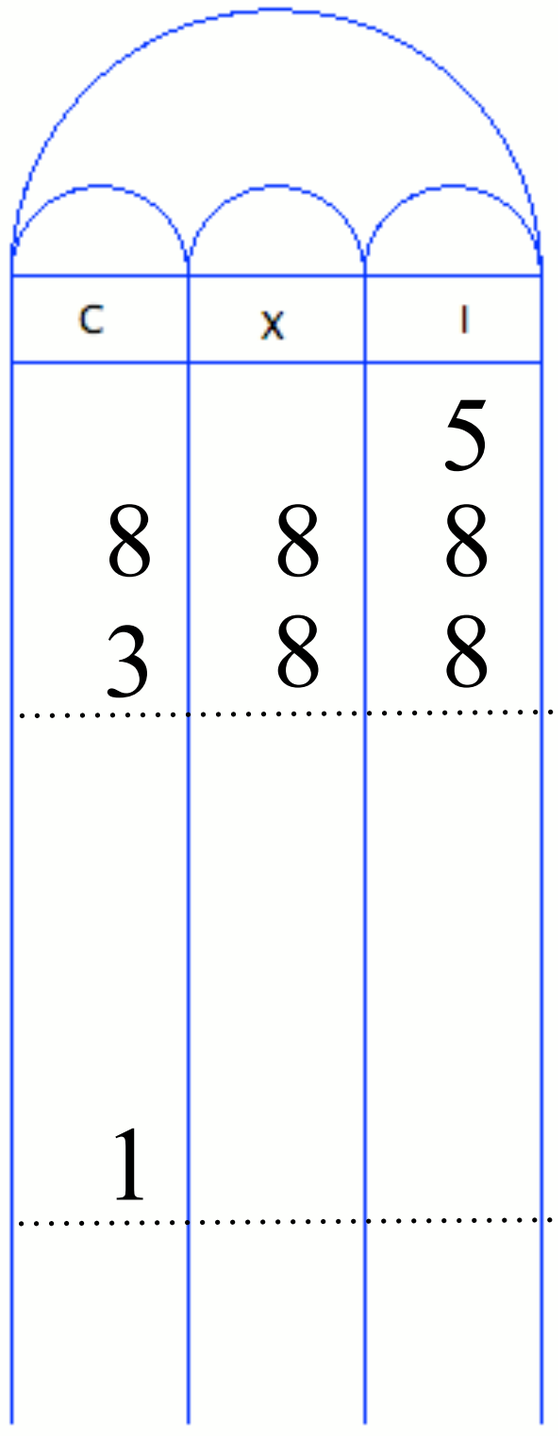
Quantos 5s em 8?...



Divisio aurea

888 : 5

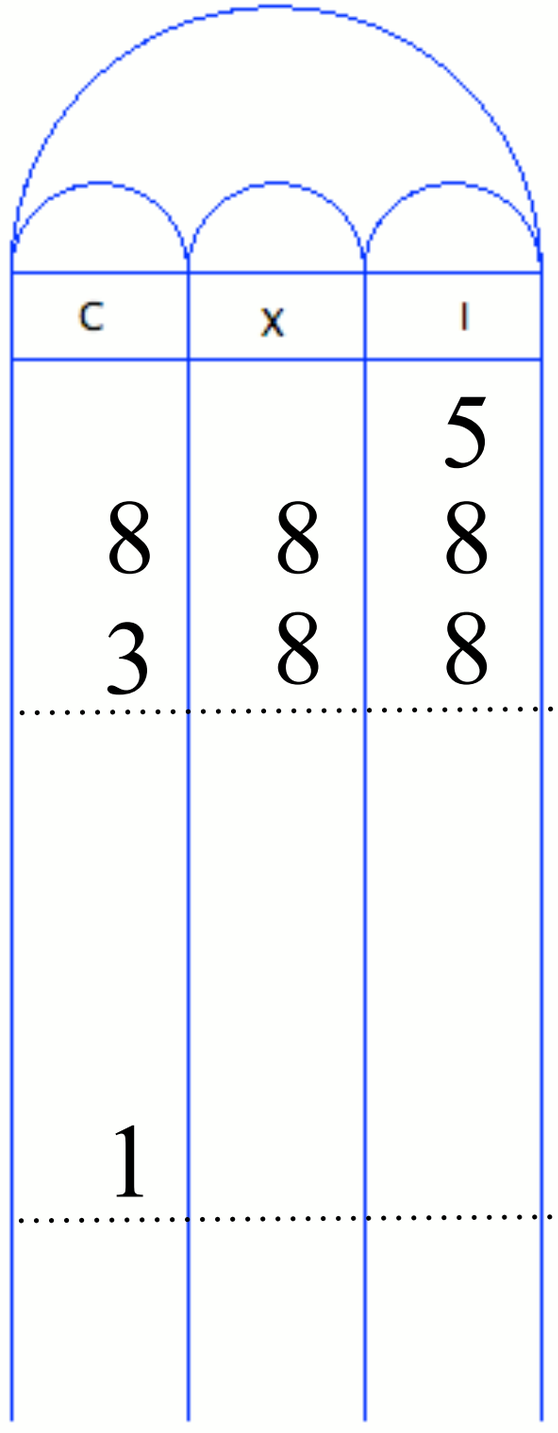
Quantos 5s em 8?...
1, com resto 3



Divisio aurea

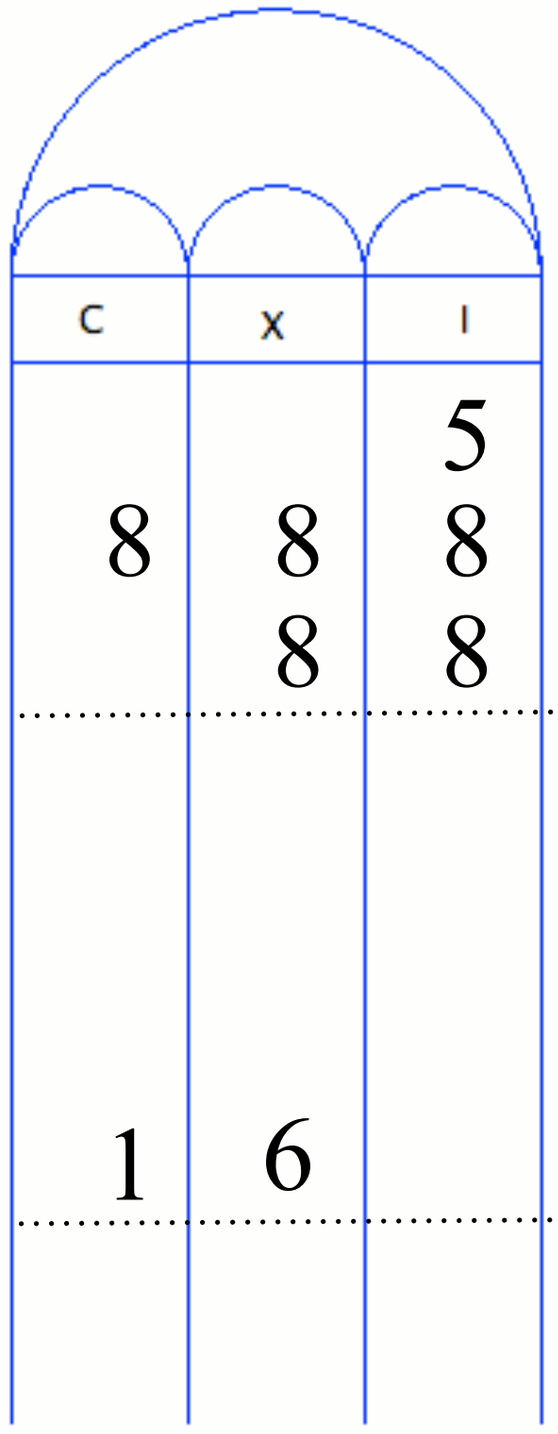
888 : 5

Quantos 5s in 3?... 6 (na coluna central, de acordo com as regras)



Divisio aurea

888 : 5



Divisio aurea

888 : 5

Quantos 5s em 8?...

