Propositiones ad acuendos iuvenes by Alcuin

The little book *Propositiones ad acuendos iuvenes* ("Problems to sharpen the young") is believed to have been produced by Alcuin of York around 800AD. When we say "produced" we are suggesting that the problems, or more accurately puzzles, that it contains are likely to have been collected by Alcuin rather than composed by him. Some of the problems were known in other cultures at an earlier period but others have not been found in any earlier work and could be composed by Alcuin himself. We still have to say something about the word "believed" in our first sentence. By this we are implying that there is no certain proof that the work is by Alcuin, but historians are almost unanimous in attributing the work to him. The only direct proof that Alcuin had indeed produced arithmetic puzzles is contained in a letter that he sent to Charlemagne in 800 which mentions that he is including with the letter:-

... certain subtle figures of arithmetic, for pleasure.

Of course, to have been produced in 800, many centuries before the advent of printing, means that the work was handed down by being copied by scribes. As always happens with this type of transmission various versions have come down to us with differences due to copying errors and also due to deliberate changes to "improve" the text. One difference is the number of puzzles, with some versions of the text containing 53 puzzles and others containing 56. The majority decision among historians is that the original text contained 56 puzzles and those with 53 are later (somewhat defective) versions. In some manuscripts the puzzles are not numbered, while in others with only 53 puzzles they have a standard numbering. We have kept to the standard numbering in our list of the puzzles so, for example, there are puzzles 11, 11(a) and 11(b).

We have chosen to give our own versions of the puzzles which we have modernised slightly to make them more accessible to people today. We have often changed the ancient units of length and money into ones with a slightly more modern look. Alcuin gives solutions in the book but often these give an answer with a verification that it is correct rather than a method of proof. We shall usually give both a modern approach to solving the problem as well as a comment on Alcuin's solution. None of the puzzles require any mathematical knowledge to solve them, although of course often they are easier if we use a little algebra. This book was produced in a period when there was little or no interest in mathematics in Europe. It is worth remembering as one reads the problems, that this was the highlight of mathematics in Europe at the time. It was written in Latin, so it was clearly intended as an amusement for the well-educated.

1. **Puzzle of the snail.**
   A snail was invited by a swallow to lunch a league away. However, it could only crawl one inch per day. How many years and days did it take for the snail to crawl to that lunch?
   [Assume that 1 league = 1500 paces, 1 pace = 5 feet, 1 foot = 12 inches and that there are 365 days in a year.]

Solution.
$1500 \times 5 \times 12 = 90000$. Divide 90000 by 365 to obtain 246 years and 210 days.

2. **Puzzle of the man walking in the street.**
A certain man walking in the street saw a group of men coming towards him, and he said to them: "Suppose there were as many more of you as you are now; and then to this half of half were added; and then half of this number were added. Then, together with myself, we would number 100." *How many men were in the group that the man saw?*

Solution.
Suppose there are x men in the group. Then

$$2x + x/2 + x/4 + 1 = 100.$$

So $x = 36$.

Alcuin, of course, doesn't use equations. He merely checks that 36 satisfies the given conditions by checking that 72 + 18 + 9 + 1 = 100. Problems of this type are very old and appear in the Rhind papyrus.

3. **Puzzle of the two men and the storks.**
Two men were walking along the street when they saw some storks. They asked each other, "How many storks are there?" Their discussion went as follows: "Suppose the number of storks was doubled, then the original number added again, and then half of a third of this sum were added. Then, together with another two, they would number 100." *How many storks did the men see?*

Solution.
Suppose there are x storks. Then

$$2x + x + 3x/6 + 2 = 100.$$

So $x = 28$.

Again, Alcuin gives the answer and checks that 28 satisfies the conditions.

4. **Puzzle of the man and the horses.**
A certain man saw some horses grazing in a field and wanted them. He said: "Suppose that they were mine, and that the number were doubled, and then a half of half of this sum were added. In this case I would have 100 *horses." How many horses did the man see grazing in the field?*

Solution.
Suppose there are x horses. Then

$$2x + {}^{2x}/_4 = 100.$$

So $x = 40$.

Alcuin gives the answer 40 and checks that it satisfies the conditions.

5. **Puzzle of the pig farmer with 100 pounds.**
A pig farmer said: "I want to buy 100 *pigs with* 100 *pounds. Now a boar costs ten pounds, a sow costs five pounds and two piglets can be bought for a pound." How many boars, sows, and piglets can the pig farmer buy so that he spends all his money?*

Solution.
Suppose the farmer buys x boars, y sows, and z pairs of piglets. Then

$$x + y + 2z = 100, \text{ and } 10x + 5y + z = 100.$$

From the second equation we see that z must be divisible by 5 so write $z = 5t$. Then

$$x + y + 10t = 100, \text{ and } 2x + y + t = 20.$$

The first equation gives $t < 10$ but, subtracting the equations gives -$x + 9t = 80$ and so $t \geq 9$ (since $9t > 80$). Hence $t = 9$, $x = 1$ and y = 9.

The pig farmer must buy 1 boar, 9 sows and 90 piglets.