C	X	I
		5
8	8	8
	8	8
	1	6

Divisio aurea

888 : 5

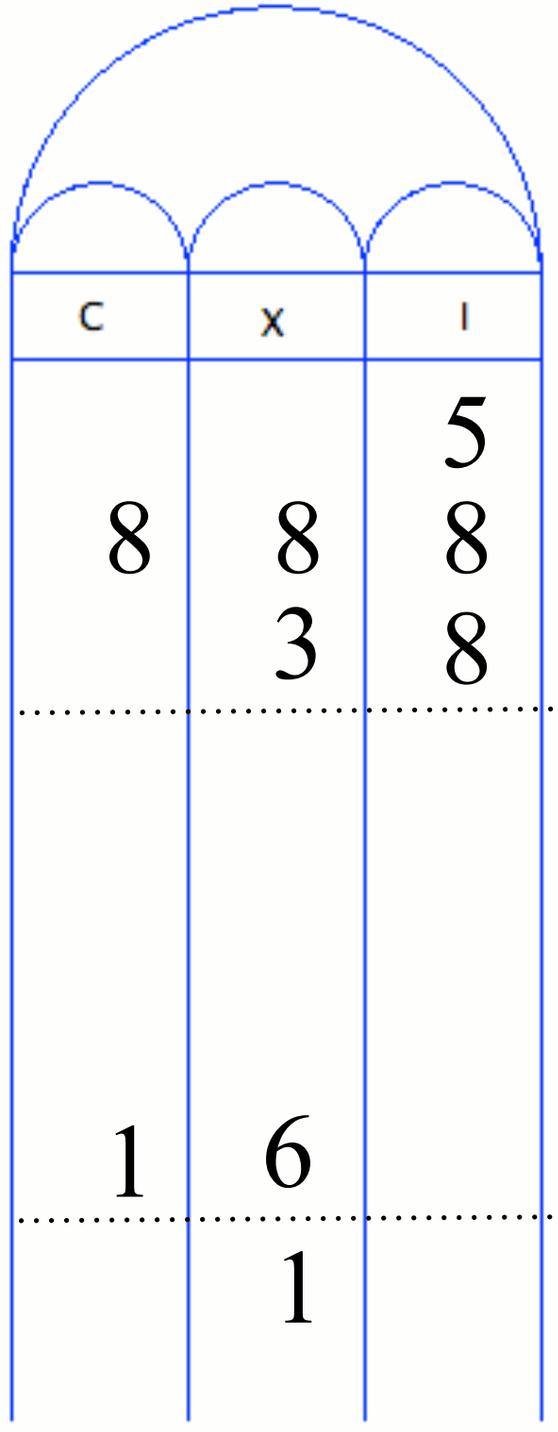
quantos 5s in 8?...
1, com resto 3

C	X	I
		5
8	8	8
	3	8
	1	6
		1

Divisio aurea

888 : 5

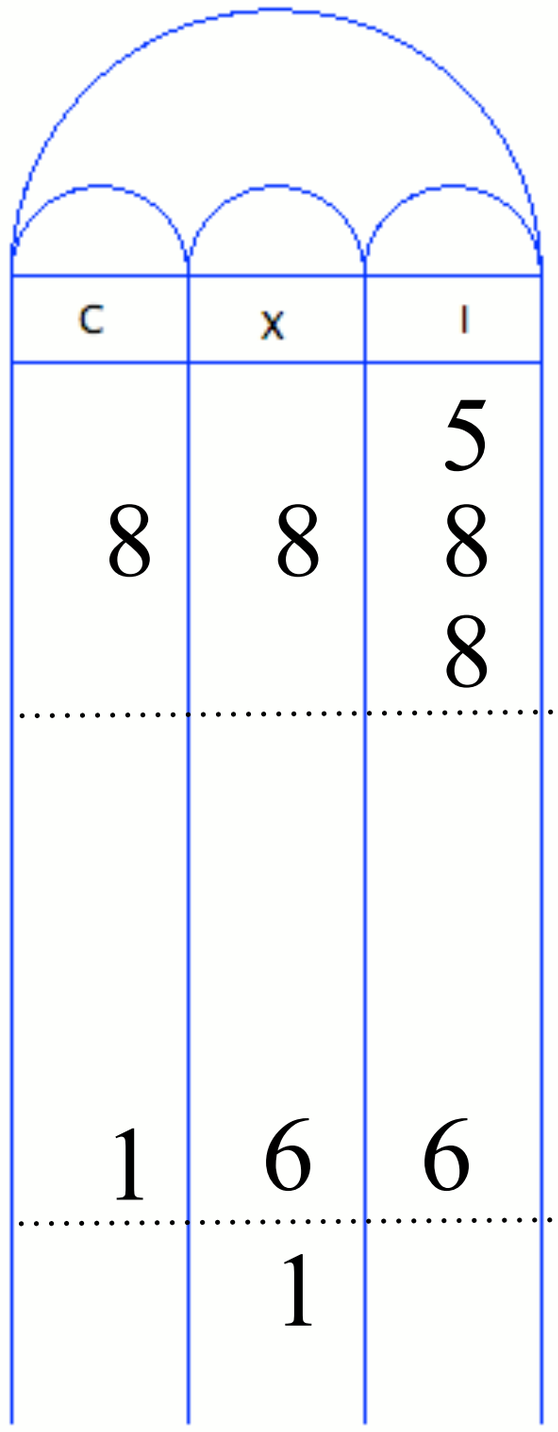
Quantos 5s in 3?...



Divisio aurea

888 : 5

Quantos 5s em 3?...
6 na coluna da direita



Divisio aurea

888 : 5

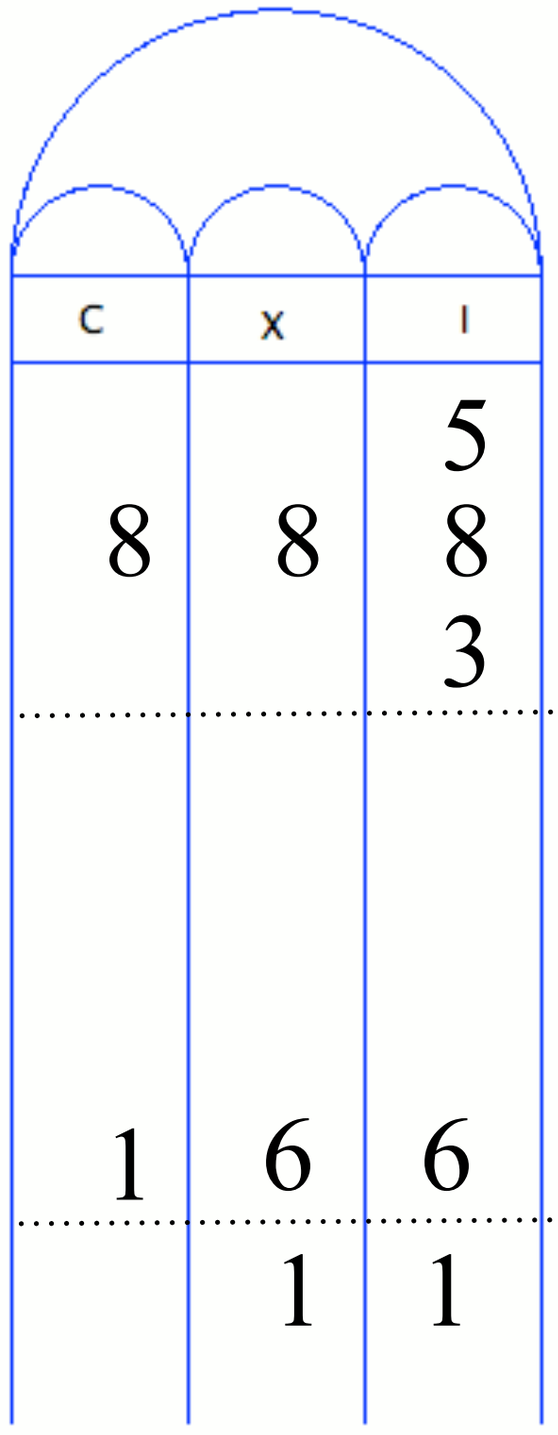
Quantos 5s em 8?...

C	X	I
8	8	5 8 8
1	6	6
	1	

Divisio aurea

888 : 5

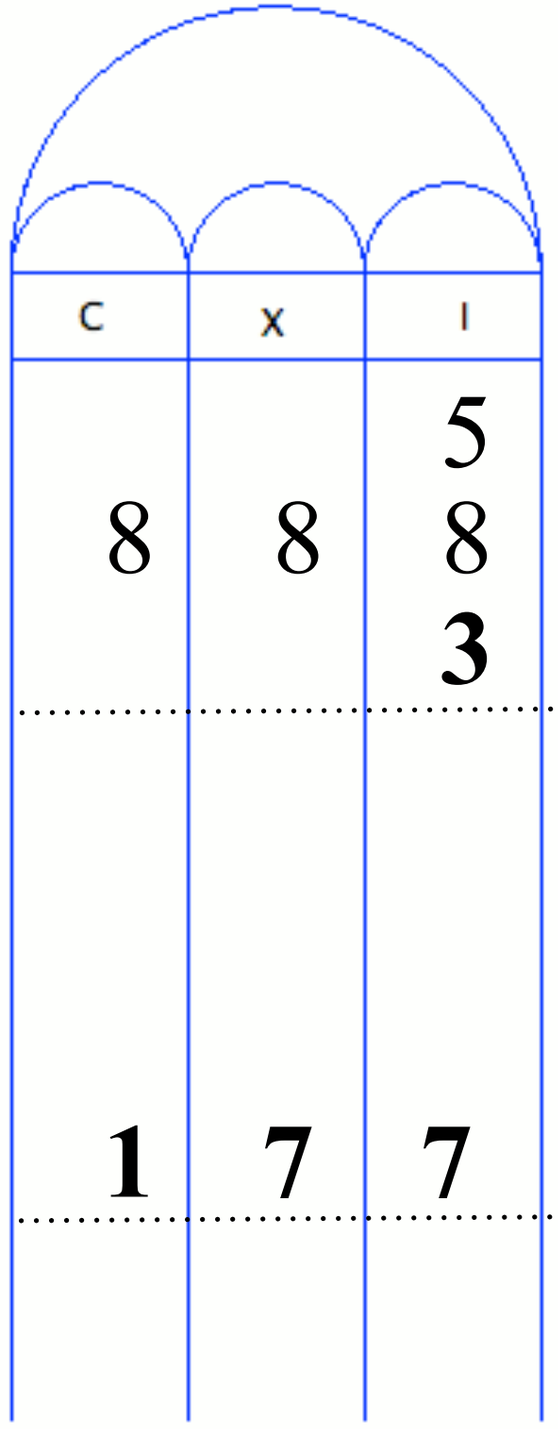
Quantos 5s in 8?...
1 com resto 3



Divisio aurea

888 : 5

Arrumando...



Bernelin descreve as operações usando numerais romanos (3 na coluna central é *xxx*).

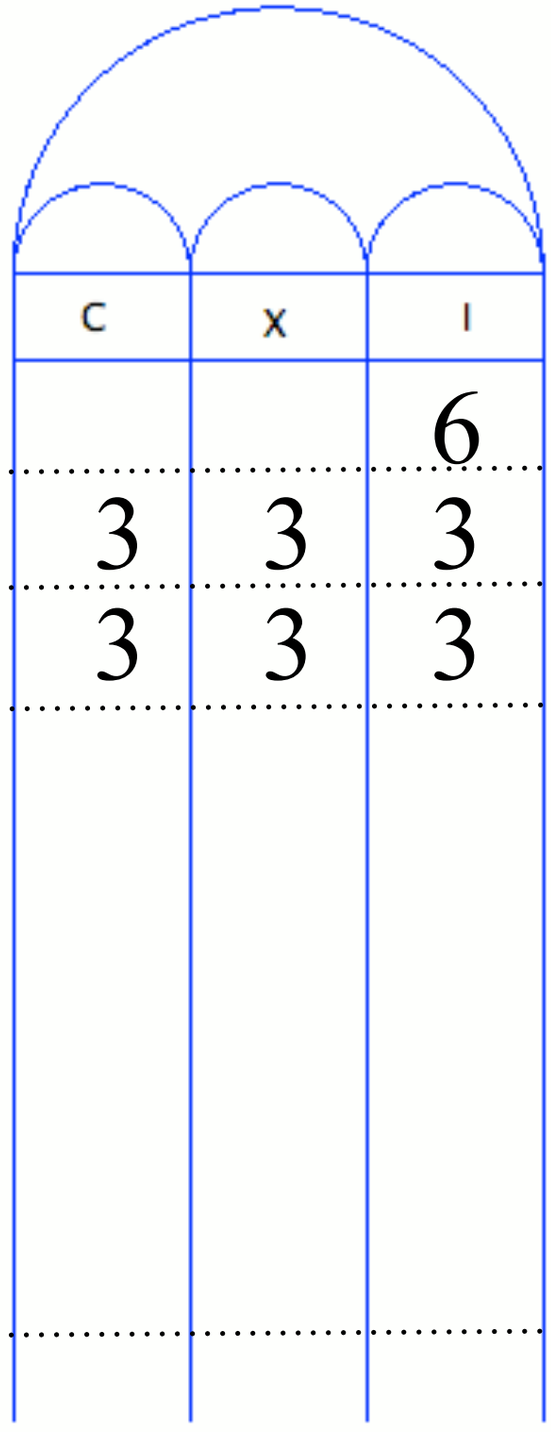
Ao referir-se a 38 diz algo como *o 3 na coluna central e o 8 na coluna da direita*.

Nunca transparece que tenha alguma ideia do sistema decimal posicional.

Divisio aurea

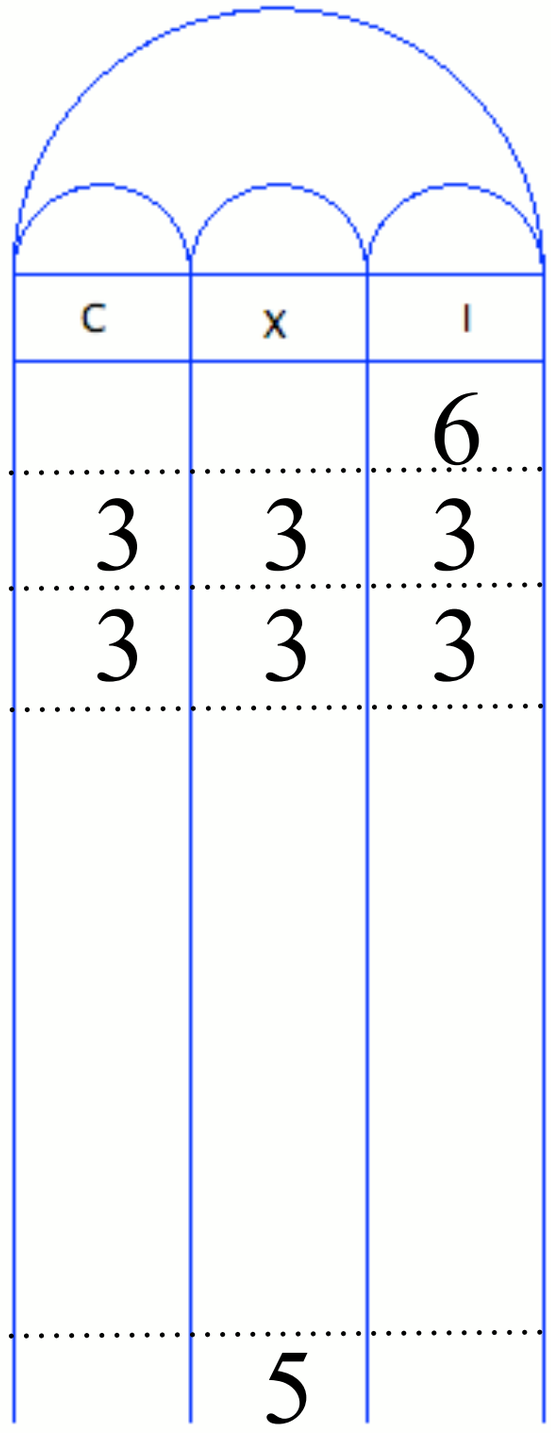
333 : 6

Quantos 6s em 3?...



Divisio aurea

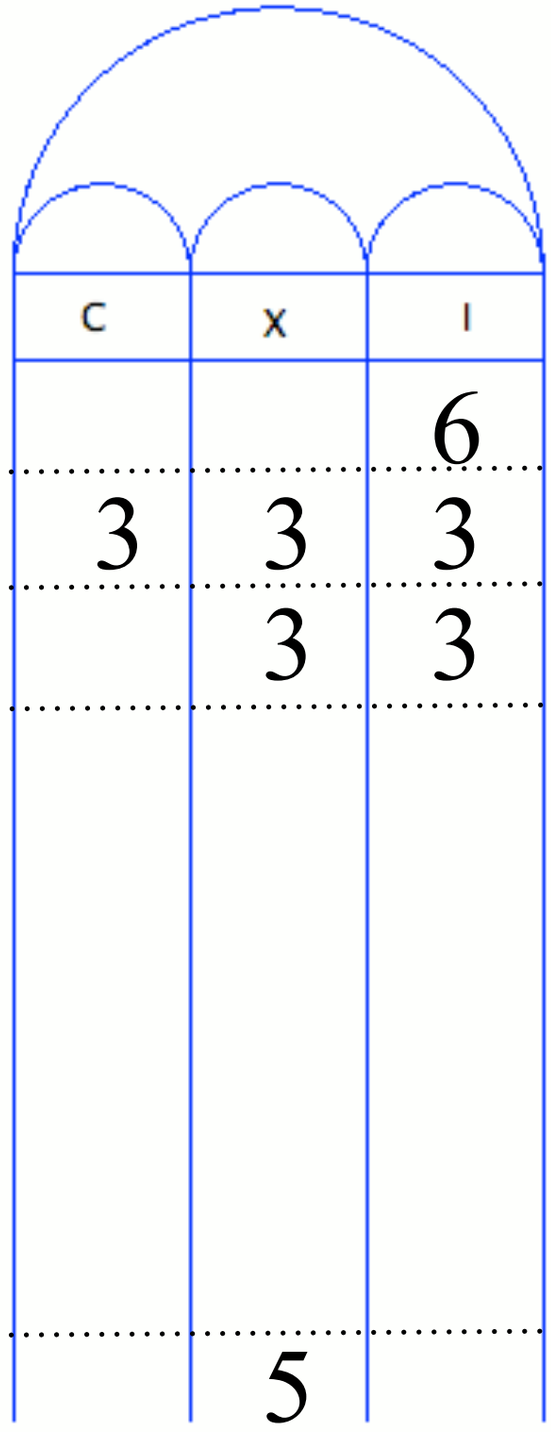
333 : 6



Divisio aurea

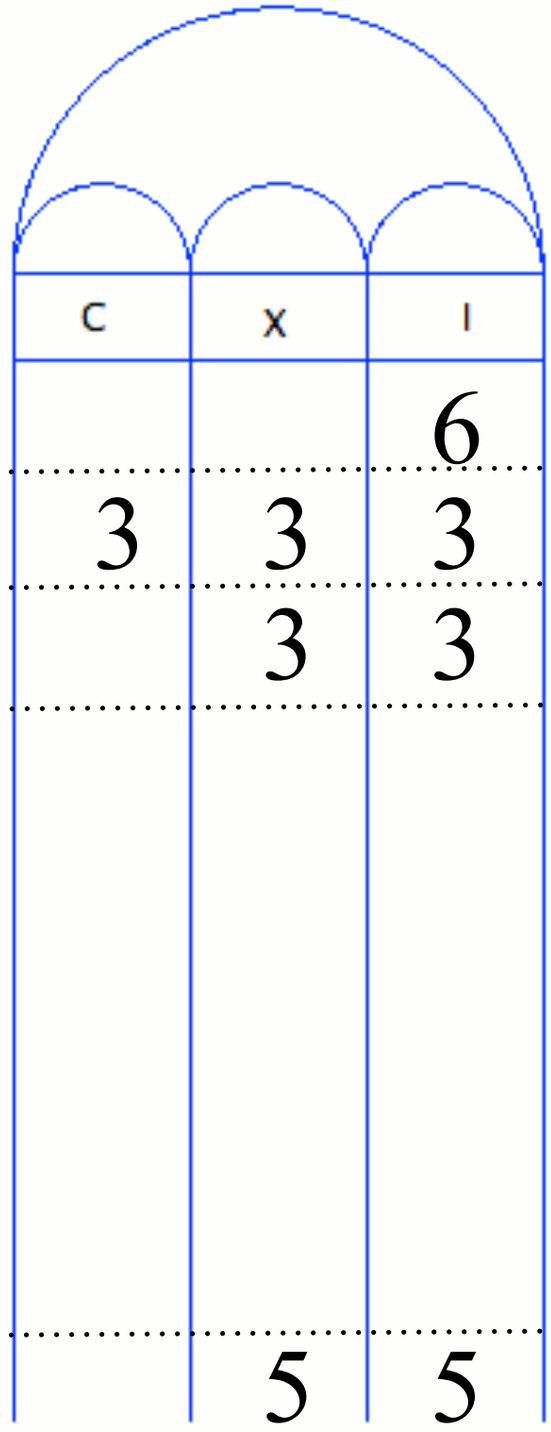
333 : 6

Quantos 6s em 3?...