Alcuin shows this answer satisfies the conditions but doesn't give any method to find it. One assumes that trial and error is intended. The first known appearance of a problem of this type is in China in the 5th century

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6. **Puzzle of the two farmers with 100 pounds.**
There were two farmers who had 100 *pounds between them, with which they bought some pigs. They bought groups of five pigs at two pounds for a group, and they intended to fatten them and to sell them at a profit. But when they saw that the time was not right to fatten the pigs, and being unable to keep them on their farms, they tried to make a profit by selling them. However, they were unsuccessful because they could only sell the pigs for what they had paid (i.e., five pigs for two pounds). When they realized this, they said to each other, "We shall divide the pigs." But by dividing and selling the pigs for as much as they had paid, they made a profit. How many pigs were there at first, and how did the men divide and sell for a profit that which they could not do together?*

Solution.
They must have bought 250 pigs. If they were able to make a profit by dividing them up differently, the pigs in the groups of 5 couldn't have been of equal value. One farmer takes 125 of the best quality pigs and the other 125 of the poorest quality pigs. The first sells 120 at 2 for a pound while the second farmer sells 120 at 3 for a pound. They therefore have their 100 pounds back and they each have 5 pigs to sell for a profit.

This may have been an original puzzle by Alcuin. Certainly no earlier versions of this type of puzzle are known.

7. **Puzzle of the plate weighing 30 pounds.**
There is a plate weighing 30 *pounds or* 600 *shillings. In it, there is gold, silver, brass and tin. It has three times are much silver as gold, three times as much brass as silver, and three times as much tin as brass. How much does each type of metal weigh?*

Solution.
Suppose there are x shillings of gold. Then there are $3x$ shillings of silver, $9x$ shillings of brass and $27x$ shillings of tin.

$$x + 3x + 9x + 27x = 40x = 600$$

so $x = 15$ and there are 15 shillings of gold, 45 shillings of silver, 135 shillings of brass and 405 shillings of tin.

As usual, Alcuin gives the answer and checks it satisfies the conditions.

8. **Puzzle of the cask with three cracks.**
There is a cask which has three cracks in it. It is filled with 7200 *pints of water. A third plus a sixth part runs out through one crack. Through another crack a third part runs out. Only a sixth part runs out through the third crack. How many pints ran out through each crack.*

Solution.

Half runs out the first crack, namely 3600 pints, $^1/_3$ through the second, namely 2400 pints, and $^1/_6$ through the third crack, namely 1200 pints.

9. **Puzzle of the cloaks.**
I have material which is 100 *feet long and* 80 *feet wide. From it, I wish to make cloaks from portions in such a way that each portion is five feet in length and four feet wide. How many cloaks can be made from the material?*

Solution.
From the 100 feet we can cut 20 at 5 feet and from the 80 we get 20 at 4 feet. So we can cut 400 portions of material 5 feet by 4 feet.

10. **Puzzle of the linen**.
I have a single linen cloth which is 60 *feet long and* 40 *feet wide. I wish to cut it into smaller portions, each being six feet in length, four feet in width, so that each piece is big enough to make a tunic. How many tunics can be made from the single linen cloth?*

Solution.

As in the previous puzzle, 10×10 which is 100 tunics.

11. **Puzzle of the two men marrying each other's sister.**
If two men should marry one another's sister. What will be the sons' relations to each other?

Solution.
The sons are cousins twice over, each having a parent who is sibling to a parent of the other, in two ways. This, and the following two "relationship" puzzles may be original to Alcuin. Certainly no earlier versions are known.

(a) **Puzzle of two men marrying each other's mother.**
If two men each take the other's mother in marriage, what would be the relationship between their sons?

Solution.
Let *A* and *B* be the two men who marry each other's mother. Then suppose that the sons of these two marriages are *S* and *T*. Then *S*'s father is *A*and *B* is his half-brother. Similarly *T*'s father is *B* and *A* is his half-brother. *S* and *T*, the two sons, are therefore both uncle and nephew to each other.

It would appear from the surviving manuscripts that Alcuin didn't give a solution to this puzzle. However, this puzzle has been seen ever since and continues to crop up from time to time. Although Alcuin doesn't ask about any of the other strange relationships that arise in this situation, the one which gives rise to a riddle and a song is as follows.
B's mother is *A*'s mother's mother-in-law, and is therefore *A*'s grandmother. Since *A* is married to *B*'s mother, he is married to his own grandmother, and so is his own grandfather.

(b) **Puzzle of a father, son, a widow and her daughter.**
If a widow and her daughter take a father and son in marriage, so that the son marries the mother and the father the daughter, what is the relationship of their sons?

Solution.
This leads to a similar situation to the previous puzzle, and the two sons are both uncle and nephew to each other. Again it would appear that Alcuin didn't give a solution. As in the previous puzzle, the father's son is his own grandfather.

12. **Puzzle of the father and his three sons.**
A certain father died and left as an inheritance to his three sons 30 *glass flasks, of which* 10 *were full of wine; another* 10 *were half full, while the last* 10 *were empty. Divide the wine and flasks so that an equal share of each should come down to the three sons, both of wine and glass.*

Solution.
Each son must get 10 glass flasks and 5 flasks worth of wine. So divide the flasks so that one son receives 10 half full flasks of wine and the other two sons each receive 5 full flasks of wine and 5 empty flasks.

13. **Puzzle of the king's army.**
A king ordered his servant to collect an army from 30 *villages as follows: He should bring back from each successive village as many*

men as he had taken there. The servant went to the first village alone; he went with one other man to the second village; he went with three other men to the third village. How many men were collected from the 30 *villages.*

Solution.
After the first village there are 2 men in the army, there are 4 after the second village, there are 8 after the third village, ... So after the nth village there will be 2^n man collected for the army. After the 30th village there will be $2^{30} = 1{,}073{,}741{,}824$ men in the army.
[The last few villages must be bigger than any city on earth to be able to supply that many men!]

14. **Puzzle of the ox.**
How many footprints are left in the last furrow by an ox which has been ploughing all day?

Solution.
This is a trick question. There are no footprints in the furrow since the plough behind the ox will obliterate them.

15. **Puzzle of the ploughman.**
How many furrows might a farmer have in his field if the ploughman shall have made three turns at each end of the field?

Solution.
The first furrow requires no turns, then after one turn he will have 2 furrows, after 2 turns there will be 3 furrows, so after 6 turns there will be 7 furrows.

16. **Puzzle of the two men leading oxen.**
Two men were leading oxen along the road when one said to the other: "Give me two oxen, and I'll have as many oxen as you." Then the other said: "If after that you give me two oxen, I'll have twice as many as you." How many oxen there were, and how many did each man have?

Solution.
Suppose the first man has x oxen and the second man has y oxen. Then

$$x + 2 = y - 2 \text{ and } 2(x + 2 - 2) = y - 2 + 2.$$

Substitute $2x = y$ into the first equation to get $x = 4$ so $y = 8$.

As usual, Alcuin merely gives the answer and then shows that it satisfies the conditions.

17. **Puzzle of the three brothers with their sisters.**
There were three men, each having a sister, who wanted to cross a river. They found only a small boat in which only two persons could cross at a time. How did they cross the river, so that none of the girls was ever left alone with a man other than her brother either in the boat or on the bank?

Solution.
Suppose $M1$ has sister $S1$, $M2$ has sister $S2$ and $M3$ has sister $S3$. Begin with all on one bank and each successive row indicates the position after a crossing has been made, with the boat marked = :

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M1, M2, M3, S1, S2, S3 =-----
 M2, M3, S2, S3         -----= M1, S1
 M2, M3, S2, S3, M1     =----- S1
 M2, M3, M1             -----= S2, S3, S1
 M2, M3, M1, S1         =----- S2, S3
 M1, S1                 -----= M2, M3, S2, S3
 M1, S1, M2, S2         =----- M3, S3
 S1, S2                 -----= M1, M2, M3, S3
 S1, S2, S3             =----- M1, M2, M3
 S2                     -----= S1, S3, M1, M2, M3
 S2, M2                 =----- S1, S3, M1, M3
                        -----= S2, M2, S1, S3, M1, M3
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This is Alcuin's solution, although of course he writes it out in words rather than the symbols we have used. It requires 11 crossings but it can actually be done with fewer crossings. See if you can find a solution with only 9 crossings. This puzzle, and the following "crossing" puzzles may be original puzzles by Alcuin. No earlier versions are known.

18. **Puzzle of the wolf, a goat and the cabbages.**
A man needed to take a wolf, a goat and a box of cabbage across a river. However, he could only find a boat which would carry two of these at a time. How did he get all of them across unharmed?
[The man can't leave the wolf and the goat together unless he is there, or the goat and the cabbage together unless he is there.]

Solution.
Denote the man, wolf, goat and cabbage by M, W, G, C respectively. Begin with all on one bank and each successive row indicates the position after a crossing has been made:

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W, G, C, M =-----
 W, C       -----= M, G
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W, C, M     =----- G
C           -----= M, W, G
C, G, M     =----- W
G           -----= M, C, W
G, M        =----- C, W
            -----= M, G, C, W
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19. **Puzzle of the heavy man and woman.**
A man and his wife, each the weight of a loaded cart, had two children each the weight of half a cart. They needed to cross a river but the boat they had could only carry the weight of one cart. Find a way of crossing so that the boat should not sink.

Solution.
Denote the man, woman and two children by M, W, $C1$, $C2$ respectively. Begin with all on one bank and each successive row indicates the position after a crossing has been made:

```
M, W, C1, C2 =-----
 M, W          -----= C1, C2
 M, W, C1      =----- C2
 M, C1         -----= W, C2
 M, C1, C2     =----- W
 M             -----= C1, C2, W
 M, C1         =----- C2, W
 C1            -----= M, C2, W
 C1, C2        =----- M, W
               -----= C1, C2, M, W
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20. **Puzzle of the river crossing.**
A man and woman who wished to cross a river. They see two children with a boat on the bank but the boat can only carry one adult or two children. How do the adults cross so that the children end up on the original bank with their boat.

Solution.
This is essentially the same problem as the preceding one. Denote the man, woman and two children by M, W, $C1$, $C2$ respectively. Begin with all on one bank and each successive row indicates the position after a crossing has been made:

```
M, W, C1, C2 =-----
 M, W          -----= C1, C2
 M, W, C1      =----- C2
 M, C1         -----= W, C2
 M, C1, C2     =----- W
 M             -----= C1, C2, W
 M, C1         =----- C2, W
 C1            -----= M, C2, W
 C1, C2        =----- M, W
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21. **Puzzle of the sheep in the field.**

There is a field which is 200 *feet long,* 100 *feet wide. I want to put sheep in it as follows: Each sheep should have an area five feet long and four feet wide. How many sheep can be put in such a place?*

Solution.