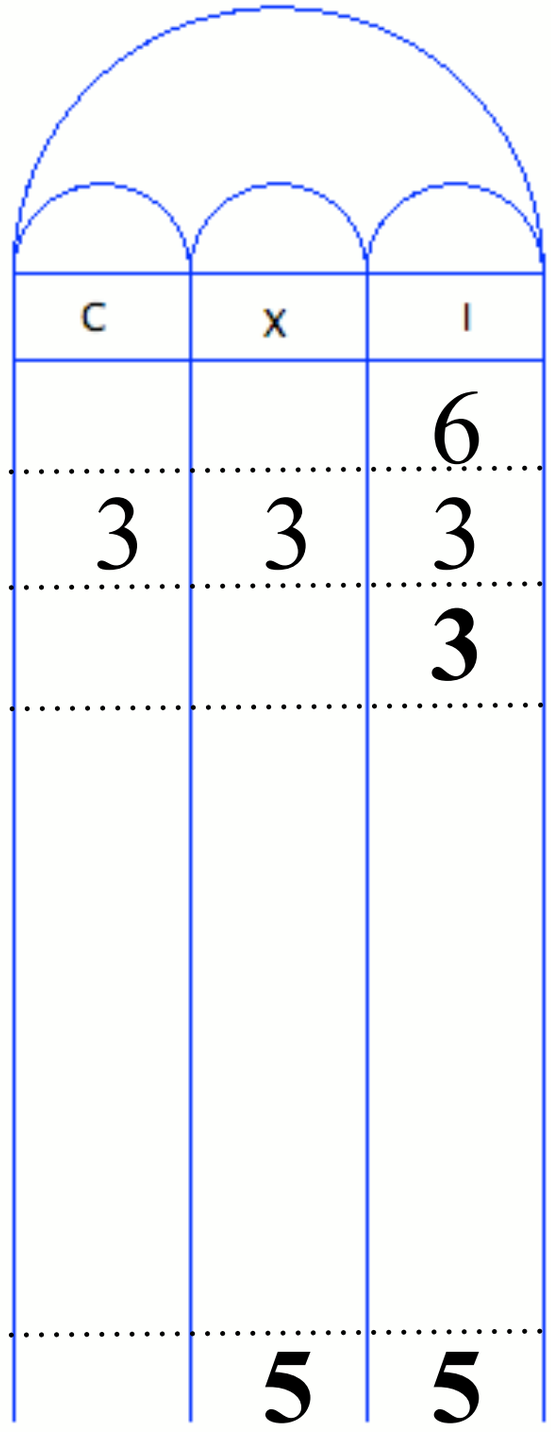
Divisio aurea

333 : 6



Divisio aurea

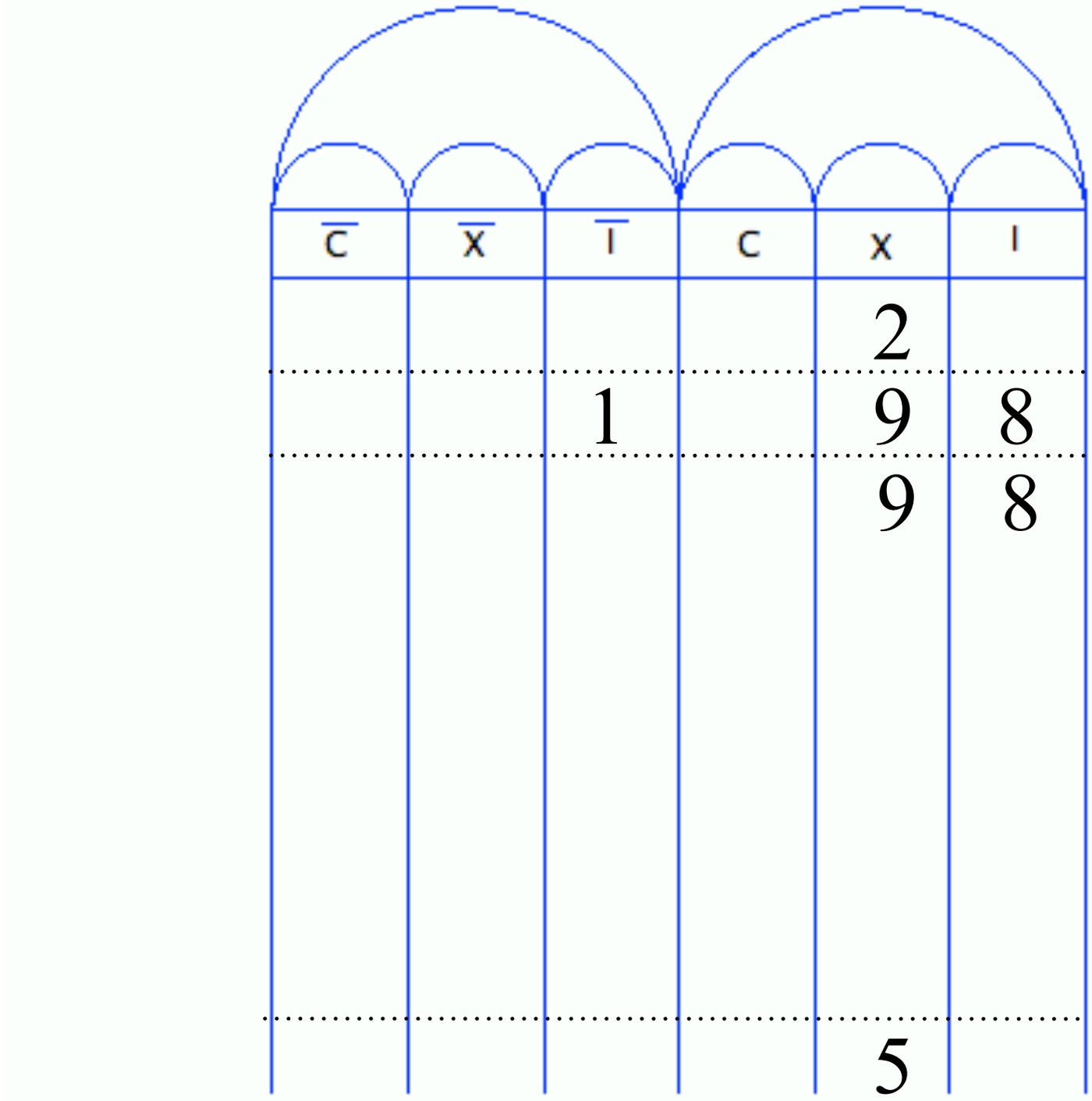
$333 = 6 \times 55 + 3$



1098 : 20

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
				2	
		1		9	8
		1		9	8

1098 : 20



1098 : 20

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
				2	
		1		9	8
				9	8
				5	4

1098 : 20

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
				2	
		1		9	8
				1	8
				5	4

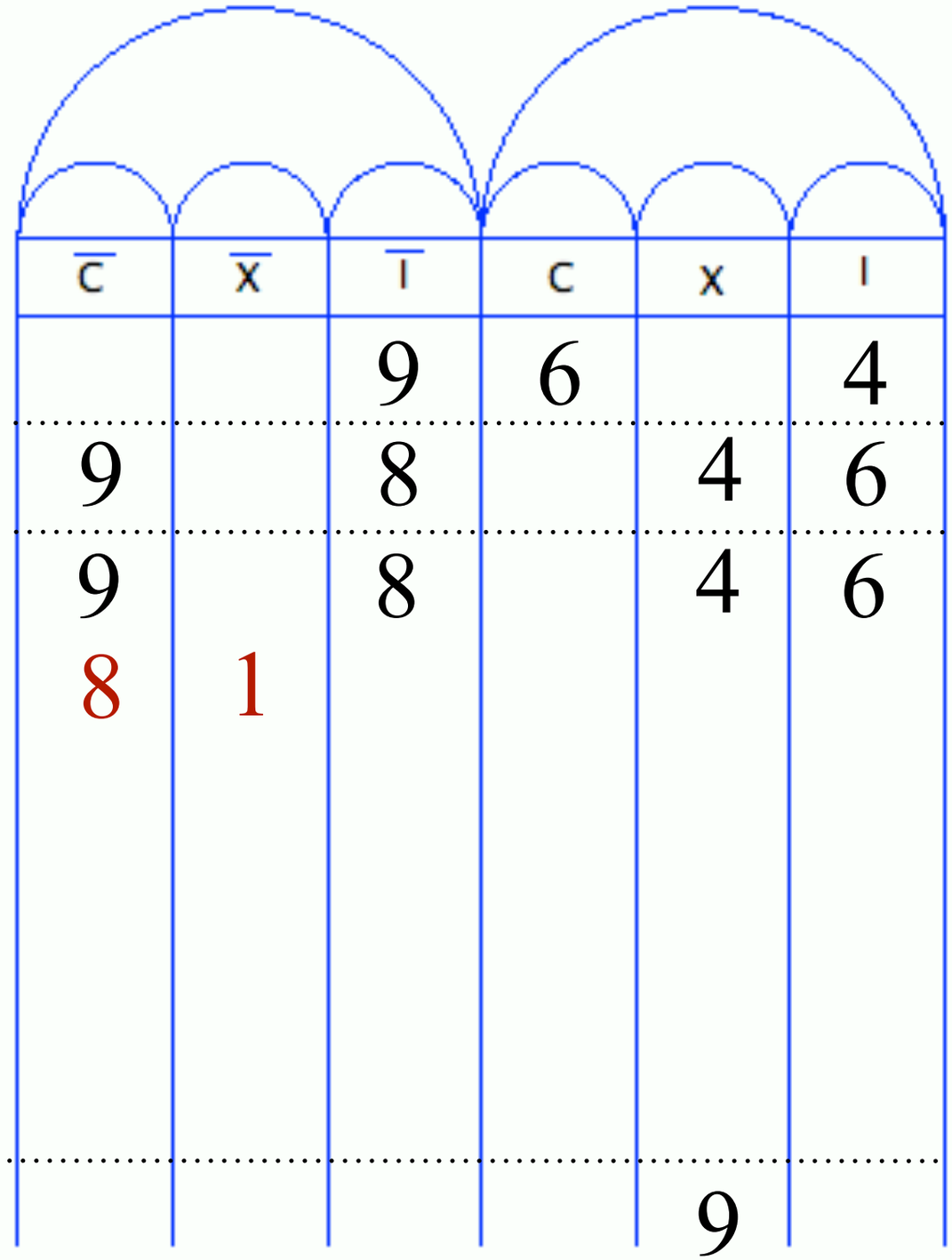
1098 =
20 x 54 + 18

908046 : 9604

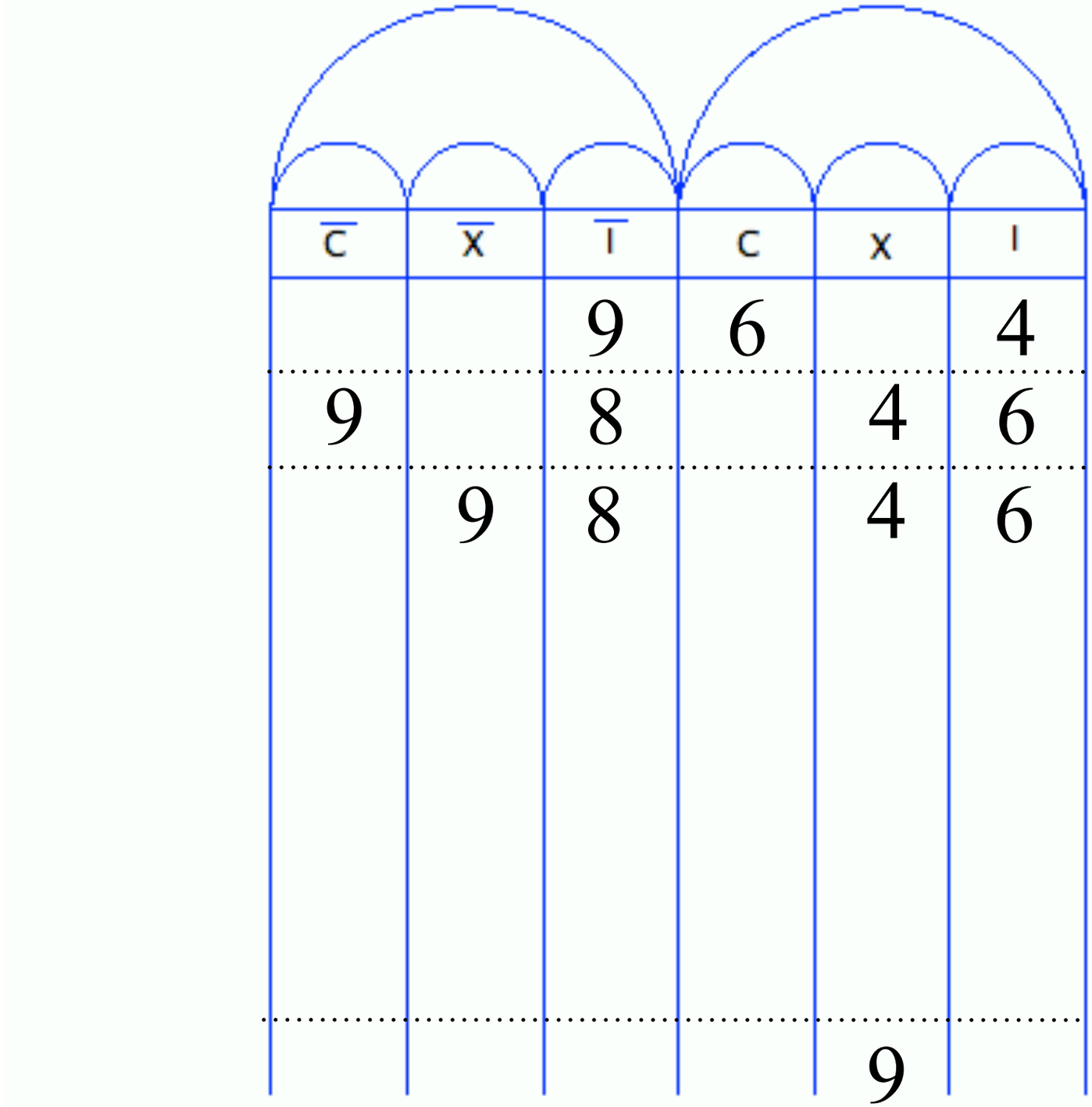
$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		9	6		4
9		8		4	6
9		8		4	6

Quantos 9s em 90?...