This is a rather easy puzzle. We can put a grid across the field by marking 5 feet between the lines on the long side and 4 feet between the lines on the shorter side. This divides the field into $40 \times 25 = 1000$ rectangles of the required size. We can therefore put 1000 sheep into the field.

22. **Puzzle of the irregular field.**

There is an irregular field which is 100 *yards on each side,* 50 *metres on one front,* 60 *yards in the middle, and* 50 *yards on the other front. How many square yards does this field enclose?*

Solution.

It is not entirely clear what shape Alcuin's field is meant to be. Let us assume that the top half of the field is a trapezium with the parallel sides of length 50 and 60 yards and the other two sides of length 50 yards. This gives 100 yards on each side but, of course, these sides are not straight. We can use Pythagoras to work out the distance between the parallel lines in the trapezium as 49.75 metres (approximately). Now add up the areas, a rectangle and 4 right-angled triangles, to get the total area as 5472.5 square metres.

Alcuin uses an approximate method, assuming that the area is the same as that of a rectangle $^1/_3$(50+50+60) yards by 100 yards. This gives 5533 square yards.

23. **Puzzle of the four-sided field.**

There is a field which is 30 *yards on one side,* 32 *yards on another,* 34 *metres in the front, and* 32 *yards on the remaining side. How many square yards are contained in such a field?*

Solution.

This question can't be solved since giving the lengths of the sides of the field does not determine it. In fact what one can determine is the maximum area the field can have. This occurs when the shape is a cyclic quadrilateral and the area is then given by Heron's formula $A = \sqrt{(s - a)(s - b)(s - c)(s - d)}$ where $s = (a + b + c + d)/2$ and a, b, c, d are the lengths of the sides. This gives (approximately) 1022 square metres.

In fact Alcuin uses a formula for the area which is incorrect, but it still gives a good approximation to the correct answer obtaining 1023 square yards.

24. **Puzzle of the triangular field.**
There is a field which is 30 *yards on one side,* 30 *yards on another, and* 18 *metres in the front. How many square yards is the area of such a field?*

Solution.
The field is an isosceles triangle so we can compute the height using Pythagoras to get 28.6 yards. The area is then 9 times 28.6 which is 257.4 square yards. Alcuin computes 9 times 30 and gets 270 square yards. Alcuin's solution appears simply wrong unless the statement of the puzzle has been corrupted..

25. **Puzzle of the round field.**
There is a round field which contains 400 *yards in its circumference. How many square yards will its area be?*

Solution.
The radius is ${}^{400}/_{2\pi} = {}^{200}/_{\pi}$. Then the area is □$\pi r^2$ = 40000/□π = 12732 square yards (approximately). Alcuin obtains 10000 square yards. How does he get that, you might ask? Well he uses the approximation $\pi = 4$. This does seem peculiar. If he had used □$\pi = 3$ one might have forgiven him!

26. **Puzzle of the dog chasing a hare.**
There is a field which is 150 *feet long. At one end stood a dog, at the other, a hare. The dog chased the hare advancing nine feet per leap, while in the same time the hare made a seven foot leap. How many feet and how many leaps did the dog take in pursuing the fleeing hare until it was caught?*

Solution.
After x leaps the dog has gone $9x$ feet and the hare $7x$ feet. So the distance between them is reduced by $2x$ feet. For the dog to catch the hare we require $2x = 150$, so $x = 75$. The dog thus travels 675 feet and the hare 525 feet in 75 leaps before the hare is caught. This type of "chasing" problem is found in China 2000 years before Alcuin.

27. **Puzzle of the four-sided city.**
There is a four-sided city which has one side of 1100 *feet, another side of* 1000 *feet, a front of* 600 *feet, and a final side of* 600 *feet. I*

want to put some houses there so that each house is 40 *feet long and* 30 *feet wide. How many houses ought the city to contain?*

Solution.
This is a strange problem. We will compute the area of the city assuming it to be an isosceles trapezium. This gives (approximately) an area of 627808.7 square feet. The area of a house is 1200 square feet so there are 523 house areas in the field. Alcuin uses an incorrect formula for the area of the field, obtaining 63000 square feet. He find this will contain 525 house areas. Of course, as well as the incorrect formula for the area of the field, this solution has another problem, namely that you can't fit that many houses into the field. Try and see how many you can fit in.

28. **Puzzle of the triangular city.**
There is a triangular city which has one side of 100 *feet, another side of* 100 *feet, and a third of* 90 *feet. Inside of this city, I want to build houses each of which is* 20 *feet in length and* 10 *feet in width. How many houses can I build in the city?*

Solution.
This problem is similar to the previous one and Alcuin's solution has all the same errors in it. It we use Pythagoras to compute the height of the triangle, it is 89.3 feet and the area of the city comes out to be 4018.63 square feet. Alcuin, with his incorrect formula, gets 4500 square feet. Now rather than divide 4500 by the area of a house (200 square feet) which is what he did in the previous puzzle, he divides 4000 by 200 and gets 20 houses. It is unclear if he has made an error or if he is somehow compensating for the fact that the houses won't fit. I can't see how to get more than 15 houses into the city.