908046 : 9604



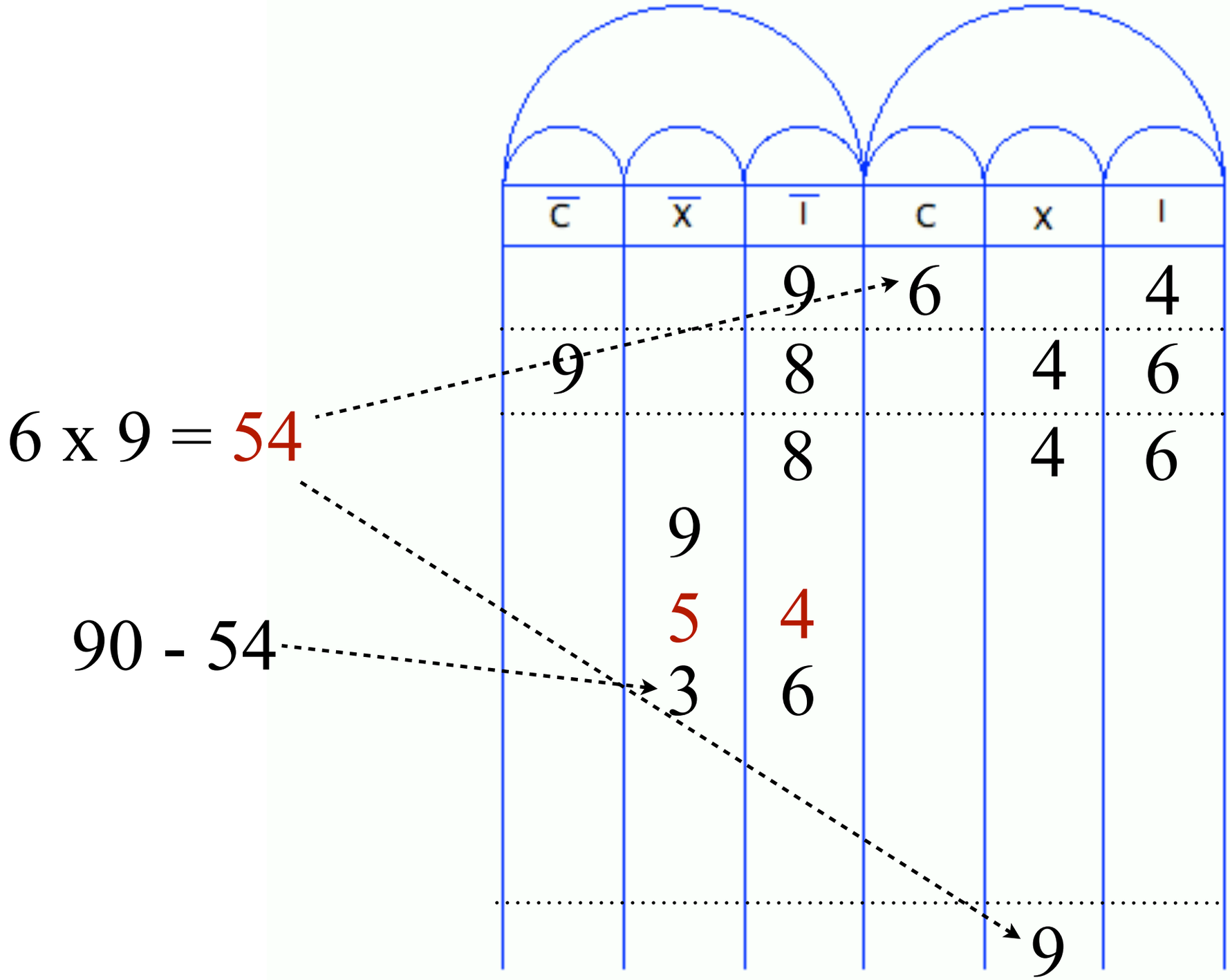
908046 : 9604



908046 : 9604

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		9	6		4
9		8		4	6
		8		4	6
	9				
	5	4			
	3	6			
				9	

908046 : 9604



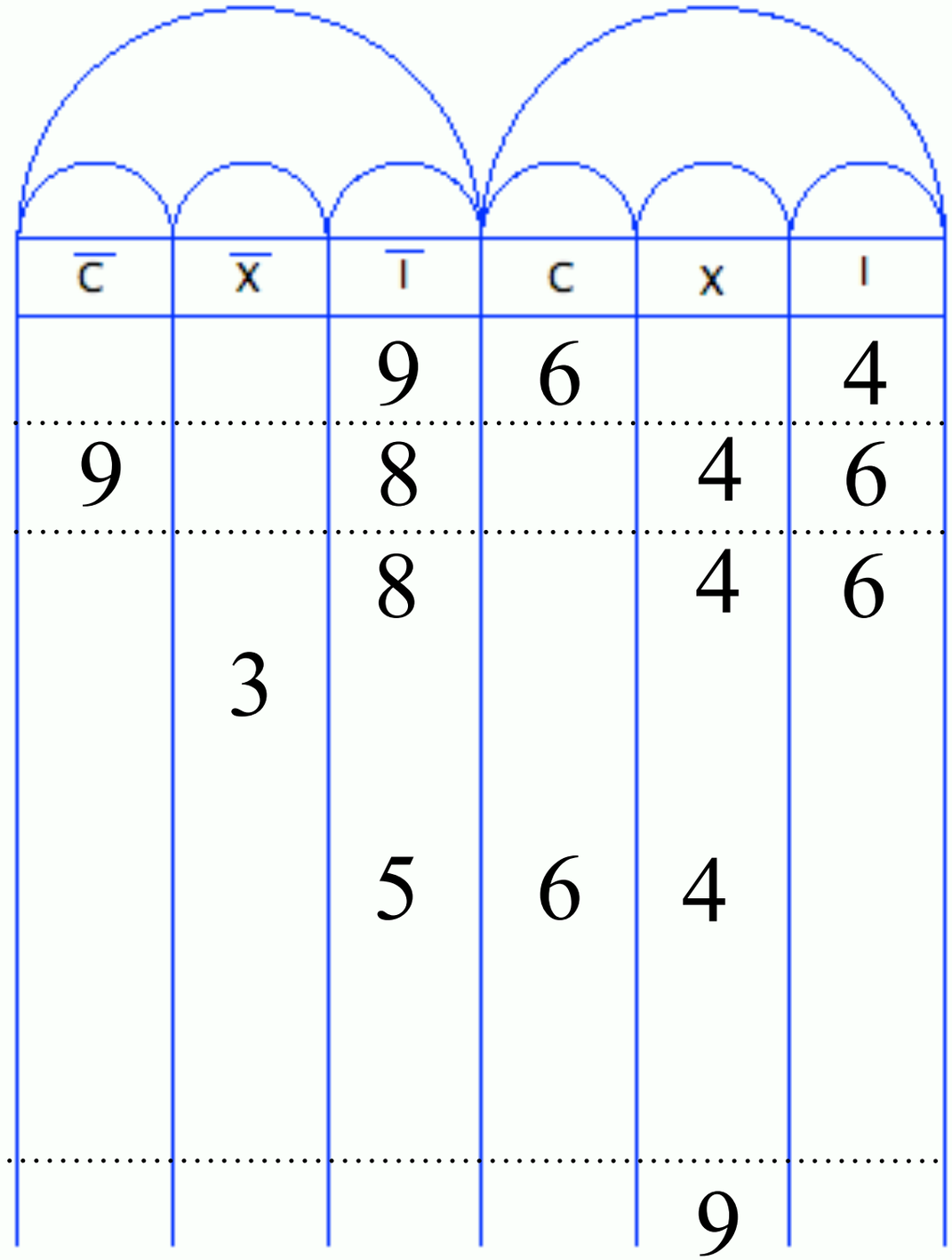
908046 : 9604

C̄	X̄	Ī	C	X	I
		9	6		4
9		8		4	6
		8		4	6
	3	6			
			3	6	
		5	6	4	
					9

4 x 9 = 36

600 - 36 = 564

908046 : 9604



908046 : 9604

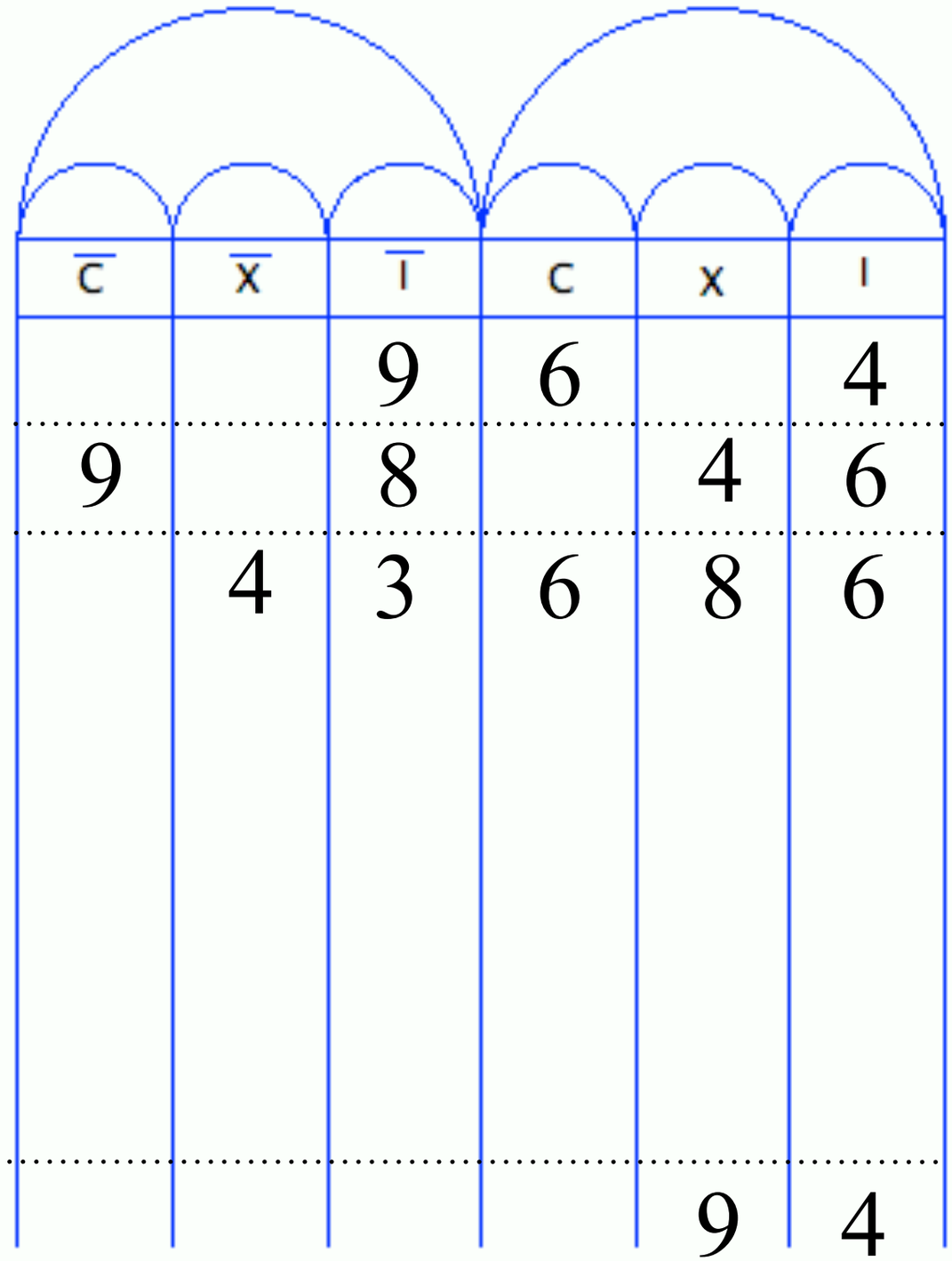
Organizando...