29. **Puzzle of the round city.**
There is a city which is 8000 *feet in circumference. How many houses could the city contain if each house is* 30 *feet long and* 20 *feet wide?*

Solution.
This time Alcuin does try to take into account the fact that the houses have to fit. However, his argument seems simply incorrect. He gets 6400 houses which, unlike the earlier puzzles of this type, is now giving a too low estimate. It is easy to fit at least 8000 houses into the city but we leave it to the reader to do better.

30. **Puzzle of the basilica.**
A basilica is 240 *feet long and* 120 *feet wide. The basilica paved with tiles* 23 *inches long and* 12 *inches wide. How many tiles are needed*

to cover the basilica?
[There are 12 inches in a foot.]

Solution.
Since the tiles are 1 foot wide we need 120 tiles in each row. The basilica is 2880 inches long and 2880/23 = 125.2. We therefore need 126 rows of tiles, each row containing 120. Therefore 15120 tiles are required in total. Alcuin gets this answer correct.

31. **Puzzle of the wine cellar.**
A wine cellar is 100 *feet long and* 64 *feet wide. How many casks can it hold, given that each cask is seven feet long and four feet wide, and given that there is an aisle four feet wide down the middle of the cellar?*

Solution.
Since we have to leave a 4 foot aisle, we will be able to get 15 casks into each row. The length of the cellar is 100 feet, and so we can fit 100/7 = 14 rows in casks into the cellar. This gives $14 \times 15 = 210$ casks.

Alcuin gets this answer correct.

32. **Puzzle of the head of the family distributing grain.**
The head of a household had 20 *servants. He ordered them to be given* 20 *measures of corn as follows. The men must receive three measures, the women must receive two measures, and the children half a measure each. How many men, women and children servants are there in the household?*

Solution.
Suppose there are x men, y women, and z children. Then

$$x + y + z = 20 \text{ and } 6x + 4y + z = 40.$$

Subtract the first equation from the second to obtain $5x + 3y = 20$. Hence y is divisible by 5, and since it must be less than 7 (since 3 times 7 is greater than 20) we must have $y = 5$. Then $x = 1$ and $z = 14$ so there is 1 man, 5 women and 14 children who are servants in the household.

As usual with this type of problem, Alcuin simply gives the answer and checks that it satisfies all the conditions.

33. **Puzzle of another head of a family distributing grain.**
A head of household had 30 *servants whom he ordered to be given* 30 *measures of corn as follows. The men should each receive three measures, the women should each receive two measures, and the children should receive a half measure each. How many men, women and children servants are there in the household?*

Solution.
Suppose there are x men, y women, and z children. Then

$$x + y + z = 30 \text{ and } 6x + 4y + z = 60.$$

Subtract the first equation from the second to obtain $5x + 3y = 30$. Hence y is divisible by 5, and since it must be less than 10 (since 3 times 10 is 30 which doesn't allow for a positive x) we must have y = 5. Then $x = 3$ and $z = 22$ so there are 3 men, 5 women and 22 children who are servants in the household.

Again with this type of problem, Alcuin simply gives the answer and checks that it satisfies all the conditions.

(a) **Another similar distributing grain puzzle**.
A gentleman has a household of 90 *persons and ordered that they be given* 90 *measures of grain. He directed that each man should receive three measures, each woman two measures, and each child half a measure. How many men, women, and children were there?*

Solution.
Suppose there are x men, y women, and z children. Then

$$x + y + z = 90 \text{ and } 6x + 4y + z = 180.$$

Subtract the first equation from the second to obtain $5x + 3y = 90$. Hence y is divisible by 5 but we now have a number of possibilities. If $y = 5$ then $x = 15$ and $z = 70$. If $y = 10$ then $x = 12$ and $z = 68$. If y = 15 then $x = 9$ and $z = 66$. If $y = 20$ then $x = 6$ and $z = 64$. If $y = 25$ then $x = 3$ and $z= 62$. These are the only 5 possible solutions with x, y, z all positive. Hence the number of servants in the household is:

3 men, 25 women and 62 children, or
6 men, 20 women and 64 children, or
9 men, 15 women and 66 children, or
12 men, 10 women and 68 children, or
15 men, 5 women and 70 children.

Alcuin only gives one of these 5 answers, namely 9 men, 15 women and 66 children. Since he doesn't give any method one cannot see how he arrived at this particular answer, but the fact that it is just 3 times the answer to the previous puzzle almost certainly indicates how he arrived at it.