C̄	X̄	Ī	C	X	I
		9	6		4
9		8		4	6
	4	3	6	8	6

9

908046 : 9604

Organizando...



908046 : 9604

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		9	6		4
9		8		4	6
	4	3	6	8	6
	3	6			
		7			
				9	4

908046 : 9604

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		9	6		4
9		8		4	6
		7	6	8	6
		2	4		
				9	4

908046 : 9604

C̄	X̄	Ī	C	X	I
		9	6		4
9		8		4	6
		7	6	8	6
		2	4		
		4	6		
				9	4

70 - 24 = 46

908046 : 9604

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		9	6		4
9		8		4	6
		4	6	8	6
			6		
				1	6
				9	4

908046 : 9604

C	X	I	C	X	I
		9	6		4
9		8		4	6
		4	6	8	6
			6		
				1	6
				7	
				9	4

86 - 16 = 70

908046 : 9604

Organizando...

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
		9	6		4
9		8		4	6
		4	6	8	
			6		
		5	2	7	
				9	4

809331 : 60780

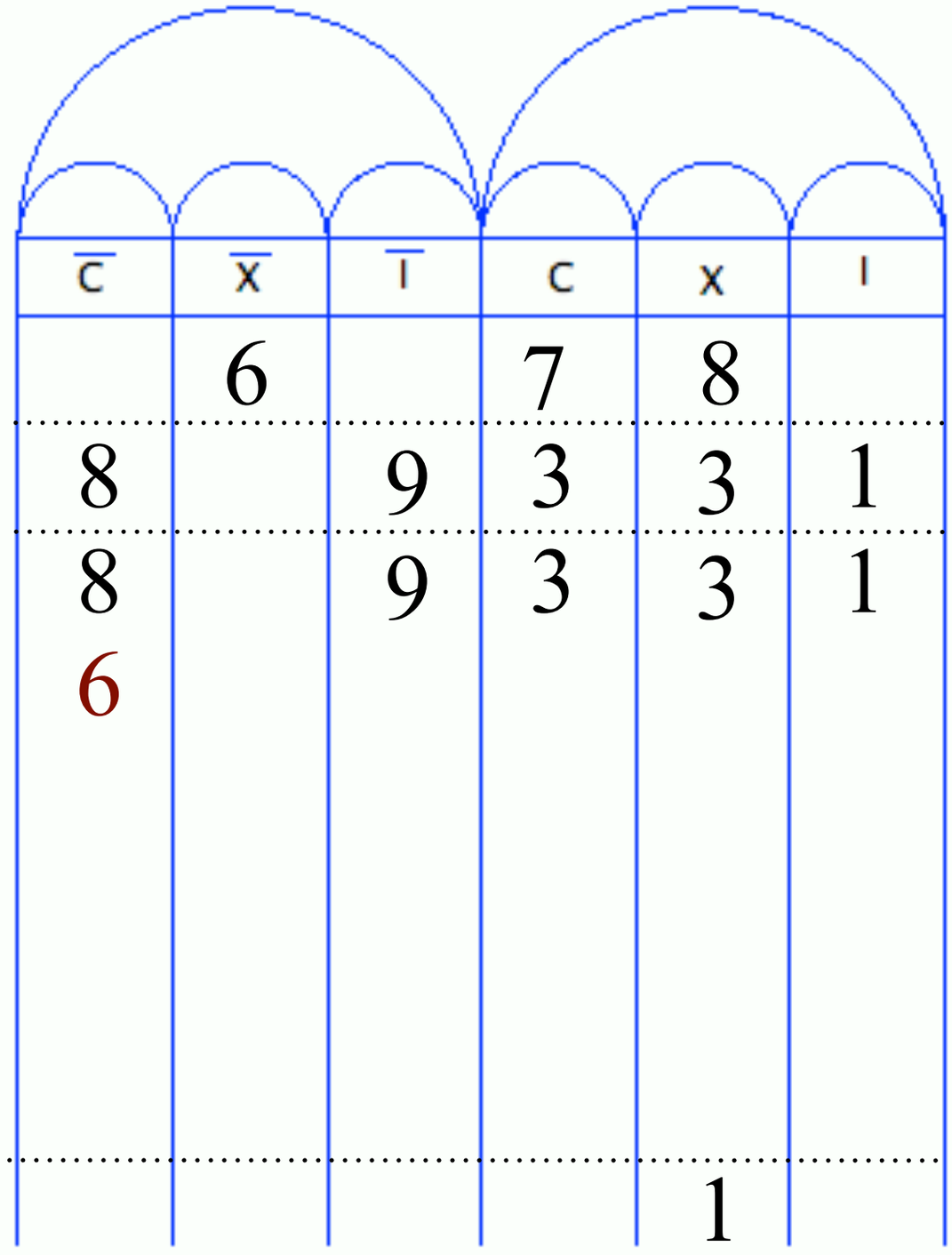
$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
	6		7	8	
8		9	3	3	1
8		9	3	3	1

809331 : 60780

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
	6		7	8	
8		9	3	3	1
8		9	3	3	1

Quantos 6s em 8?...

809331 : 60780



809331 : 60780

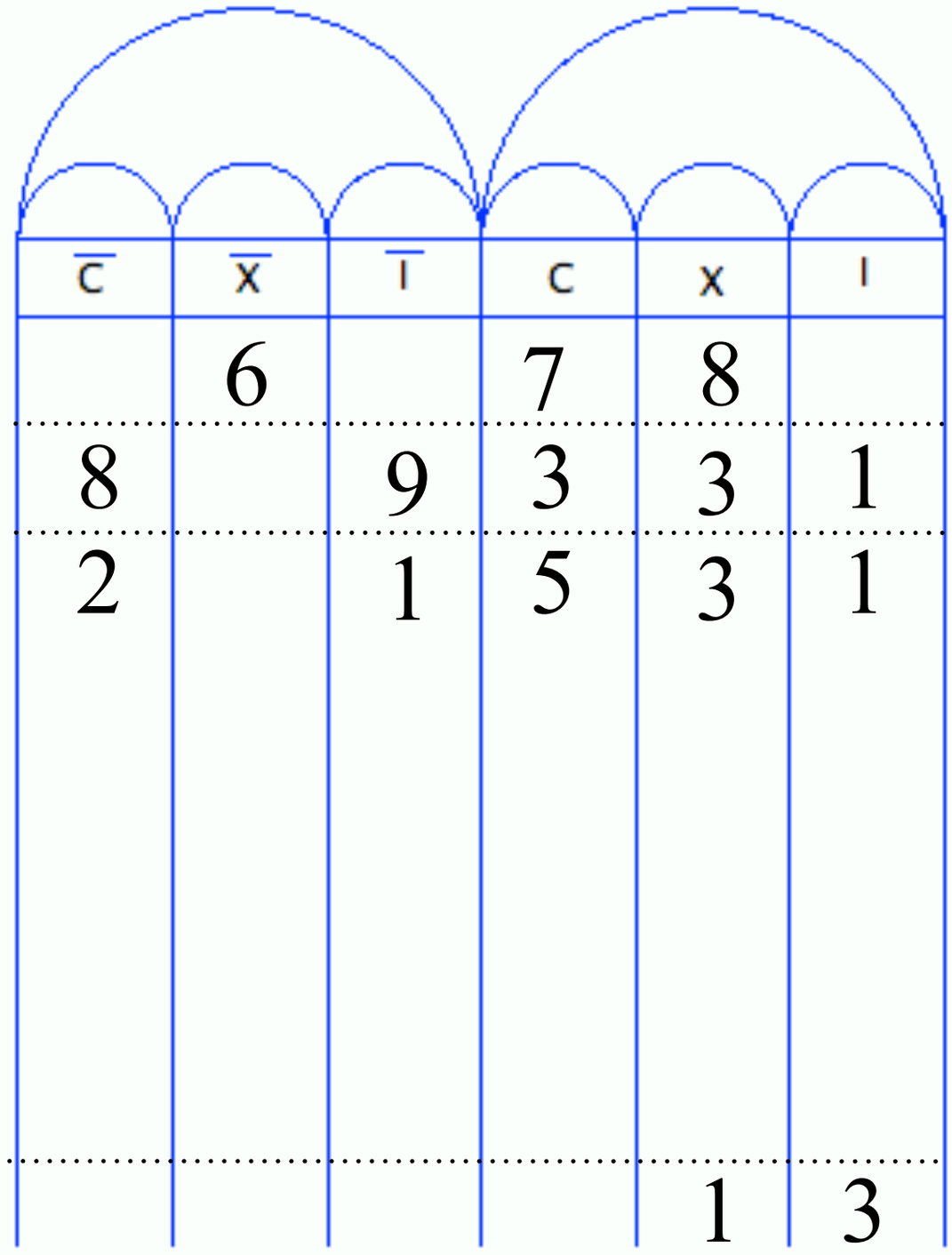
$\overline{c}$	$\overline{x}$	$\overline{I}$	c	x	I
	6		7	8	
8		9	3	3	1
2		9	3	3	1
		7			
				1	

809331 : 60780

$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
	6		7	8	
8		9	3	3	1
2		2	3	3	1
			8		