34. **Another puzzle about distributing grain.**
A head of a household had 100 *servants. He ordered that they be given* 100 *measures of corn as follows. The men should receive three measures, the women should receive two measures, and the children should receive half a measure each. How many men, women, and children servants are there in the household?*

Solution.
This is getting a little repetitive! Suppose there are x men, y women, and z children. Then

$$x + y + z = 100 \text{ and } 6x + 4y + z = 200.$$

Subtract the first equation from the second to obtain $5x + 3y = 100$. Hence y is divisible by 5 but we now have a number of possibilities. If $y = 5$ then $x = 17$ and $z = 78$. If $y = 10$ then $x = 14$ and $z = 76$. If $y = 15$ then $x = 11$ and $z = 74$. If $y = 20$ then $x = 8$ and $z = 72$. If $y = 25$ then $x = 5$ and$z = 70$. If $y = 30$ then $x = 2$ and $z = 68$. These are the only 6 possible solutions with x, y, z all positive. Hence the number of servants in the household is:

2 men, 30 women and 68 children, or
5 men, 25 women and 70 children, or
8 men, 20 women and 72 children, or
11 men, 15 women and 74 children, or
14 men, 10 women and 76 children, or
17 men, 5 women and 78 children.

Alcuin only gives one of these 6 answers, namely 11 men, 15 women and 74 children.

35. **Puzzle of the dying man's will.**

A certain father died and left behind children, a pregnant wife, and 960 *pounds from his estate. However, in his will, he stipulated that if a son should be born to his wife, then the son should receive three quarters of the inheritance. In this case, the mother should get a quarter of his estate. However, if a daughter were born, she should receive seven twelfths of his estate, and in this case the mother would receive five twelfths. But his wife gave birth to twins, one boy and one girl. How much did the mother, son and daughter each receive?*

Solution.

This problem is impossible without further information. The original problem that Alcuin is basing this puzzle on might be based on Islamic law or Alcuin may be assuming the reader will base the solution on Roman law. However, his solution assumes that the each will receive the average of the two possibilities, namely:

> the mother receives $^1/_2(^1/_4 + ^5/_{12}) = ^1/_3$ of 960 pounds, which is 320 pounds,
> the son receives $^1/_2(^3/_4 + 0) = ^3/_8$ of 960 pounds, which is 360 pounds,
> the daughter receives $^1/_2(^7/_{12} + 0) = ^7/_{24}$ of 960 pounds, which is 280 pounds.

Most historians suggest that Alcuin didn't understand the Roman law of inheritance!

36. **Puzzle of an old man greeting a boy.**

An old man greeted a boy as follows "May live for a long time - as long as you have already lived, and then an amount equal to your age at that time, and then three times as much. And if God will grant you one more year than that, and you shall live to be 100." *How old was the boy at the time the old man greeted him?*

Solution.

Suppose the boy were x years old when the old man greeted him. Then $2(x + x)$ multiplied by 3 gives 99. Hence $4x = 33$, and $x = 8$ years 3 months. Alcuin gives the answer and checks that it satisfies the conditions of the puzzle.

37. **Puzzle of the man building a house.**

A man wanted to build a house. He employed six workmen, of whom five were master builders and one was an apprentice. It was agreed *between the man who wanted to build the house and the workmen he employed, that a total of* 25 *pounds should be given to them per day as pay, and that the apprentice should receive half what the master builders receive. How much did each of them receive per day?*

Solution.
Suppose each master builder receives x pounds. then

$$5x + {}^{x}/_{2} = 25.$$

This gives $11x = 50$, so $x = 4$ pounds and ${}^{6}/_{11}$ of a pound. This is what each master builder receives, while the apprentice receives 2 pounds and${}^{3}/_{11}$ of a pound. Alcuin first divides 22 pounds of which each master builder receives 4 pounds, while the apprentice receives 2 pounds. Then he divides the remaining 3 pounds, giving the correct fractions to each man.

38. **Puzzle of the man buying 100 animals.**
A certain man bought 100 *various animals for* 100 *pounds. The cost was three pounds per horse, one pound per cow, and one pound per* 24*sheep. How many horses, cows and sheep were there?*

Solution.
Suppose there are x horses, y cows, and z sheep. Then

$$x + y + z = 100 \text{ and } 3x + y + {}^{z}/_{24} = 100.$$

Subtract the first from the second to get $2x = ({}^{23}/_{24})z$ or $48x = 23z$. Thus x is divisible by 23 and, by the second equation, cannot be as large as 46. Hence $x = 23$, $z = 48$ and $y = 29$. Therefore the man buys 23 horses, 29 cows, and 48 sheep. As usual, Alcuin gives the answer and checks it works.

39. **Puzzle of the Oriental merchant.**
A certain merchant bought 100 *assorted animals for* 100 *pounds on a trip to the Orient. He paid five pounds for each camel, one pound for each ass, and one pound for* 20 *sheep. How many camels, asses and sheep did the merchant buy?*

Solution.
Suppose there are x camels, y asses, and z sheep. Then

$$x + y + z = 100 \text{ and } 5x + y + {}^{z}/_{20} = 100.$$

Subtract the first from the second to get $4x = (^{19}/_{20})z$ or $80x = 19z$. Thus x is divisible by 19 and, by the second equation, cannot be larger than 19. Hence $x = 19$, $z = 80$, $y = 1$. Therefore the man buys 19 camels, 1 ass, and 80 sheep. As usual, Alcuin gives the answer and checks it works.