1

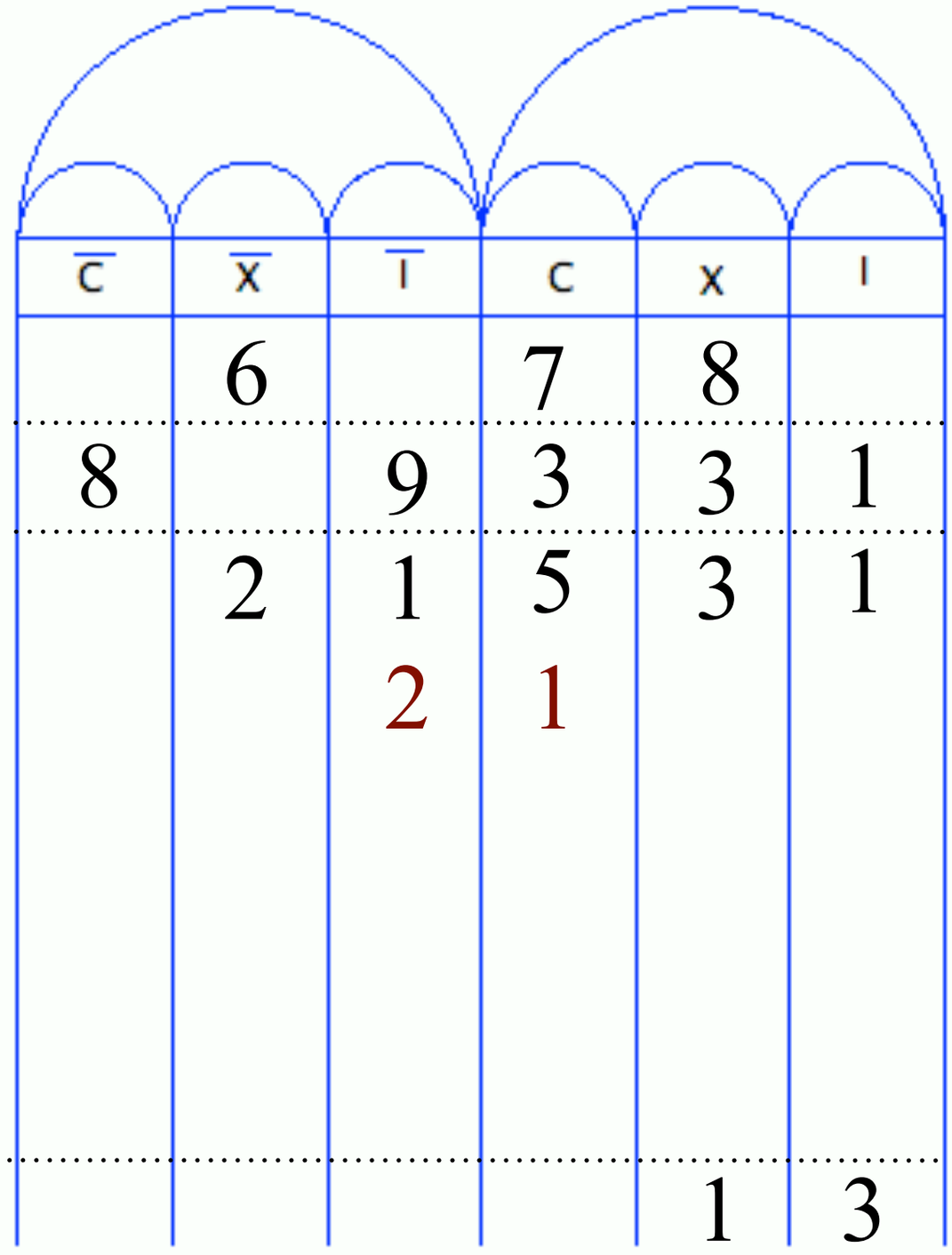
809331 : 60780



809331 : 60780

C	X	I	C	X	I
	6		7	8	
8		9	3	3	1
2		1	5	3	1
1	8				
				1	3

809331 : 60780



809331 : 60780

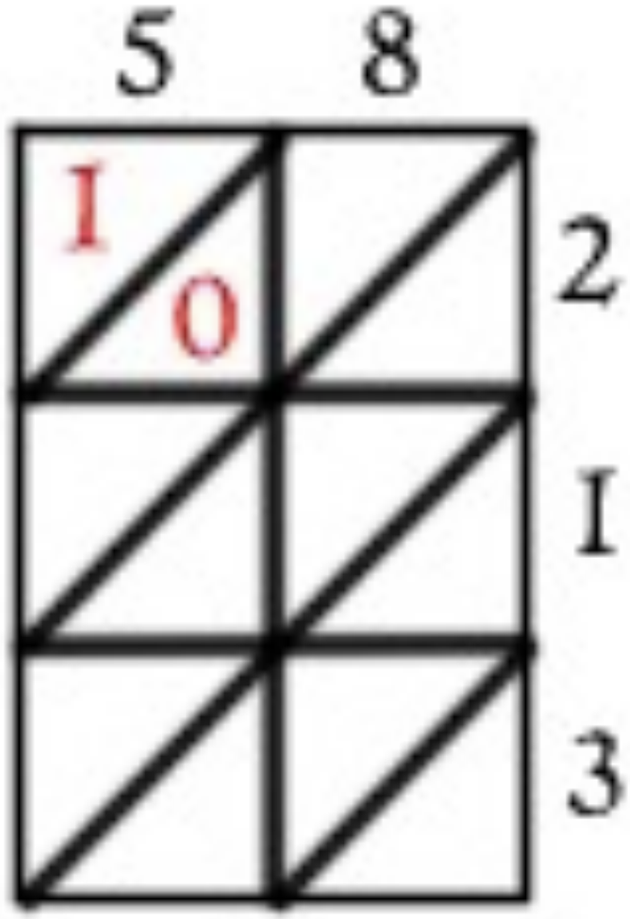
$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
	6		7	8	
8		9	3	3	1
	1	9	4	3	1
			2	4	
				1	3

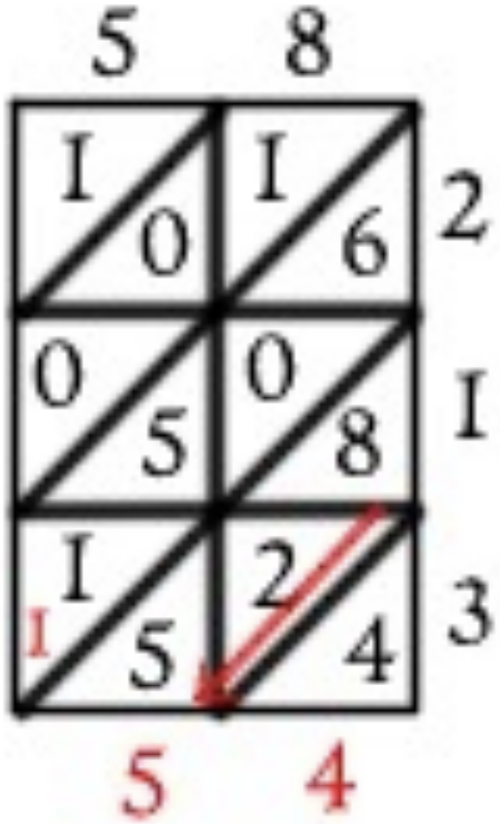
809331 : 60780

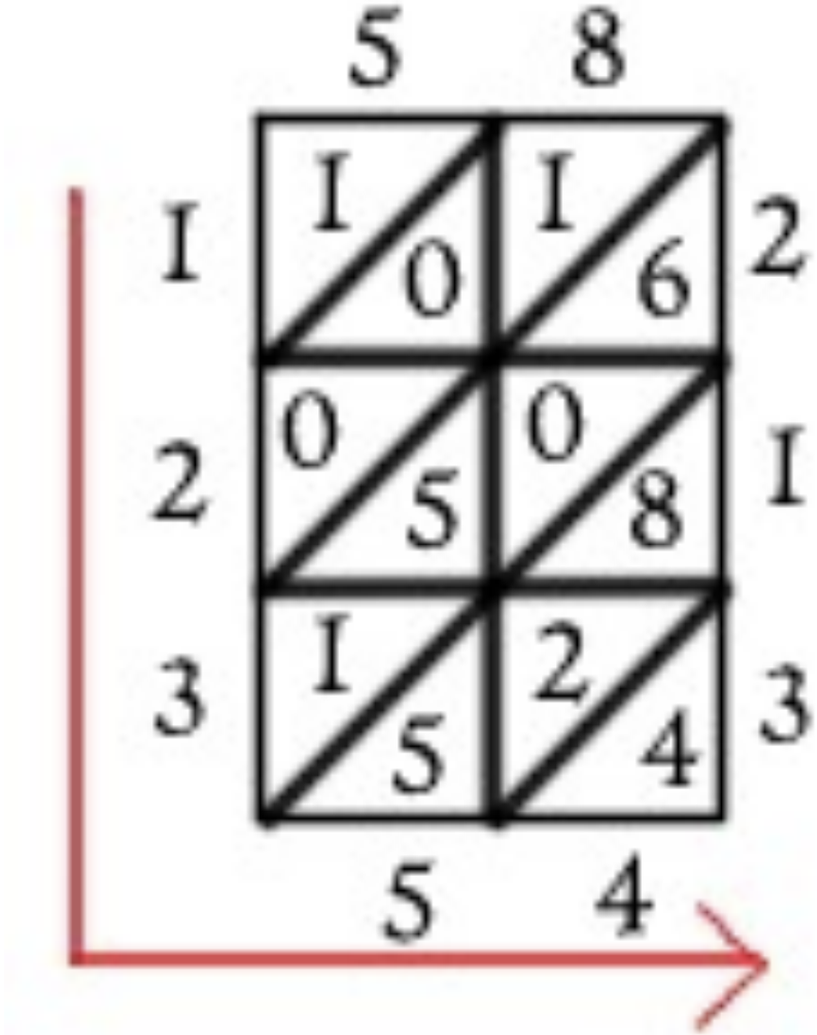
$\overline{C}$	$\overline{X}$	$\overline{I}$	C	X	I
	6		7	8	
8		9	3	3	1
	1	**9**	**1**	**9**	**1**
				1	**3**

Lattice multiplication, also known as **gelosia multiplication**, **sieve multiplication**, **shabakh**, **Venetian squares**, or the **Hindu lattice**, is a method of multiplication that uses a lattice to multiply two multi-digit numbers. It is the same as regular long multiplication, but it breaks the process into smaller steps, which some practitioners find easier to use.

The method had already arisen by medieval times, and has been used for centuries in many different cultures. It is still being taught in certain curricula today.


5
8
I
0
2
I
3


5 8
I 0
I 6
2
0 5
0 8
I
I 5
2 4
3
I
5 4


5 8
I
I 0
I 6
2
2
0 5
0 8
I
3
I 5
2 4
3
5 4

$7 \times 1 =$ 7

$7 \times 2 =$ 14

$7 \times 3 =$ 21

$7 \times 4 =$ 28

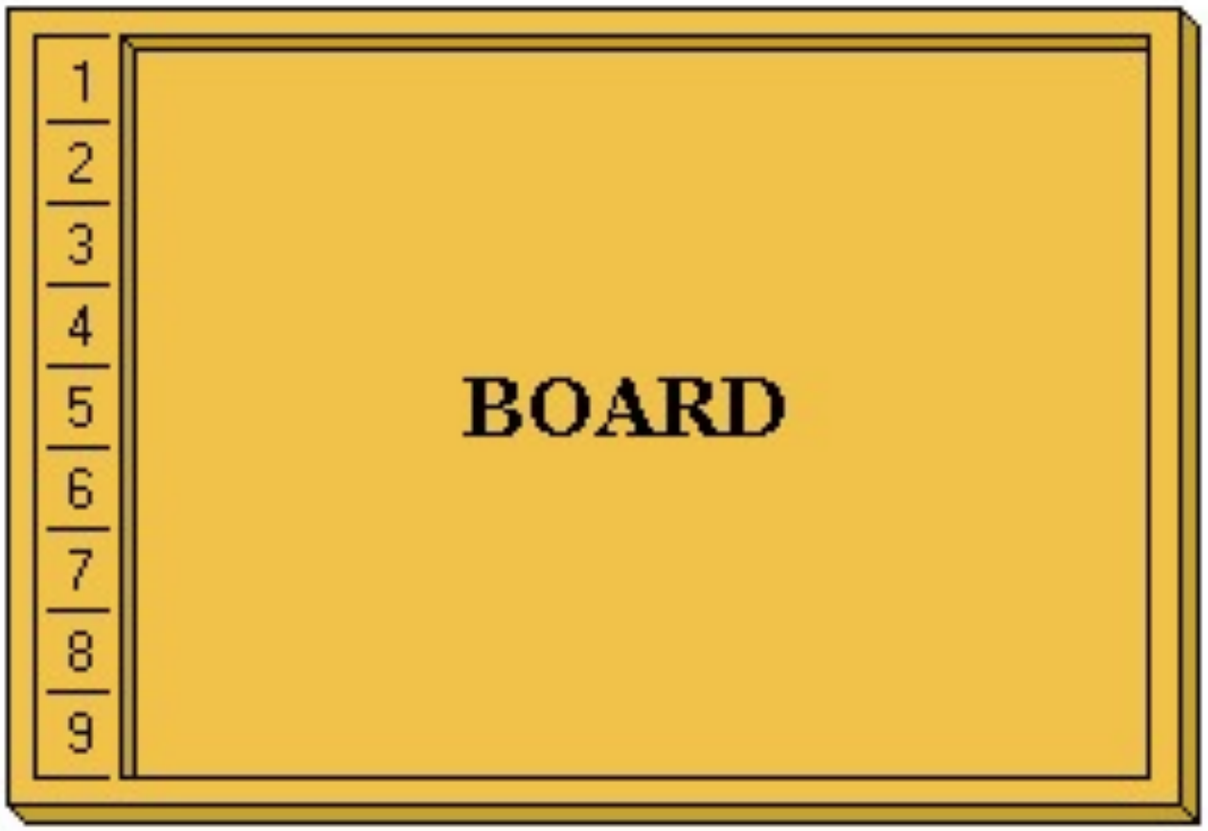
$7 \times 5 =$ 35

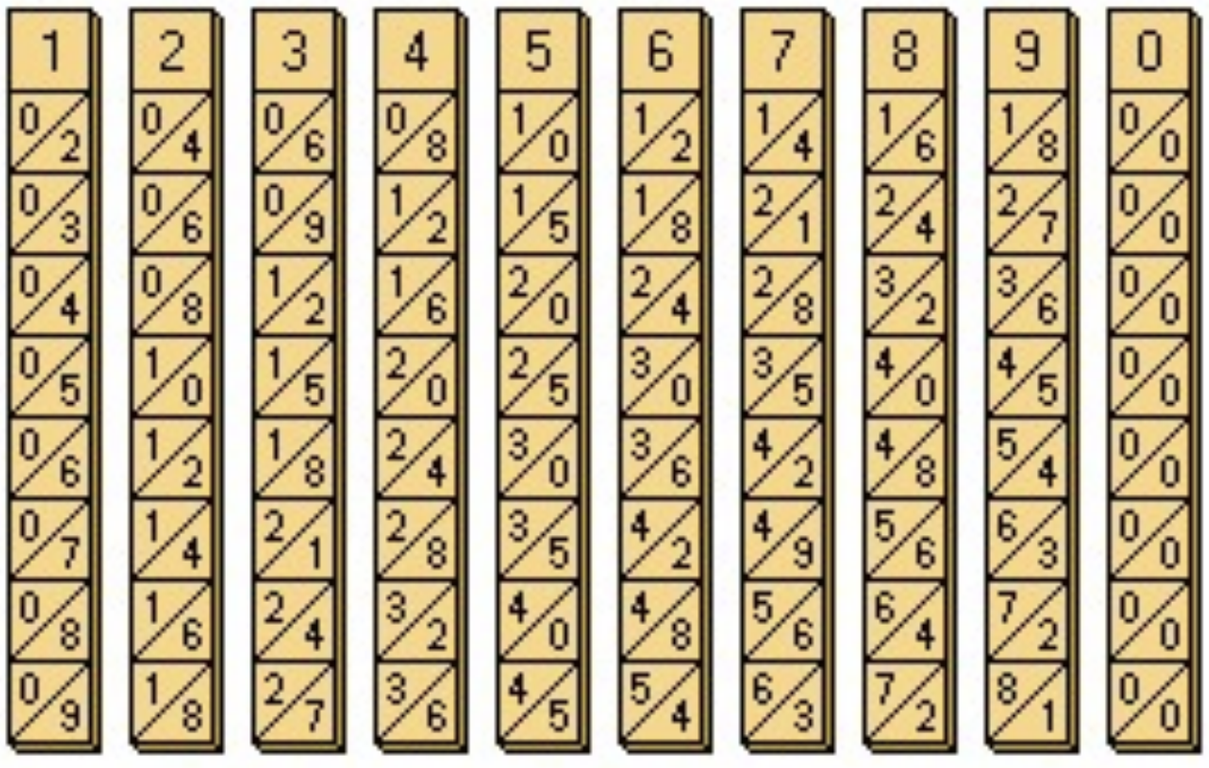
$7 \times 6 =$ 42

$7 \times 7 =$ 49

$7 \times 8 =$ 56

$7 \times 9 =$ 63





SET OF RODS

425 x 6 = ?

1	4	2	5
2	0 8	0 4	1 0
3	1 2	0 6	1 5
4	1 6	0 8	2 0
5	2 0	1 0	2 5
6	2 4	1 2	3 0
7	2 8	1 4	3 5
8	3 2	1 6	4 0
9	3 6	1 8	4 5

→

6	2 4	1 2	3 0

6	2 4 + 1	2 + 3	0
=2	=5	=5	=0

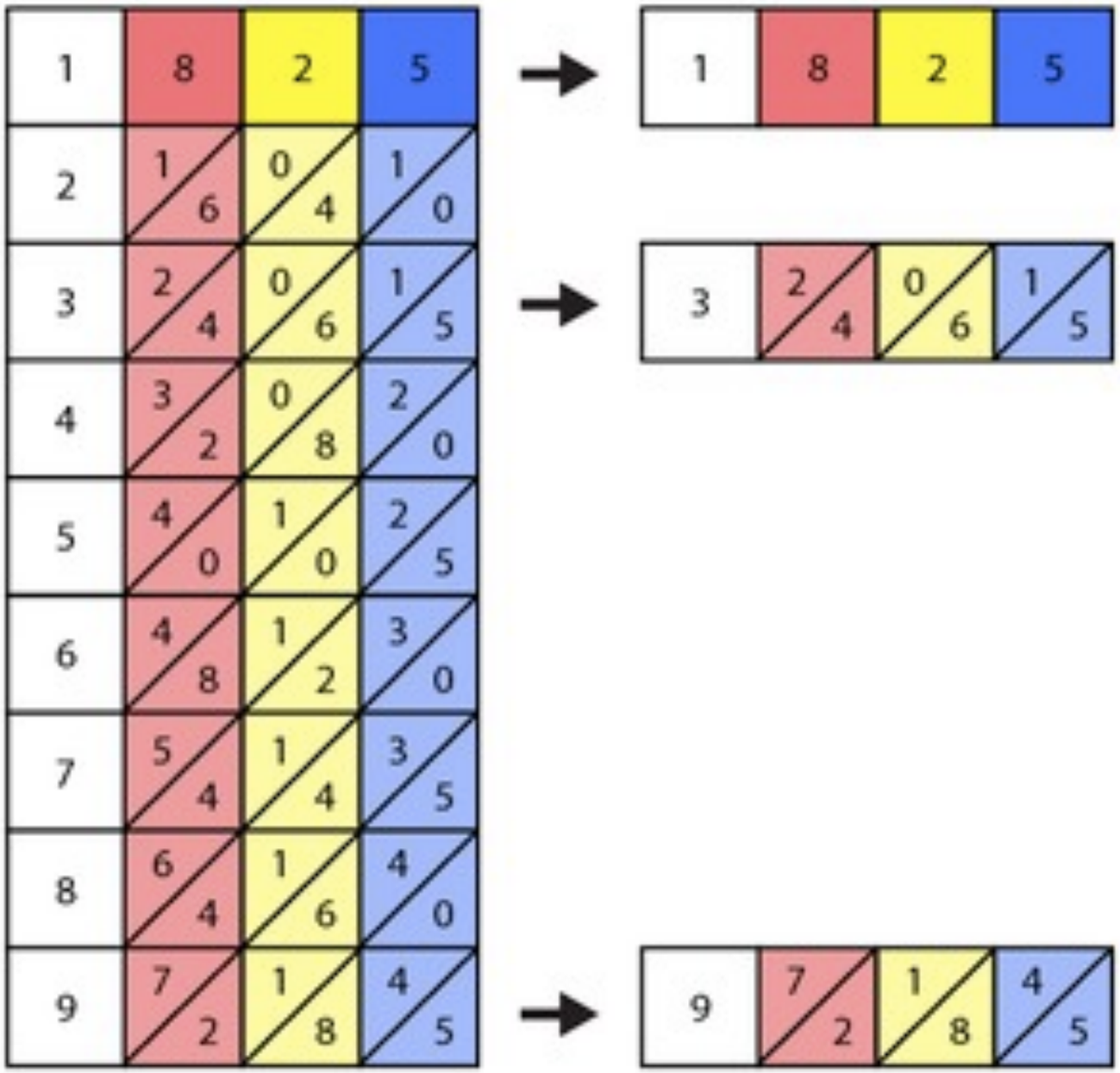
6785 x 8 = ?

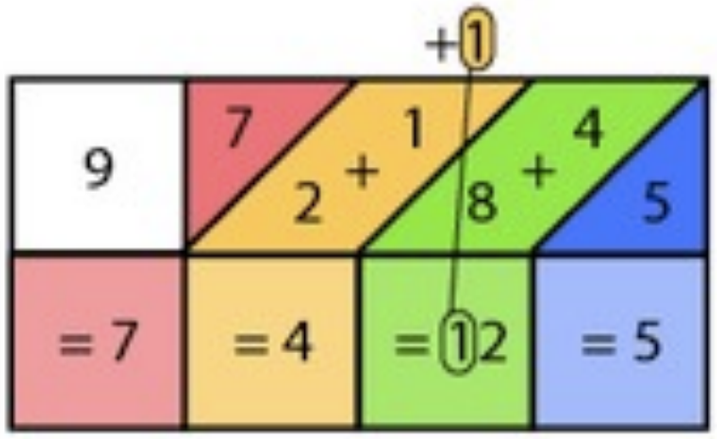
1	6	7	8	5
2	12	14	16	10
3	18	21	24	15
4	24	28	32	20
5	30	35	40	25
6	36	42	48	30
7	42	49	54	35
8	48	56	64	40
9	54	63	72	45

→

8	48	56	64	40

825 x 913 = ?


1 8 2 5
2 1 6 0 4 1 0
3 2 4 0 6 1 5
4 3 2 0 8 2 0
5 4 0 1 0 2 5
6 4 8 1 2 3 0
7 5 4 1 4 3 5
8 6 4 1 6 4 0
9 7 2 1 8 4 5
1 8 2 5
3 2 4 0 6 1 5
9 7 2 1 8 4 5



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Row 3: 2475

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Obrigado.