40. **Puzzle of the man and the grazing sheep.**

A certain man saw from sheep grazing on the mountainside and said, "I wish I had that number of sheep, and then just as many more, plus a half of half of this number, and then another half of the last amount added. Then if I took these sheep back to my home together with me there would be, counting myself, 100." *How many sheep did the man see grazing?*

Solution.

Suppose he saw x sheep grazing on the mountainside. Then we get

$$(x + x) + {}^{x}/_{2} + {}^{x}/_{4} + 1 = 100.$$

This gives $(^{11}/_{4})x = 99$, so $x = 36$. Hence the man saw 36 sheep grazing on the mountainside.

Alcuin gives the answer 36 and checks it works.

41. **Puzzle of the sow and the pigsty.**

A certain farmer built a large square enclosure in which he placed a sow. The sow gave birth to seven piglets in the centre of the sty. The offspring, along with the mother, the eighth pig, each gave birth to another seven piglets in the first of the four corners of the sty. Next the sow and all the offspring each give birth to seven more piglets in the second corner. The same happens in the third corner, and then in the fourth corner. Finally the sow and all the offspring each give birth to seven more piglets in the centre of the sty. How many pigs, including the mother, were in the sty by this time?

Solution.

We have to ignore the biological difficulties in this puzzle. All the piglets are female and produce young without the presence of males! This puzzle is about powers of 8. After the sow has 7 piglets in the centre of the sty, there are 8 pigs in total. Each of these has 7 piglets in the first corner so, at this stage there are 64 pigs. After each has had a piglet in the second corner there will be $8^3 = 512$ pigs. Then 8^4 pigs after the third corner, and 8^5 pigs after the fourth corner.

Again each produces a litter of 7 in the centre of the sty so at this final stage there will be $8^6 = 262144$ pigs in the sty. Good job it was a large sty!

Alcuin solves this puzzle correctly but appears to make a numerical error at the 8^5 stage getting 32788 instead of 32768. This is not a simple copying error in the manuscript since 32788 is multiplied by 8 to get the final number. Several surviving manuscripts have different incorrect answers to this puzzle, all seemingly due to arithmetic errors.

42. **Puzzle of the hundred steps.**

There is a ladder which has 100 *steps. One pigeon sat on the first step, two pigeons on the second, three pigeons on the third, four on the fourth, five on the fifth, and so on up to the hundredth step. How many pigeons were there in total on the ladder?*

Solution.

The total number of pigeons is the sum of the first 100 natural numbers. The sum of the first n natural numbers is $n(n+1)/2$, so we get the answer 5050. Hence there are 5050 pigeons on the ladder.

Alcuin gives a good solution to this puzzle. He notes that the first and 99th step contain 100 pigeons in total. Similarly the 2nd and 98th step contain 100 pigeons in total. Continue until we get to the 49th step and the 51st step which total to 100 pigeons. At this stage we have counted 4900 pigeons but we still have to add the 50 pigeons on the 50th step and the 100 pigeons on the 100th step. This makes 5050 pigeons in all.

43. **Puzzle of the pigs.**

A certain man had 300 *pigs. He ordered all of them slaughtered in three days, but with an uneven number being killed each day. He wished the same thing to be done with* 30 *pigs. What odd number of pigs out of* 300 *or* 30 *were to be killed on each of the three days?*

Solution.

Of course the problem has no solution since the sum of three odd numbers can never be even.

Alcuin knows that the problem is insoluble. He suggests this is a good problem to give to children who have been naughty!

44. **Puzzle of the boy greeting his father.**

A certain boy addressed his father, saying, "Hello, father!" His father answered, "Hello, my son. May you live to twice your present age and then at that time three times the age you will then be. If I gave you one of my years to add to this, then you will live to be 100 *years old." How old was the boy at the time?*

Solution.

Suppose the boy is x years old. Then $3(2x) + 1 = 100$. Hence $x = 16.5$ years. The boy is therefore 16 years and 6 months old.

Alcuin gives the answer 16 years and 6 months and checks it works.

45. **Puzzle of the pigeons.**

A pigeon sitting in a tree saw some other pigeons flying by and said to them, "Suppose there were as many of you again and then the same number were added again. Then, along with me, you would number 100*." How many pigeons were there flying by?*

Solution.

Suppose there are x pigeons flying by. Then $x + x + x + 1 = 100$. This gives $x = 33$ so there were 33 pigeons flying by.

Alcuin gives the answer 33 pigeons and checks it works.

46. **Puzzle of the man who was robbed.**

A certain man was walking along the street when he found a small bag containing 2 *talents. A crowd of people noticed that he had found a purse and said to him: "Friend, give us a part of your discovery." But the man shook his head and said he didn't want to give them any of the money. Those in the crowd rushed at him, emptying the money out of the bag, and each grabbing* 50 *gold shillings. The man was left with* 50*gold shillings after the crowd left. How many men were there in the crowd?*

[A talent is worth 75 pounds and there are 72 gold shillings in each pound.]

Solution.

One talent is 5400 gold shillings, so there were 10800 gold shillings in the bag. If there were 50 gold shillings left after each person in the crowd took 50 gold shillings, then 10750 gold shillings were taken. There must have been 10650/50 = 215 people in the crowd.

Alcuin calculates that there are 216 portions of 50 gold shillings.

47. **Puzzle of the bishop with 12 loaves.**
A certain bishop ordered 12 *loaves of bread to be divided amongst the clergy. He stipulated that each priest should receive two loaves, each deacon should receive half a loaf and each reader should receive a quarter of a loaf. It turned out that the number of clerics and the number of loaves were the same. How many priests, deacons and readers must there have been?*

Solution.
Suppose there were x priests, y deacons and z readers. Then

$$x + y + z = 12 \text{ and } 2x + {}^{y}/_{2} + {}^{z}/_{4} = 12.$$

Multiply the second equation by 4 and subtract twice the first equation from it to obtain $6x - z = 24$. Since $6x > 24$, x must be at least 5. But the second equation shows that x can't be as big as 6 (for then y and z would have to be 0 or negative) Hence $x = 5$, $y = 1$ and $z = 6$. Thus there were 5 priests, 1 deacon and 6 readers.

48. **Puzzle of the man meeting students.**
A certain man met some students and asked them, "*How many of you are there in your school?*" *One of the students replied:* "*I do not want to tell you directly but I'll tell you how to work it out. You double the number of students, then triple that number, then divide that number into four parts. If you add me to one of the quarters, there will be* 100." *How many students are in the school?*

Solution.
Suppose there are x students in the school. Then

$$3(2x)/_{4} + 1 = 100.$$

Thus $x = 66$ and so there are 66 students in the school.

Alcuin writes, "Twice 33 is 66. This is the number." He then checks that 66 satisfies the conditions. Perhaps when he writes, "Twice 33 is 66" this is a clue to how he worked it out.

49. **Puzzle of the carpenters.**

Seven carpenters each made seven wheels. How many carts did they build?

Solution.

This puzzle assumes that a cart requires 4 wheels. Then, since the carpenters made 49 wheels, they have enough wheels for 12 carts. There will be one wheel left over. This is precisely Alcuin's solution.

50. **Puzzle of the flasks of wine.**

I ask a question to which anyone may reply. How many pints
do 100 *measures of wine contain, and how many cups*
do 100 *measures contain?*
[A measure contains 48 pints, and a pint contains 6 cups.]

Solution.

There are 4800 pints and 28800 cups in 100 measures. There seems little to work out in this puzzle.

51. **Puzzle of the man dividing flasks of wine.**

A certain dying man had four flasks of wine which he wanted to divide between his four sons. In the first flask there were 40 *measures of wine; in the second there were* 30 *measures of wine, in the third there were* 20 *measures of wine, and in the fourth there were* 10 *measures of wine. Calling his servant, he said to him, "Divide these four flasks containing wine amongst my four sons in such a way that each son receives an equal portion of wine and flasks." The servant had no means of measuring wine and no container other than the four flasks. How did he carry out the dying man's wishes?*

Solution.

The total amount of wine is 40 + 30 + 20 + 10 = 100 measures. Hence each son must receive 25 measures of wine. The servant has, therefore, to find a way of getting 25 measures into each flask without being able to measure it. He takes the 2 flasks, one containing 40 measures and the other containing 10 measures. He tips wine from one to the other until each has an equal amount of wine. These now contain 25 measures each. Similarly he take the 2 flasks containing 30 measures and 20 measures, and tips wine from one to the other until each has an equal amount of wine. These now contain 25 measures each and the problem is solved.

52. **Puzzle of the head of the house.**

A certain head of household ordered that 90 *measures of grain be* *taken from one of his houses to another* 30 *leagues away. This load of grain can be carried by a camel in three trips. Given that the camel eats one measure of grain for each league it goes* (*the camel only eats when carrying a load*)*, how many measures were left after the grain was transported to the second house?*

Solution.
This is a nice puzzle. It appears at first that nothing will be left. The camel can carry 30 measures of grain per trip and will eat all 30 measures by the time it reaches the second house. But this is not the way the transporting is done. Alcuin gives a solution in which the camel makes three trips to a point 20 leagues from the start and moves the grain to here at a cost of 60 measures. Then one final camel trip of 10 leagues is used to move the remaining 30 measures at a cost of 10 measures. Thus there are 20 measures left at the end.

In fact one can do better. Move all the grain to a spot 10 leagues from the start (three camel trips at a cost of 30 measures) then move the remaining grain to a spot 25 leagues from the start (two camel trips at a cost of 30 measures) and then move the remaining 30 measures to the end (one camel trip at a cost of 5 measures). Then there are 25 measures left at the end.

53. **Puzzle of the abbot and twelve monks.**
A certain abbot of a monastery was in charge of 12 *monks. He asked his treasurer to give an equal share of* 204 *eggs to each of the monks. The monks consisted of* 5 *priests,* 4 *deacons and* 3 *readers? How many eggs were given to priests? How may to deacons? How many to readers?*

Solution.
There seems to be something wrong with this problem since there is little to do given the way it is posed. Each monk will receive $^{204}/_{12}$ = 17 eggs. Therefore 85 eggs go to priests, 68 to deacons and 51 to readers. Perhaps the best puzzle we can set the reader is to ask for a more interesting puzzle which comes up with the given answer.

References (10 books/articles)

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